

EE4036

Chapter 1

Radio-wave Propagation

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Propagation in Free Space – Mode of Propagation

Consider first an average power P_T , assumed to be radiated equally in all directions (isotropically). This will spread out spherically as it travels away from the source, so that at distance d , the power density in the wave, which is the power per unit area of wavefront, will be

$$P_{Di} = \frac{P_T}{4\pi d^2} \quad \text{W/m}^2 \quad (1.2.1)$$

This is so because $4\pi d^2$ is the surface area of the sphere of radius d , centered on the source. P_{Di} stands for isotropic power density.

Propagation in Free Space – Mode of Propagation

- Let G_T be the maximum directivity gain of the transmitting antenna; then the power density along the direction of maximum radiation will be

$$\begin{aligned} P_D &= P_{Di} G_T \\ &= \frac{P_T G_T}{4\pi d^2} \end{aligned} \quad (1.2.2)$$

Propagation in Free Space – Mode of Propagation

A receiving antenna can be positioned so that it collects maximum power from the wave. When so positioned, let P_R be the power delivered by the antenna to the load (receiver) under matched conditions; then the antenna can be considered as having an effective area (or aperture) A_{eff} , where

$$\begin{aligned} P_R &= P_D A_{\text{eff}} \\ &= \frac{P_T G_T}{4\pi d^2} A_{\text{eff}} \end{aligned} \quad (1.2.3)$$

It can be shown that for any antenna, the ratio of maximum directivity gain to effective area is

$$\frac{A_{\text{eff}}}{G} = \frac{\lambda^2}{4\pi} \quad (1.2.4)$$

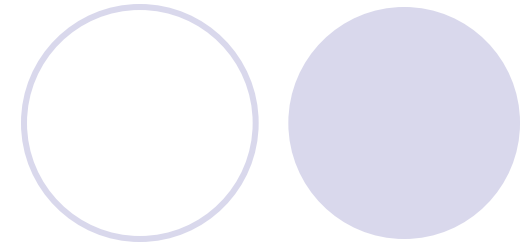
Propagation in Free Space – Mode of Propagation

Here, λ is the wavelength of the wave being radiated. Letting G_R be the maximum directivity gain of the receiving antenna, we have

$$\frac{P_R}{P_T} = G_T G_R \left(\frac{\lambda}{4\pi d} \right)^2 \quad (1.2.5)$$

This is the fundamental equation for free-space transmission. Usually it is expressed in terms of frequency f , in megahertz, and distance d , in kilometers.

Propagation in Free Space – Mode of Propagation



As shown $\lambda f = c$, and on substituting this in Eq. (1.2.5) and doing the arithmetic, which is left as an exercise for the reader, the result obtained is

$$\frac{P_R}{P_T} = G_T G_R \frac{0.57 \times 10^{-3}}{(df)^2} \quad (1.2.6)$$

By expressing the power ratios in decibels, Eq. (1.2.6) can be written as

$$\left(\frac{P_R}{P_T} \right)_{\text{dB}} = (G_T)_{\text{dB}} + (G_R)_{\text{dB}} - (32.5 + 20 \log_{10} d + 20 \log_{10} f) \quad (1.2.7)$$

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The third term in parentheses on the right-hand side of Eq. (1.2.7) is the loss, in decibels, resulting from the spreading of the wave as it propagates outward from the source. It is known as the transmission path loss, L . Thus

$$L = (32.5 + 20 \log_{10} d + 20 \log_{10} f)_{\text{dB}} \quad (1.2.8)$$

where d is in kilometers and f in megahertz.

Equation (1.2.7) then becomes

$$\left(\frac{P_R}{P_T} \right)_{\text{dB}} = (G_T)_{\text{dB}} + (G_R)_{\text{dB}} - (L)_{\text{dB}} \quad (1.2.9)$$

Propagation in Free Space – Mode of Propagation

EXAMPLE 1.2.1

In a satellite communications system, free-space conditions may be assumed. The satellite is at a height of 36,000 km above earth, the frequency used is 4000 MHz, the transmitting antenna gain is 15 dB, and the receiving antenna gain is 45 dB. Calculate (a) the free-space transmission loss and (b) the received power when the transmitted power is 200 W.

SOLUTION (a)
$$L = 32.5 + 20 \log_{10} 36,000 + 20 \log_{10} 4000$$
$$= \mathbf{196 \text{ dB}}$$

(b)
$$\left(\frac{P_R}{P_T} \right)_{\text{dB}} = 15 + 45 - 196$$
$$= -136 \text{ dB}$$

This is a power ratio of 0.25×10^{-13} , and since $P_T = 200 \text{ W}$,

$$P_R = 200 \times 0.25 \times 10^{-13}$$
$$= 5 \times 10^{-12} \text{ W} = \mathbf{5 \text{ pW}}$$

Propagation in Free Space – Mode of Propagation

Frequently, it is required to know the electric field strength of the wave at the receiving antenna. E is given in terms of power density P_D and wave impedance Z_0 as

$$E = \sqrt{Z_0 P_D} \quad (1.2.10)$$

Also, Z_0 is given as

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}} \quad (1.2.11)$$

Propagation in Free Space – Mode of Propagation

The free-space values are as follows: $\mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$, and $\epsilon = \epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$. Substituting these in Eq. (1.2.11) gives

$$Z_0 = 120\pi \Omega \quad (1.2.12)$$

The field strength may now be found by substituting Eqs. (1.2.2) and (1.2.12) in Eq. (1.2.10)

$$E = \frac{\sqrt{30P_T G_T}}{d} \text{ V/m} \quad (1.2.13)$$

Propagation in Free Space – Mode of Propagation

This is the fundamental equation that gives the field strength at the receiving antenna, for free-space propagation conditions. A receiving antenna has an effective length ℓ_{eff} (analogous to effective area) such that the open circuit EMF of the antenna E_s is given by

$$V_s = E\ell_{\text{eff}} \quad (1.2.14)$$

Propagation in Free Space – Mode of Propagation

EXAMPLE 1 .2.2

Calculate the open-circuit voltage induced in a $\frac{1}{2}\lambda$ dipole when 10 W at 150 MHz is radiated from another $\frac{1}{2}\lambda$ dipole 50 km distant. The antennas are positioned for optimum transmission and reception.

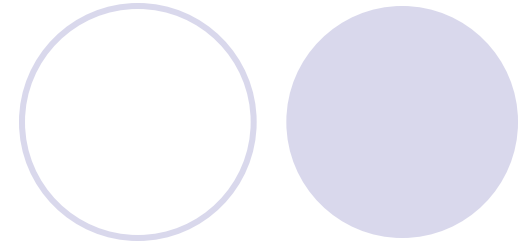
SOLUTION

$$\begin{aligned}\lambda &= \frac{3 \times 10^8}{150 \times 10^6} \\ &= 2 \text{ m}\end{aligned}$$

For a $\frac{1}{2}\lambda$ dipole, the maximum gain is 1.64 : 1, and the effective length is λ/π . Therefore,

$$\begin{aligned}V_s &= \frac{\sqrt{30 \times 10 \times 1.64}}{50 \times 10^3} \cdot \frac{2}{\pi} \\ &= 282 \mu\text{V}\end{aligned}$$

Propagation in Free Space – Mode of Propagation



Equation (1.2.13) is sometimes expressed in terms of the field strength at unit distance, E_0 . Thus, at $d = 1$ m,

$$E = E_0 = \frac{\sqrt{30P_T G_T}}{1} \text{ V/m} \quad (1.2.15)$$

and therefore Eq. (1.2.13) can be written as

$$E = \frac{E_0}{d} \text{ V/m} \quad (1.2.16)$$

Propagation in Free Space – Mode of Propagation

Note that both E and E_0 are in units of V/m, although Eq. (1.2.16) may tend to suggest that E was in units of (V/m)/m, or V/m². Equation (1.2.16) really expresses a proportionality, and written in full it would be

$$E = E_0 \times \frac{1 \text{ (m)}}{d \text{ (m)}} \quad \text{V/m} \quad (1.2.17)$$

Propagation in Free Space – Microwave System

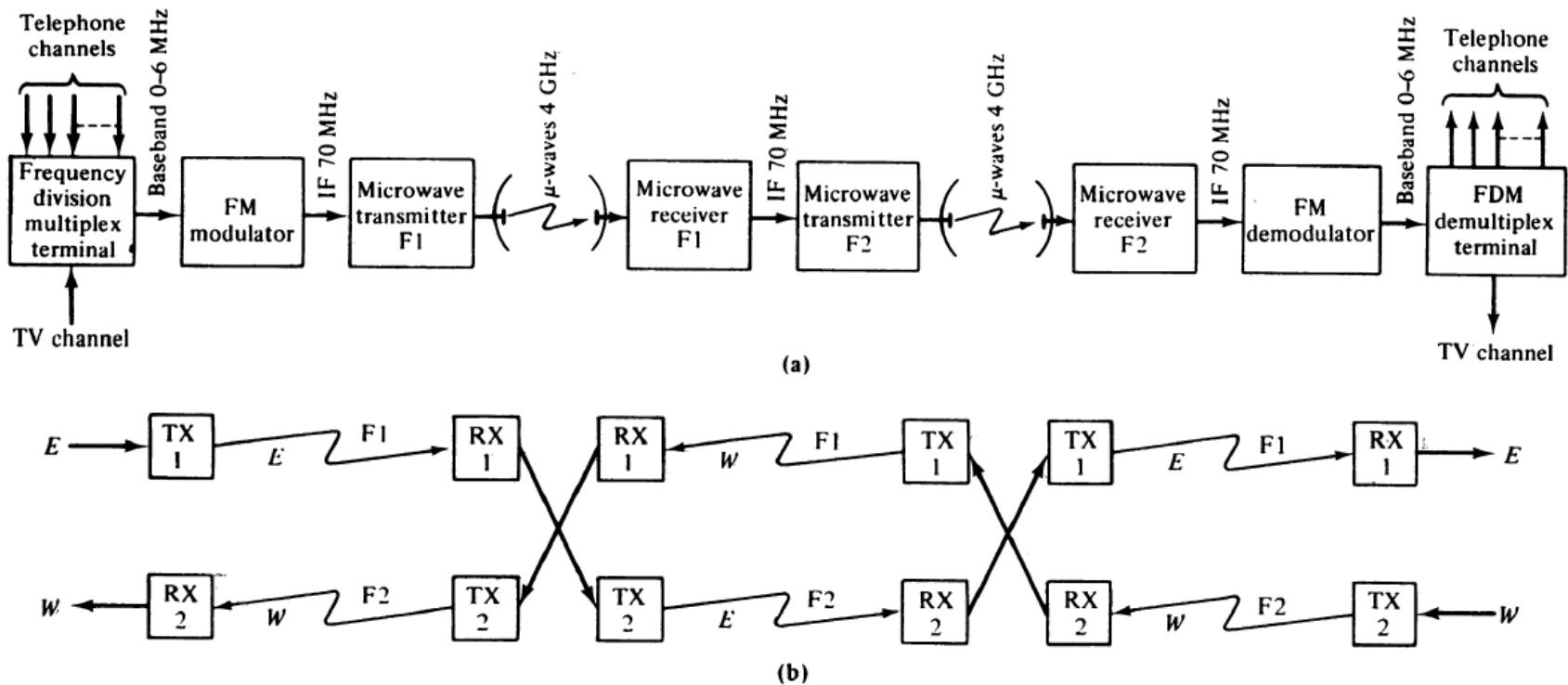


Figure 1.2.1 Microwave relay system. (a) One-way channel showing the equipment used on the route. (b) Two-way channel pair showing frequency interchanges at intermediate repeater stations.

Tropospheric Propagation – Mode of Propagation

Considering first the effect of the earth's surface, a much-simplified model has been developed which successfully describes electromagnetic propagation over a wide range of practical circumstances and is illustrated in Fig. 1.3.1(a). In this simplified picture the earth is assumed to be flat, and the space wave reaching the receiver has two components: the direct wave, which follows a ray path s_d , and a ground-reflected wave, which follows a ray path s_i . The reflected wave travels a greater distance than the direct wave, and although this has negligible effect on the amplitude, it does introduce a phase difference which is highly significant. Let Δs be the path difference; then, since a phase angle of 2π radians corresponds to a path length of one wavelength (λ), the phase angle corresponding to Δs is

$$\phi_s = \frac{2\pi}{\lambda} \Delta s \quad (1.3.1)$$

Tropospheric Propagation – Mode of Propagation

The problem now is to find Δs , and this can be solved from the geometry of Fig. 1.3.1(b). h_T is the height of the transmitting antenna above ground, h_R is the height of the receiving antenna, and d is the distance along the flat earth between the two. From triangle FBD [Fig.1.3.1(b)],

$$s_i^2 = (h_T + h_R)^2 + d^2$$

From triangle ABC ,

$$s_d^2 = (h_T - h_R)^2 + d^2$$

Tropospheric Propagation – Mode of Propagation

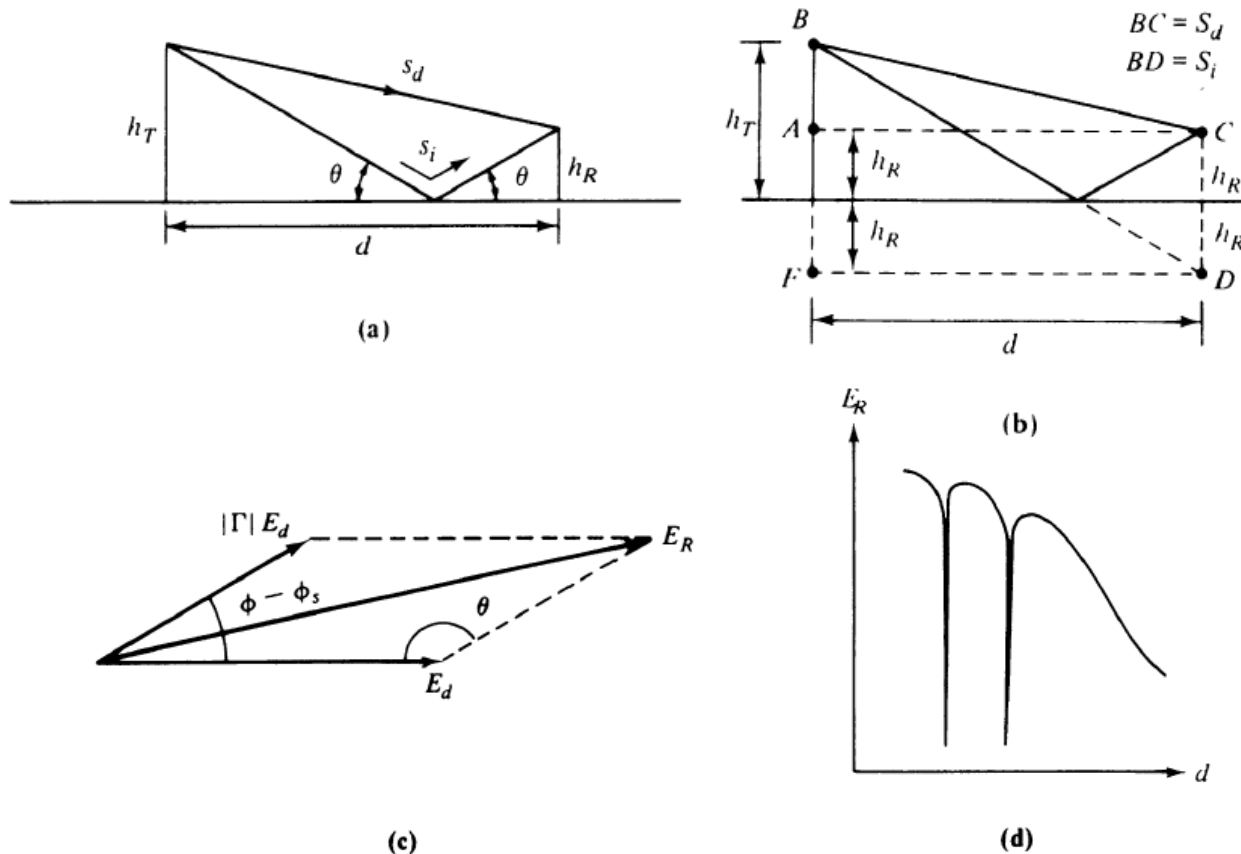
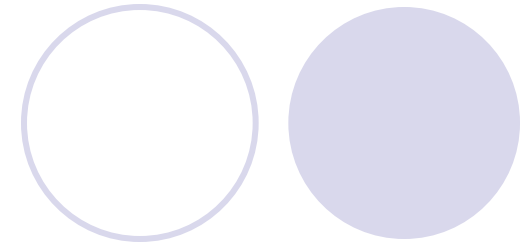


Figure 1.3.1 (a) Simplified model for tropospheric propagation ray paths. (b) Geometry used to determine phase difference. (c) Field strength phasor diagram at the receiver. (d) Sketch of field strength modulus versus distance.

Tropospheric Propagation – Mode of Propagation



Therefore,

$$\begin{aligned} s_i^2 - s_d^2 &= (h_T + h_R)^2 - (h_T - h_R)^2 \\ &= 4h_T h_R \end{aligned} \quad (1.3.2)$$

Also, applying the formula for the difference of two squares,

$$\begin{aligned} s_i^2 - s_d^2 &= (s_i + s_d)(s_i - s_d) \\ &= (s_i + s_d) \Delta s \end{aligned}$$

For most practical purposes, $s_i \cong s_d \cong d$. Therefore,

$$s_i^2 - s_d^2 \cong (2d) \Delta s \quad (1.3.3)$$

Tropospheric Propagation – Mode of Propagation

Equating Eqs. (1.3.2) and (1.3.3), the result is

$$2d \Delta s = 4h_T h_R$$

Therefore,

$$\Delta s = \frac{2h_T h_R}{d} \quad (1.3.4)$$

Substituting this in Eq. (1.3.1) gives

$$\phi_s = \frac{4\pi h_T h_R}{\lambda d} \quad (1.3.5)$$

Tropospheric Propagation – Mode of Propagation

Let the electric field reflection coefficient at the point of reflection be represented by $\Gamma = |\Gamma|/\phi$, analogously to the voltage reflection coefficient introduced in Section 13.8. The amplitude of the reflected wave at the receiving point will be $|\Gamma|E_d$, where E_d is the amplitude of the direct wave. The phase of the reflected wave relative to the direct wave is $(\phi - \phi_s)$, since it is known that ϕ_s is a phase lag because of the longer path length for the reflected wave, and a positive value of ϕ is assumed to be a phase lead. The phasor diagram for the field strength at the receiving point is therefore as shown in Figure 1.3.1(c). The angle θ is equal to $180^\circ - (\phi - \phi_s)$, and applying the cosine rule to the triangle to find the resultant field strength gives

$$\begin{aligned} E_R^2 &= E_d^2 + |\Gamma|^2 E_d^2 - 2|\Gamma|E_d^2 \cos \theta \\ &= E_d^2(1 + |\Gamma|^2 + 2|\Gamma| \cos(\phi - \phi_s)) \end{aligned}$$

Therefore,

$$E_R = E_d \sqrt{1 + |\Gamma|^2 + 2|\Gamma| \cos(\phi - \phi_s)} \quad (1.3.6)$$

Tropospheric Propagation – Mode of Propagation

For a wide range of conditions it is found that the reflection coefficient is equal to -1 ; that is, the reflection occurs with negligible amplitude change but 180° phase change. For this particular condition, Eq. (1.3.6) reduces to

$$\begin{aligned} E_R &= E_d \sqrt{1 + 1 + 2 \cos(180 - \phi_s)} \\ &= E_d \sqrt{2(1 - \cos \phi_s)} \end{aligned} \quad (1.3.7)$$

Using the identity $2 \sin^2 A = 1 - \cos 2A$ allows this to be written as

$$E_R = 2E_d \sin\left(\frac{\phi_s}{2}\right) \quad (1.3.8)$$

Tropospheric Propagation – Mode of Propagation

Equation (1.2.6) gives E_d as

$$E_d = \frac{E_0}{d}$$

Thus, substituting Eqs. (1.2.16) and (1.3.5) in Eq. (1.3.8) results in

$$E_R = \frac{2E_0}{d} \sin\left(\frac{2\pi h_T h_R}{\lambda d}\right) \quad (1.3.9)$$

Tropospheric Propagation – Mode of Propagation

The variation of E_R with d is sketched in Fig. 1.3.1(d). The sharp dips toward zero occur where the sine term goes to zero. When d is large, so that the angle within the brackets of Eq. (1.3.8) is small (less than about 0.5 rad or about 30°), the approximate form

$$E_R \cong \frac{2E_0}{d} \left(\frac{2\pi h_T h_R}{\lambda d} \right)$$

or

$$E_R = E_0 \frac{4\pi h_T h_R}{\lambda d^2} \quad (1.3.10)$$

may be used.

Tropospheric Propagation – Mode of Propagation

EXAMPLE 1.3.1

In a VHF mobile radio system, the base station transmits 100 W at 150 MHz, and the antenna is 20 m above ground. The transmitting antenna is a $\frac{1}{2}\lambda$ dipole for which the gain is 1.64. Calculate the field strength at a receiving antenna of height 2 m at a distance of 40 km.

SOLUTION

$$\lambda = \frac{300 \times 10^6}{150 \times 10^6}$$

$$= 2 \text{ m}$$

$$E_0 = \sqrt{30 \times 100 \times 1.64} \quad [\text{Eq. (1.2.15)}]$$

$$= 70 \text{ V/m}$$

The angle $2\pi h_T h_R / \lambda d$ is very much less than 0.5 rad, as an approximate calculation will show. Therefore, Eq. (1.3.10) can be used:

$$E_R = \frac{70 \times 4 \times \pi \times 20 \times 2}{2 \times (40 \times 10^3)^2}$$

$$= 11 \text{ } \mu\text{V/m}$$

Tropospheric Propagation – Radio Horizon

The curvature of the earth has a more important effect in that it presents a horizon that limits the range of transmission. This range is greater than the optical range because the effect of the earth's atmosphere is to cause a bending of the radio wave, which carries it beyond the optical horizon. Figure 1.3.2(a) shows a typical radio-wave ray path, and Fig. 1.3.2(b) shows how the path can be considered straight by assigning a greater radius to the earth than it actually has. For standard atmospheric conditions the increase in radius has been worked out at $\frac{4}{3}$, so that

$$a' = \frac{4}{3}a \quad (1.3.11)$$

where a is the earth's actual radius and a' is the fictitious radius that accounts for refraction.

Tropospheric Propagation – Radio Horizon

From Fig. 1.3.2(b),

$$(a')^2 + d_1^2 = (a' + h_T)^2$$

Therefore,

$$d_1^2 = 2a'h_T + h_T^2 \quad (1.3.12)$$

But, since $a' \gg h_T$,

$$d_1^2 \approx 2a'h_T \quad (1.3.13)$$

Tropospheric Propagation – Radio Horizon

Similarly,

$$d_2^2 \approx 2a'h_R \quad (1.3.14)$$

The maximum radio range $d_{\max}$ is

$$\begin{aligned} d_{\max} &\approx d_1 + d_2 \\ &= \sqrt{2a'h_T} + \sqrt{2a'h_R} \end{aligned} \quad (1.3.15)$$

Tropospheric Propagation – Radio Horizon

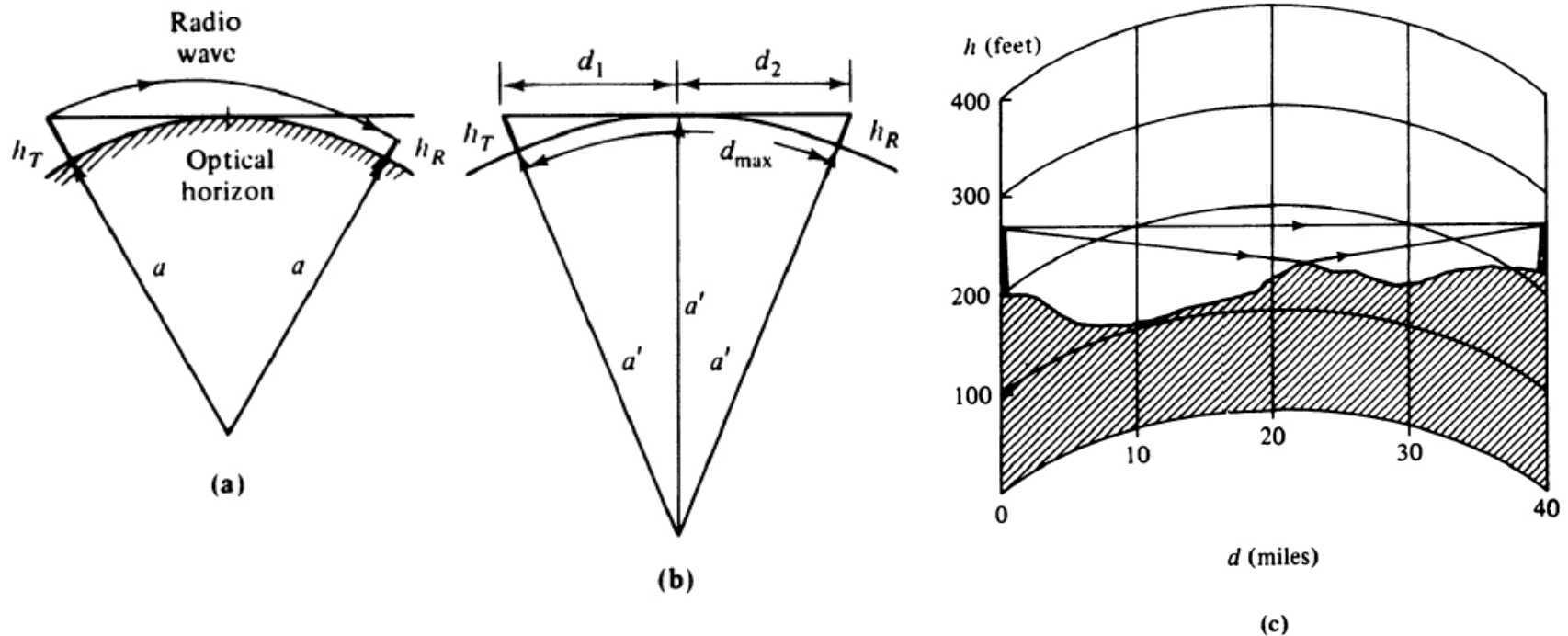


Figure 1.3.2 (a) Curvature of ray path resulting from change of refractive index of the air. (b) Equivalent straight line ray path for fictitious earth radius a' . (c) Example of a contour map for radio path planning.

Tropospheric Propagation – Radio Horizon

Substituting in the numerical values, $a' = \frac{4}{3} \times 3960$ miles, and expressing h_T and h_R in feet results in the useful expression

$$d_{\max}(\text{miles}) = \sqrt{2h_T(\text{ft})} + \sqrt{2h_R(\text{ft})} \quad (1.3.16)$$

Alternatively, in metric units,

$$d_{\max}(\text{km}) = \sqrt{17h_T(\text{m})} + \sqrt{17h_R(\text{m})} \quad (1.3.17)$$

Tropospheric Propagation – Radio Horizon

EXAMPLE 1.3.2

Calculate the maximum range for a tropospheric transmission for which the antenna heights are 100 ft and 60 ft.

SOLUTION

$$\begin{aligned}d_{\max}(\text{miles}) &= \sqrt{200} + \sqrt{120} \\ &= \mathbf{25.1 \text{ miles}}\end{aligned}$$

The phenomenon of *diffraction* will extend the range in practice somewhat beyond the radio horizon.

Tropospheric Propagation – Contour Maps and Super- and Subrefractions

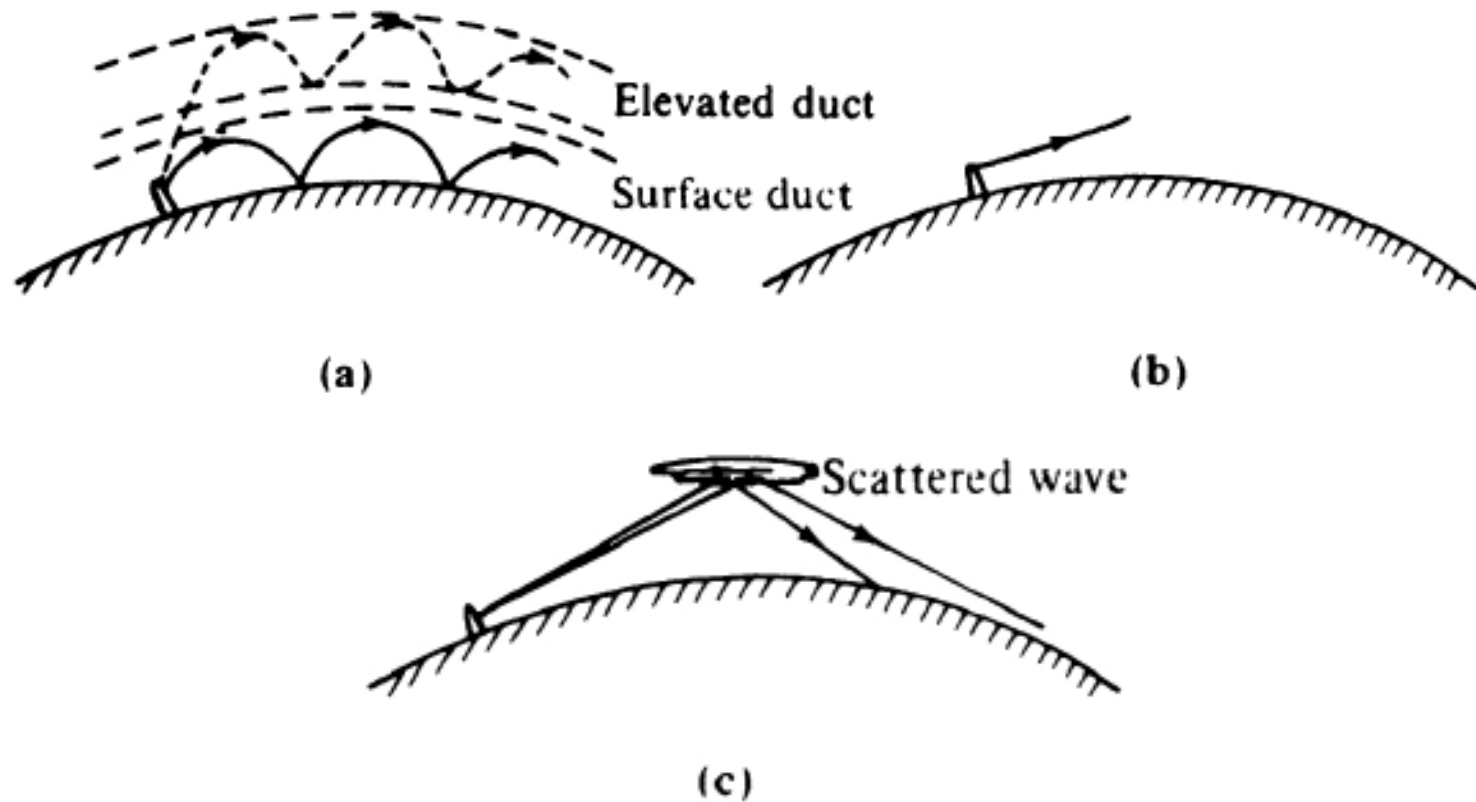


Figure 1.3.3 (a) Superrefraction. (b) Subrefraction. (c) Tropospheric scatter propagation.

Tropospheric Propagation – VHF / UHF Radio Systems

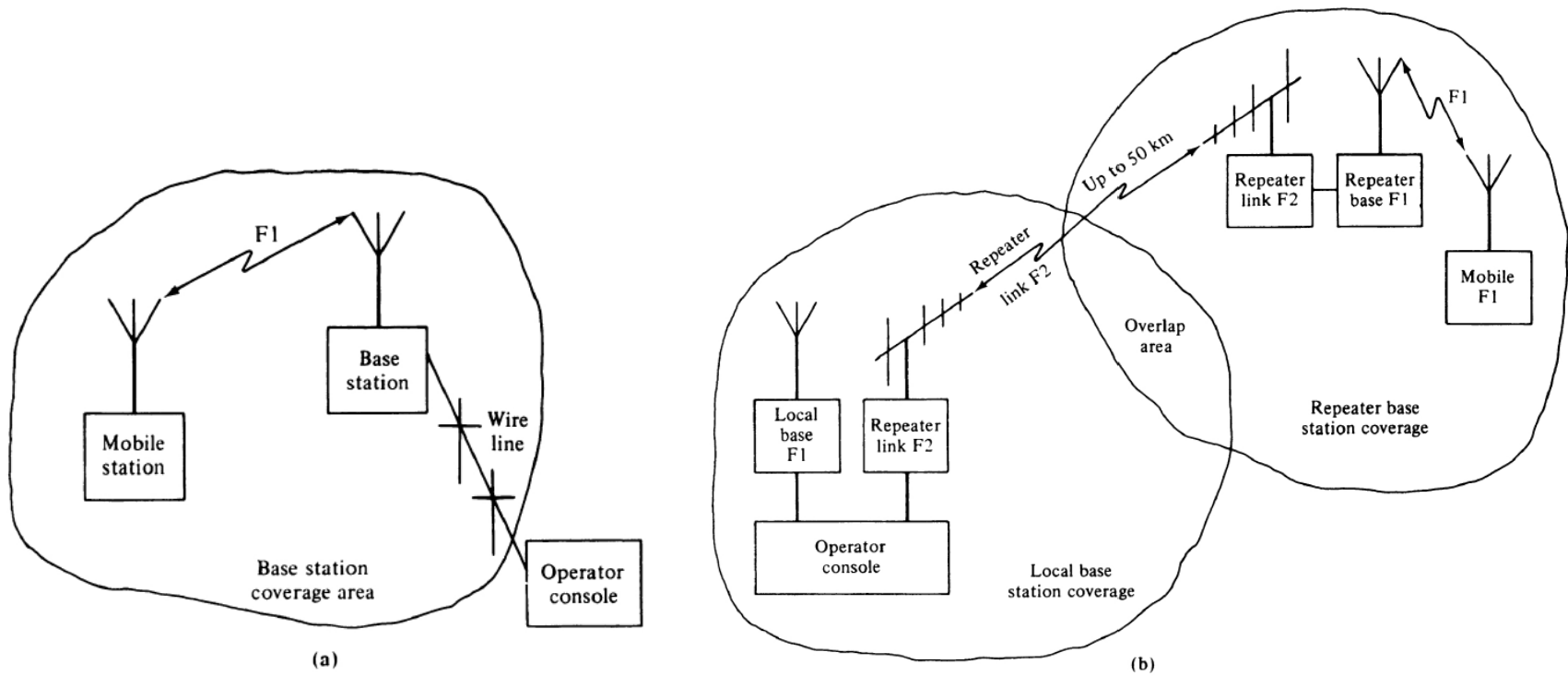


Figure 1.3.4 (a) Simple mobile dispatch system. (b) Dispatch system with a repeater station to extend coverage area.

Ionospheric Propagation – Plasma Frequency and Critical Frequency

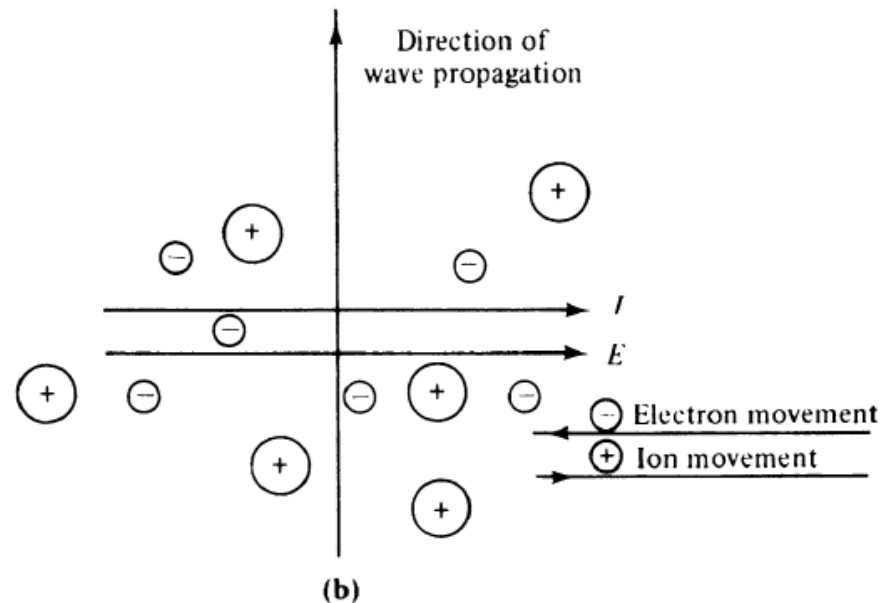
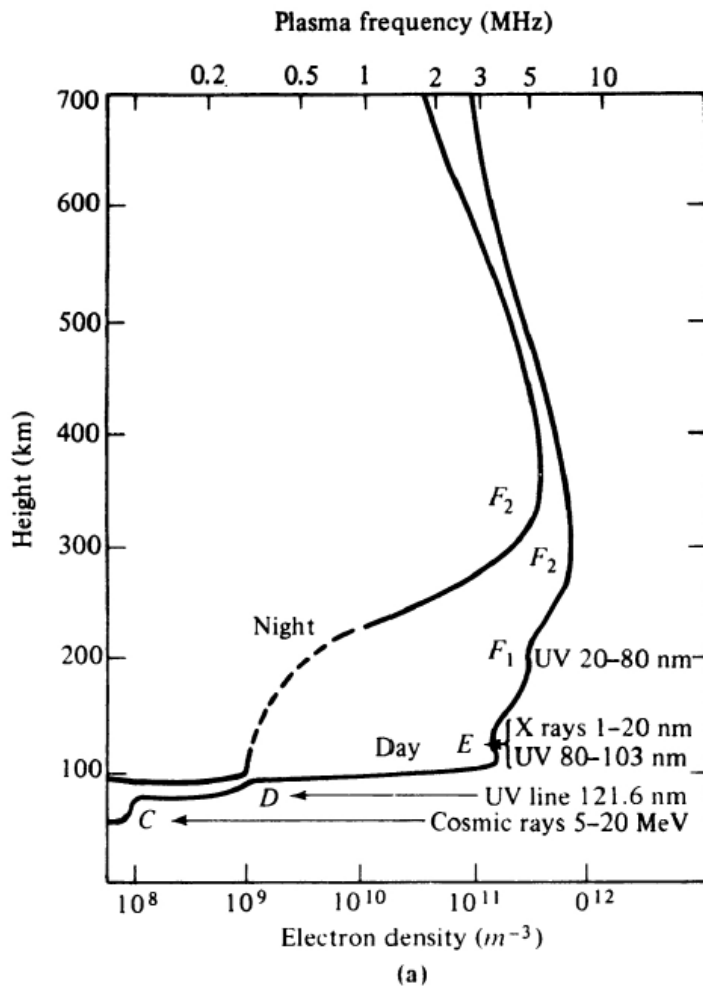


Figure 1.4.1 (a) Typical electron density distributions for summer noon and midnight conditions at mid-latitudes and the principal ionizing radiations at each level. Plasma frequency f_N is related to electron density m^{-3} by $f_N = 9\sqrt{N}$. The plasma frequency at the peak of a layer is called the critical frequency f_o . (Reprinted by permission of the IEEE.) (b) Electric field E and electric current density J vectors in an ionospheric layer.

Ionospheric Propagation – Plasma Frequency and Critical Frequency

The electron cloud will oscillate in the electric field of the wave, but with a phase retardation of 90° (for a sinusoidal wave) because of the electron mass inertia. This motion of the electron cloud produces a space current; in addition, the electric field has its own capacitive displacement current, which leads the field by 90° . The space current is therefore in phase opposition to the displacement current, and it appears to reduce the relative permittivity (dielectric constant) of the ionized medium, which is then given by

$$\epsilon_r = 1 - \frac{Nq_e^2}{\epsilon_0 m \omega^2} \quad (1.4.1)$$

where N = electron density, m^{-3}

m = electron rest mass = 9.11×10^{-31} kg

q_e = electron charge magnitude = 1.6×10^{-19} C

ω = angular frequency of wave, rad/s

ϵ_0 = permittivity of free space = 8.854×10^{-12} F/m

Ionospheric Propagation – Plasma Frequency and Critical Frequency

The angular velocity of the wave can have a value that makes ϵ_r equal to zero, and this is termed the *plasma angular velocity*, ω_N . From Eq. (1.4.1), ω_N is seen to be

$$\omega_N^2 = \frac{Nq_e^2}{m\epsilon_0} \quad (1.4.2)$$

or

$$f_N^2 = \frac{Nq_e^2}{(2\pi)^2 m\epsilon_0} \quad (1.4.3)$$

Ionospheric Propagation – Plasma Frequency and Critical Frequency

Putting in the numerical values for the constants gives

$$f_N = 9\sqrt{N} \quad (1.4.4)$$

Equation (1.4.1) can now be rewritten as

$$\epsilon_r = 1 - \frac{f_N^2}{f^2} \quad (1.4.5)$$

Ionospheric Propagation – Plasma Frequency and Critical Frequency

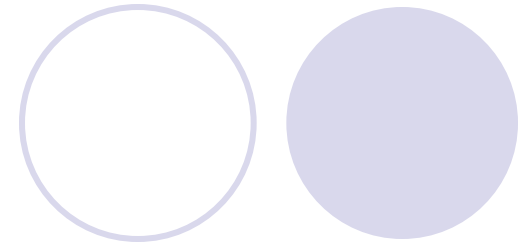
The significance of f_N is that when a wave of this frequency reaches a region of electron density N , as given by Eq. (1.4.4), the relative permittivity, as given by Eq. (1.4.5), is seen to be zero. This, in turn, means that the total displacement current density is zero, and hence the *effective* electric field is zero. This can be accounted for in terms of a reflected wave that exactly cancels the incident wave at the point of reflection. Of course, it should be possible to receive this reflected wave, which is exactly what happens in short-wave radio communications via the ionosphere.

The highest-frequency wave that will be reflected from a given layer will be determined by the maximum electron density of that layer and will be given by

$$f_0 = 9\sqrt{N_{\max}} \quad (1.4.6)$$

Here f_0 is known as the *critical frequency*.

Ionospheric Propagation – Phase and Group Velocities



An interesting point arises in connection with the velocities associated with the wave. The phase velocity has already been shown to be

$$v_p = \frac{c}{\sqrt{\epsilon_r}} \quad (1.4.7)$$

Hence, in the ionosphere, when the wave reaches a height such that ϵ_r is zero, v_p becomes infinite! Now, the energy in a wave travels at the group velocity v_g , and it can be shown that for an ionized layer

$$v_p v_g = c^2 \quad (1.4.8)$$

The wavefront in the ionosphere is a step function that will propagate energy at the group velocity, and hence, from Eq. (1.4.8), when the phase velocity is infinite, the group velocity is zero, so that the energy ceases to be propagated upward.

Ionospheric Propagation – Secant Law and Maximum Usable Frequency

When a wave enters an ionized layer at an oblique angle of incidence i , it follows a curved ray path [Fig. 1.4.2(a)]. The phase velocity v_p at any given height can be determined by application of Snell's law of refraction for optics, the details of which will be omitted here. The result is that

$$\frac{\sin i}{c} = \frac{\sin r}{v_p} \quad (1.4.9)$$

Here r is the angle of refraction at the height where v_p occurs, as shown in Fig. 1.4.2(a).

At the apex of the path, $r = 90^\circ$, and therefore

$$v_p = \frac{c}{\sin i} \quad (1.4.10)$$

Ionospheric Propagation – Secant Law and Maximum Usable Frequency

It can be seen that as the angle of incidence i approaches zero the phase velocity approaches infinity, in agreement with the results in the previous section.

As already shown,

$$v_p = \frac{c}{\sqrt{\epsilon_r}}$$

It follows, therefore, that

$$\sqrt{\epsilon_r} = \sin i$$

Ionospheric Propagation – Secant Law and Maximum Usable Frequency

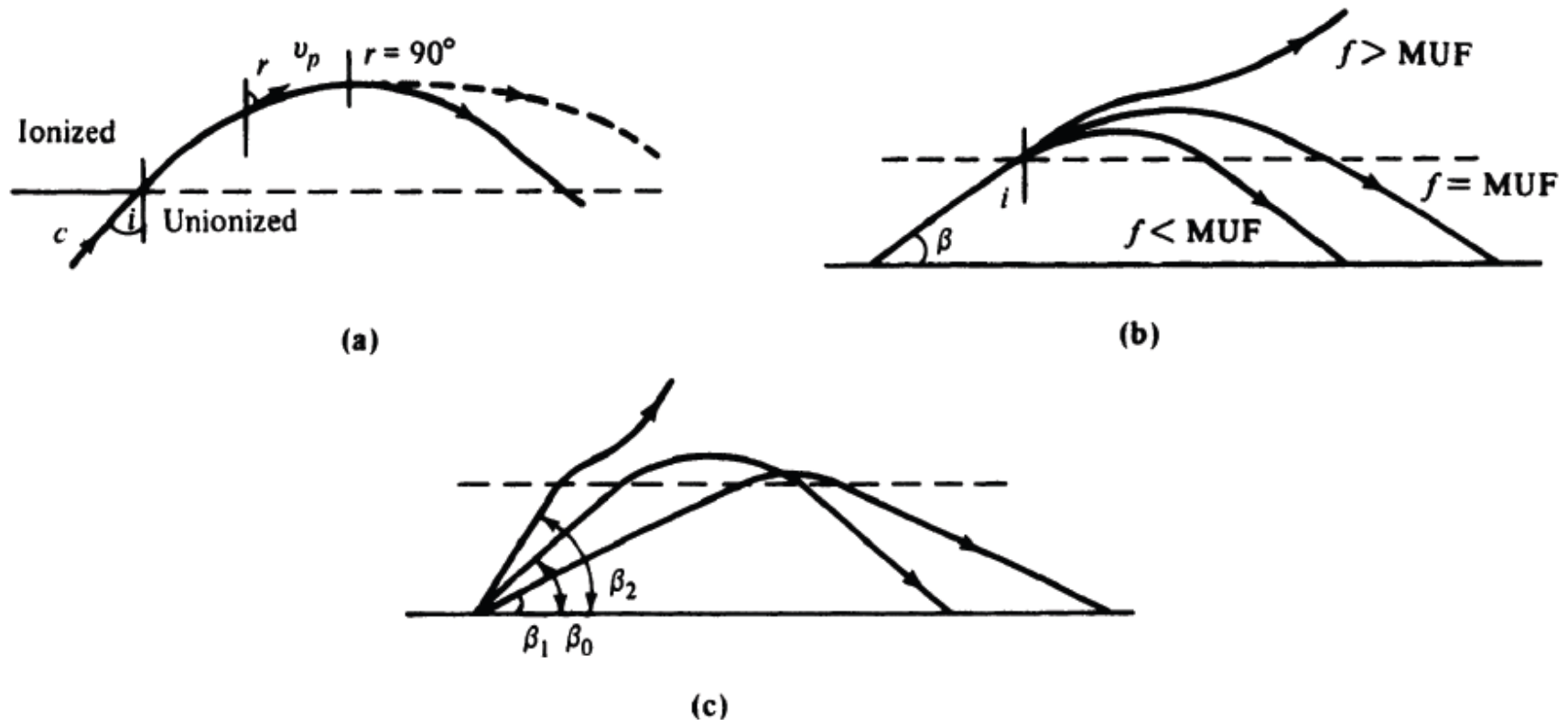


Figure 1.4.2 (a) Curved path of a wave in the ionosphere entering at oblique incidence. (b) Ray paths for fixed angle of incidence and varying frequency. (c) Ray paths for fixed frequency and varying angle of incidence.

Ionospheric Propagation – Secant Law and Maximum Usable Frequency

Substituting Eq. (1.4.5) for ϵ_r ,

$$\left(1 - \frac{f_N^2}{f^2}\right) = \sin^2 i$$

from which the reader should be able to derive that

$$f = f_N \sec i \quad (1.4.11)$$

This is known as the *secant law*. The highest frequency that can be used will be determined by $N_{\max}$, and hence f_0 ; this is known as the *maximum usable frequency* (MUF):

$$\text{MUF} = f_0 \sec i \quad (1.4.12)$$

Ionospheric Propagation – Virtual Height

A wave traveling in a curved path has a horizontal component of group velocity v_h given by [see Fig. 1.4.3(a)]

$$v_h = v_g \sin r \quad (1.4.13)$$

From Eq. (1.4.8),

$$v_g = \frac{c^2}{v_p}$$

Therefore,

$$v_h = c^2 \frac{\sin r}{v_p}$$

Ionospheric Propagation – Virtual Height

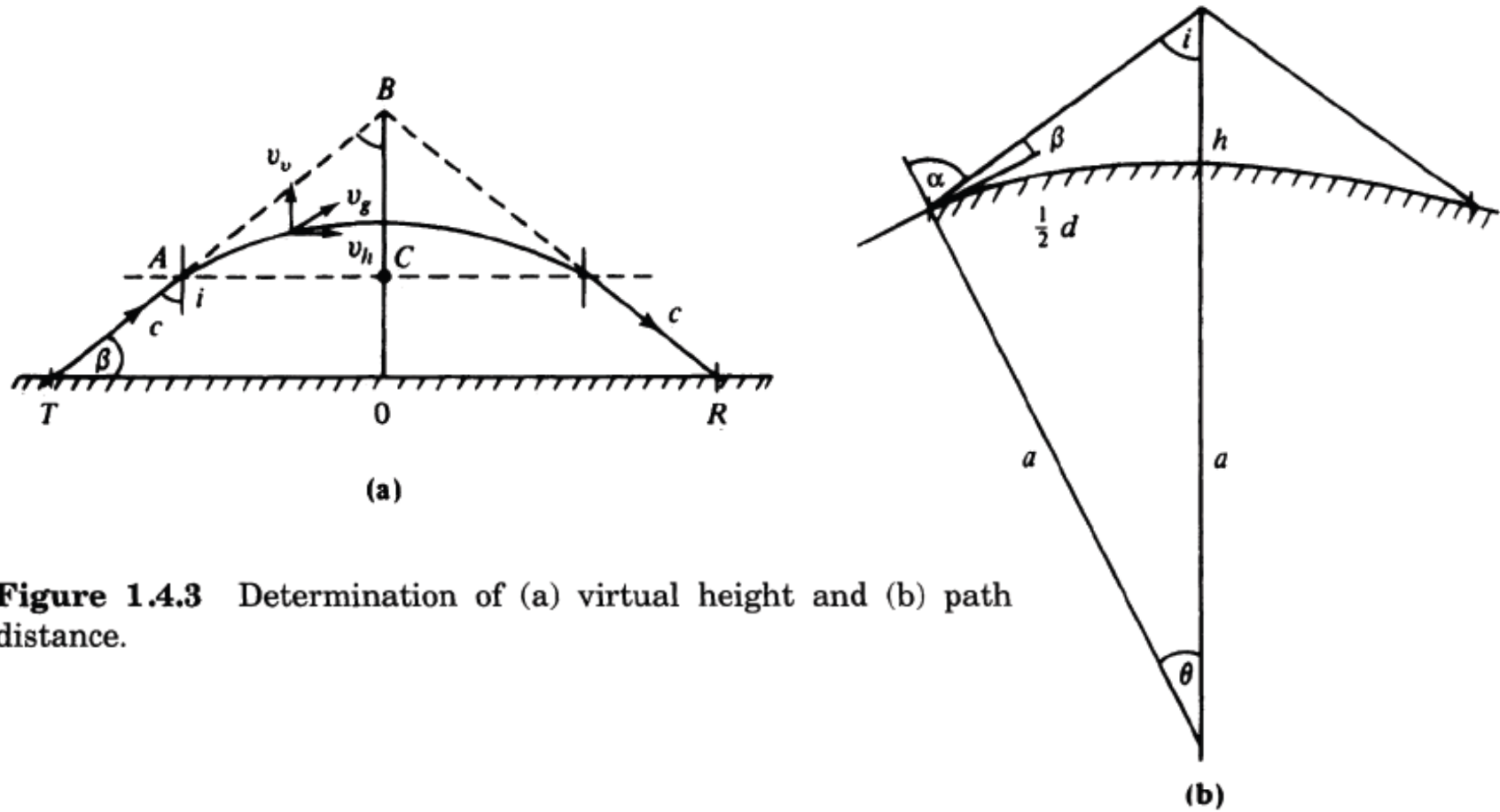


Figure 1.4.3 Determination of (a) virtual height and (b) path distance.

Ionospheric Propagation – Virtual Height

Substituting for $(\sin r)/v_p$ from Snell's law, Eq. (1.4.9),

$$\begin{aligned} v_h &= c^2 \frac{\sin i}{c} \\ &= c \sin i \end{aligned} \quad (1.4.14)$$

This shows that the horizontal component of group velocity is constant and independent of height in the ionized layer. Therefore, the time t required for the wavefront to reach the highest point in the path will be

$$t = \frac{AC}{v_h} \quad (1.4.15)$$

where AC is the horizontal distance from point of entry to the vertical dropped from the highest point [Fig. 1.4.3(a)].

Ionospheric Propagation – Virtual Height

Substituting from Eq. (1.4.14) for v_h , Eq. (1.4.15) becomes

$$t = \frac{AC}{c \sin i}$$

But from Fig. 15.4.3(a), $AC = AB \sin i$, and therefore

$$t = \frac{AB}{c} \tag{1.4.16}$$

Ionospheric Propagation – Virtual Height

For flat-earth assumptions and assuming that the ionospheric conditions are symmetrical for the incident and reflected waves, the transmission-path distance TR , Fig 1.4.3(a), is

$$TR = \frac{2h}{\tan \beta} \quad (1.4.17)$$

Where the curvature of the earth is taken into account, the transmission-path distance can be determined from the geometry of Fig. 1.4.3(b):

$$\begin{aligned} \frac{\sin i}{a} &= \frac{\sin(180 - \alpha)}{a + h} \\ &= \frac{\sin \alpha}{a + h} \end{aligned} \quad (1.4.18)$$

Ionospheric Propagation – Virtual Height

Also,

$$180 - \alpha = 180 - (i + \theta)$$

Therefore,

$$i = \alpha - \theta \quad (1.4.19)$$

Substituting Eq. (1.4.19) in Eq. (1.4.18),

$$\frac{\sin(\alpha - \theta)}{a} = \frac{\sin \alpha}{a + h}$$

Therefore,

$$\theta = \alpha - \sin^{-1}\left(\frac{a}{a + h} \sin \alpha\right) \quad (1.4.20)$$

Ionospheric Propagation – Virtual Height

In terms of the angle of elevation, Eq. (1.4.20) becomes

$$\theta = (90 - \beta) - \sin^{-1}\left(\frac{a}{a+h} \cos \beta\right) \quad (1.4.21)$$

Also, from Fig. 1.4.3(b), the arc length

$$\frac{d}{2} = a\theta \quad (1.4.22)$$

where the angle θ is in radians. Substituting for θ from Eq. (1.4.21) and using radian measure for all angles,

$$d = 2a\left[\left(\frac{\pi}{2} - \beta\right) - \sin^{-1}\left(\frac{a}{a+h} \cos \beta\right)\right] \quad (1.4.23)$$

Ionospheric Propagation – Virtual Height

EXAMPLE 1.4.1

Calculate the transmission-path distance for an ionospheric transmission that utilizes a layer of virtual height 200 km. The angle of elevation of the antenna beam is 20° .

SOLUTION The flat-earth approximation [Eq. (1.4.17)], gives

$$\begin{aligned}d &= \frac{2 \times 200}{\tan 20^\circ} \\&= 1100 \text{ km}\end{aligned}$$

Using Eq. (15.4.23),

$$\begin{aligned}d &= 2 \times 6370 \left[(1.57 - 0.349) - \sin^{-1} \left(\frac{6370}{6570} \cos 20^\circ \right) \right] \\&= \mathbf{966 \text{ km}}\end{aligned}$$

Ionospheric Propagation – Virtual Height

Measurement of virtual height is usually carried out by means of an instrument known as an *ionosonde*. The basic method is to transmit vertically upward a pulse-modulated radio wave with a pulse duration of about 150 μs . The reflected signal is received close to the transmission point, and the time T required for the round trip is measured. The virtual height is then

$$h = \frac{cT}{2}, \quad \text{where } c = \text{speed of light} \quad (1.4.24)$$

The ionosonde will have facilities for sweeping over the radio-frequency range; typically, it will sweep from 1 to 20 MHz in 3 min. It will also have facilities for automatic plotting of virtual height against frequency, the resultant graph being known as an *ionogram* (Figure 1.4.4). The ionogram shows two critical frequencies, f_0F_2 and f_xF_2 , which will be explained in the next section.

Ionospheric Propagation – Virtual Height

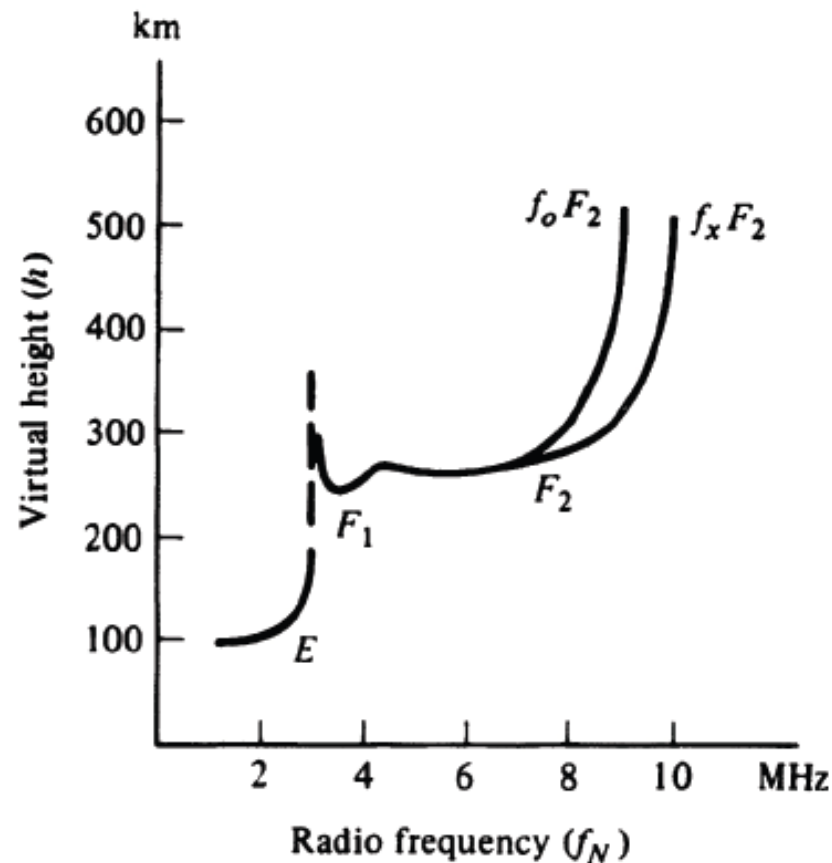


Figure 1.4.4 Sketch of an ionogram.

Ionospheric Propagation – Effect of Earth's Magnetic Field

The gyrofrequency can be shown to be equal to

$$f_g = \frac{q_e}{2\pi m} B \quad (1.4.25)$$

where q_e is the charge of the electron, m is its mass, and B is the magnetic field strength. Substituting values for q_e and m and using an average value of 0.5×10^{-4} T for the earth's magnetic field results in a gyrofrequency of 1.4 MHz.

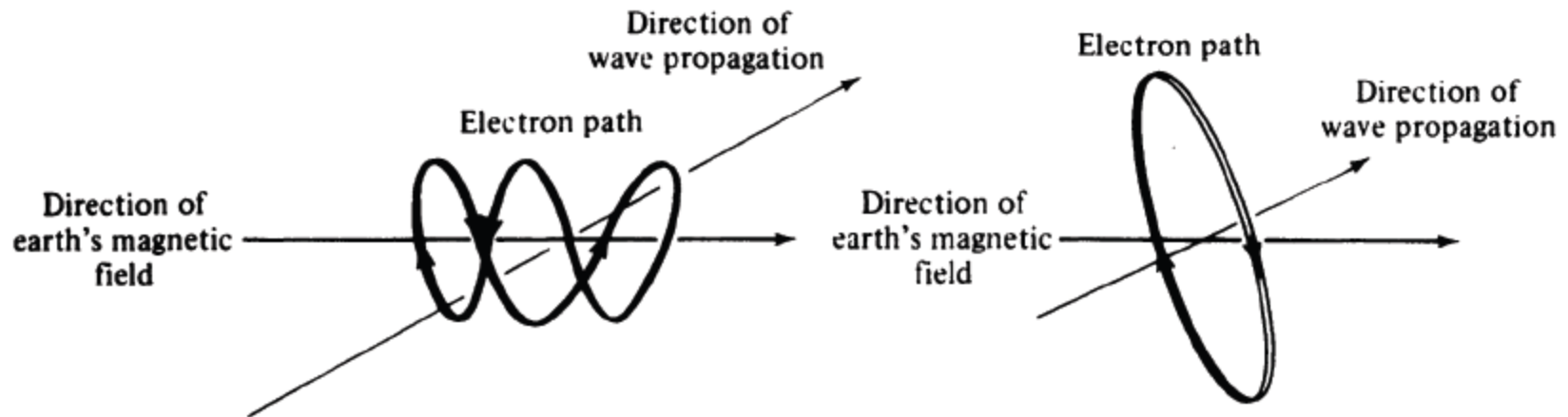


Figure 1.4.5 (a) Helical path of an electron moving in the earth's magnetic field. (b) Electron path at gyrofrequency.

Ionospheric Propagation – Service Range

The maximum possible range is reached when the critical ray coincides with the glancing ray [Fig. 1.4.6(b)], when the glancing ray is returned from the greatest virtual height h_m . The geometry of Fig. 1.4.6(b) is similar to that of Fig. 1.4.3(b) with β equal to zero and h being replaced by h_m . The angle θ is

$$\theta = \cos^{-1} \frac{a}{a + h_m}$$

and, from Eq. (1.4.22),

$$\begin{aligned} d &= 2a\theta \\ &= 2a \cos^{-1} \frac{a}{a + h_m} \end{aligned} \quad (1.4.26)$$

Ionospheric Propagation – Service Range

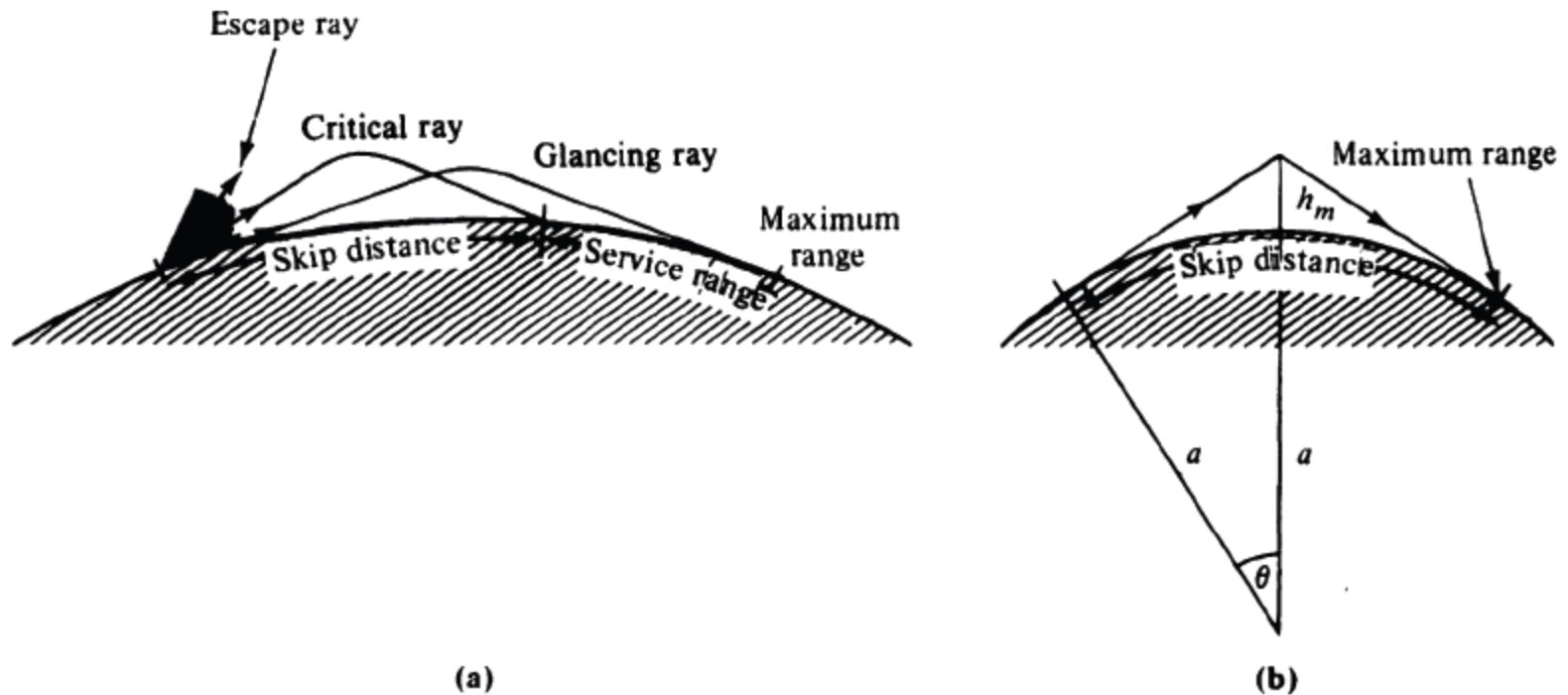


Figure 1.4.6 (a) Service range showing skip distance. (b) Maximum range.

Ionospheric Propagation – Summary of Layer

The main features of the various layers are summarized here, with values obtained from Fig. 1.4.1(a). Frequencies, heights, and so on, will of course vary considerably, as already discussed, and the values are presented simply to give an order-of-magnitude comparison for the various layers.

C and D layers. Virtual height, 60 to 80 km. Reflect low and very low frequencies (see Section 14.6), but for HF communications the D layer, in particular, introduces attenuation.

E layer. Virtual height, ~ 110 km. Critical frequency, ~ 4 MHz. Maximum single-hop range, ~ 2350 km.

F₁ layer. Virtual height, ~ 180 km. Critical frequency, ~ 5 MHz. Maximum single-hop range, ~ 3000 km.

F₂ layer. Virtual height, ~ 300 km daytime, ~ 350 km nighttime. Critical frequency, ~ 8 MHz daytime, ~ 6 MHz nighttime. Maximum single-hop range, ~ 3840 km daytime, ~ 4130 km nighttime.

Surface Wave – Mode of Propagation

The electric field strength as given by Eq. (1.2.13) can be modified to take into account the various factors affecting propagation by introducing an attenuating factor A , such that

$$E = \frac{A\sqrt{30P_T G_T}}{d} \quad \text{V/m} \quad (1.5.1)$$

It is normal practice to evaluate E for standard conditions of 1 kW radiated from a short unipole antenna, which has a power gain of 3. Equation (1.5.1) then becomes

$$E = 300 \frac{A}{d} \quad \text{V/m} \quad (1.5.2)$$

Surface Wave – Ground Wave

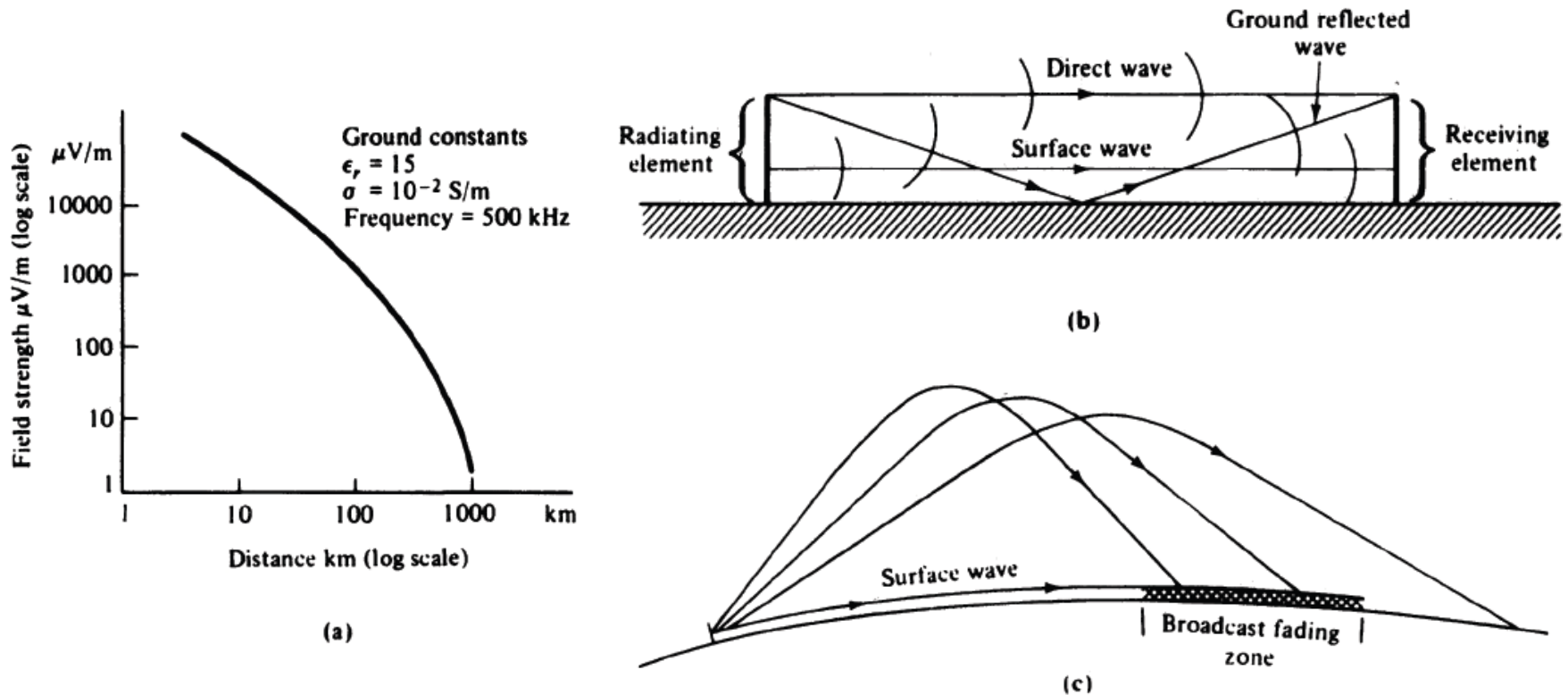


Figure 1.5.1 (a) Example of variation of surface wave field strength with distance. (b) Ground wave. (c) Broadcast fading zone.

Low Frequency Propagation and Very Low Frequency Propagation

Radio navigational systems make extensive use of the low and very low frequency bands. The highly stable phase characteristics of the propagation allow the phase delay in a wave as a function of distance to be accurately known in advance, so that it can be measured by receiving equipment on the ship, aircraft, and so on, to determine position. The most common method is to use two or more transmissions to generate a pattern of *lines of position*, which can then be superimposed upon a map. Referring to Fig. 1.6.1(a), the signals received at position X , will be

$$A \sin(\omega_a t - \beta_a |s_1|) \quad \text{from transmitter } A \quad (1.6.1)$$

$$B \sin(\omega_b t - \beta_b |s_2|) \quad \text{from transmitter } B \quad (1.6.2)$$

Low Frequency Propagation and Very Low Frequency Propagation

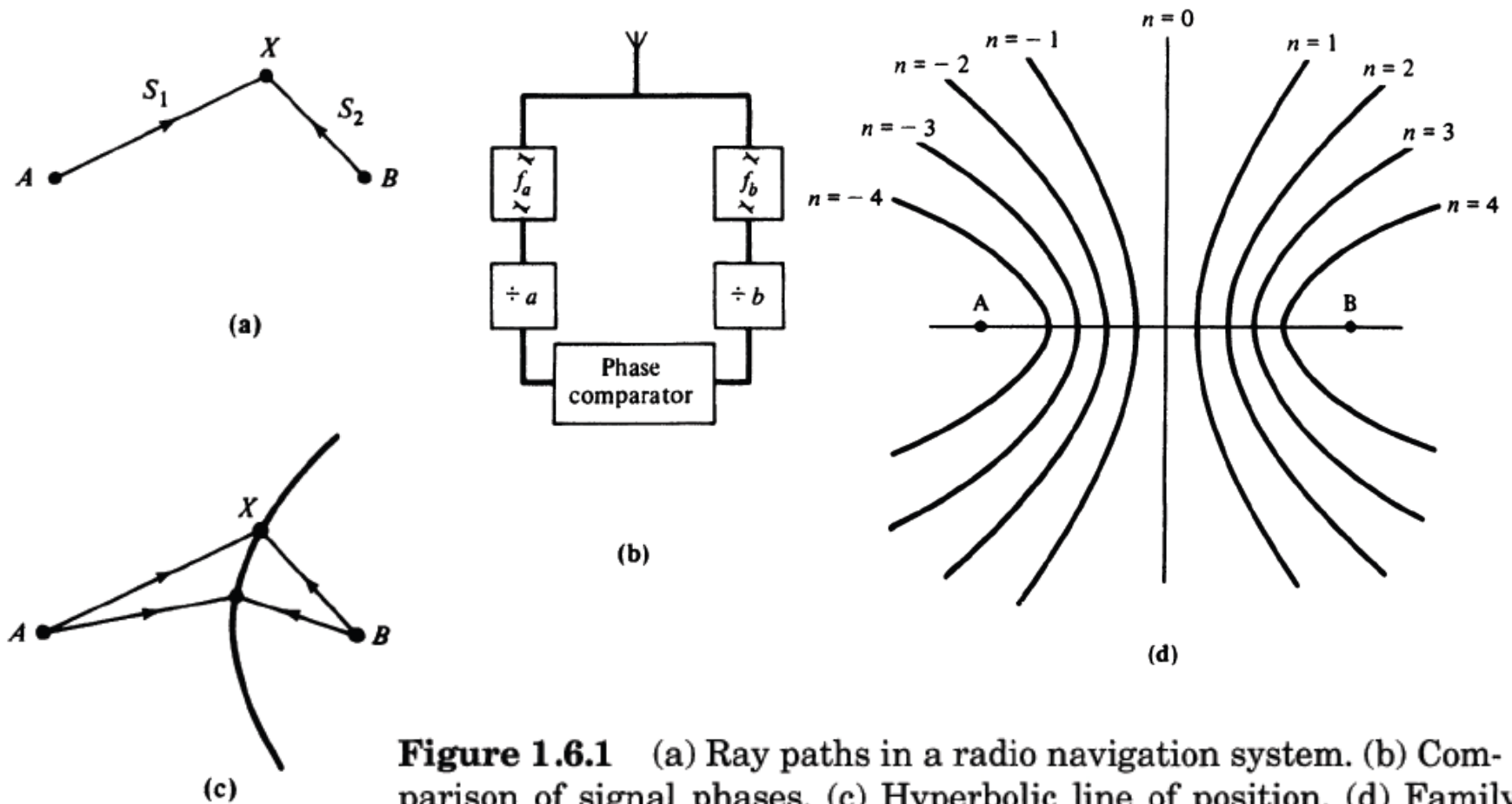


Figure 1.6.1 (a) Ray paths in a radio navigation system. (b) Comparison of signal phases. (c) Hyperbolic line of position. (d) Family of lines of position.

Low Frequency Propagation and Very Low Frequency Propagation

By making ω_a and ω_b multiples of a common generating frequency ω_0 to which both are synchronized, it becomes possible to compare the phase of the two signals. The phase angles are $\beta_a|s_1|$ and $\beta_b|s_2|$. Let $\omega_a = 2\pi af_0$ and $\omega_b = 2\pi bf_0$; then, by putting both signals through frequency dividers at the receiver, as shown in Fig. 1.6.1(b), and comparing phases, a measure of $|s_1| - |s_2|$ is obtained. The phase of the signals after frequency division will be

$$\text{phase of signal A} = \frac{\beta_a|s_1|}{a} \quad (1.6.3)$$

$$\text{phase of signal B} = \frac{\beta_b|s_2|}{b} \quad (1.6.4)$$

Low Frequency Propagation and Very Low Frequency Propagation

Now, $\beta_a = 2\pi/\lambda_a$ and $\lambda_a = \lambda_0/\alpha$, so the phase angle of signals A and B , after frequency division, become

$$\text{phase of signal } A = \frac{2\pi}{\lambda_0}|s_1| \quad (1.6.5)$$

$$\text{phase of signal } B = \frac{2\pi}{\lambda_0}|s_2| \quad (1.6.6)$$

Therefore, the phase difference Δ is

$$\Delta = \frac{2\pi}{\lambda_0}(|s_1| - |s_2|) \quad (1.6.7)$$

Low Frequency Propagation and Very Low Frequency Propagation

In effect, the phase difference goes through a series of maxima and minima with change in distance, just like that which occurs for standing waves on transmission lines. The two signals will be in phase when $\Delta = n2\pi$, where n is an integer. It follows, therefore, that

$$|s_1| - |s_2| = n\lambda_0 \quad (1.6.8)$$

Along the base line joining A and B , the spacing is narrowest. Let L be the width of a lane on the base line. Then, on moving from line n to $n + 1$ along the base line, the total change in the path difference will be $2L$, since $|s_1|$ increases by L and $|s_2|$ decreases by L . In terms of wavelength, the change will be $(n + 1)\lambda_0 - n\lambda_0 = \lambda_0$. Hence

$$\begin{aligned} 2L &= \lambda_0 \\ L &= \frac{1}{2}\lambda_0 \end{aligned} \quad (1.6.9)$$

Low Frequency Propagation and Very Low Frequency Propagation

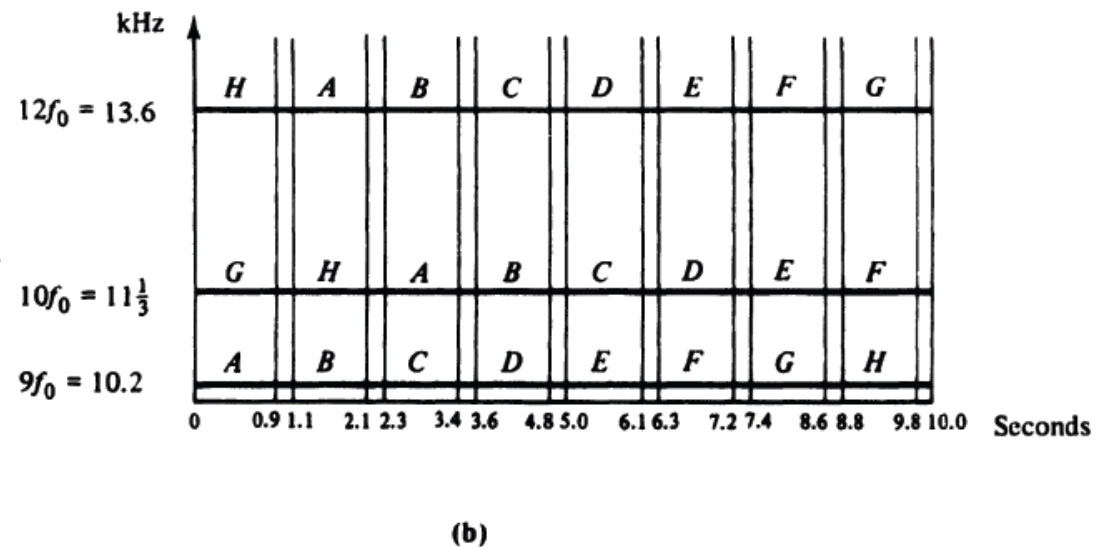
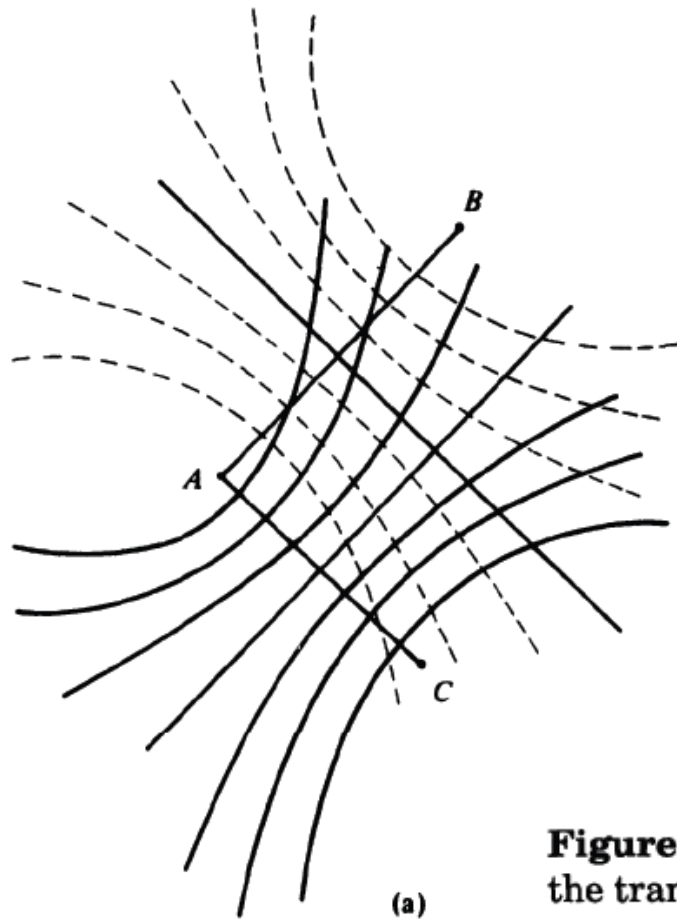


Figure 1.6.2 (a) Intersecting lines of position. (b) Chart showing the transmission schedule for the Omega navigation system.

Extremely Low Frequency Propagation

As has been pointed out in the case of VLF propagation, the propagation around the earth may be considered to take place in a waveguide mode, the earth and the ionosphere forming the waveguide boundaries. Because of the very long wavelengths, the transmitting antenna has to be mounted horizontally. The length of the antenna is very much less than a wavelength. The situation is sketched in Fig. 1.7.1. It is found that the far field consists of a horizontal magnetic field and a vertical electric field, and propagation in the earth-ionosphere waveguide is by means of a quasi-TEM mode. A leakage field also occurs, directed into the surface of the earth, and subsurface communications utilize this leakage field. The leakage field is a plane TEM wave that follows a propagation law similar to that for the voltage on a transmission line; that is, the electric field at depth z is related to $E(0)$, the electric field at the surface, by

$$E(z) = E(0)e^{-\gamma z} \quad (1.7.1)$$

Extremely Low Frequency Propagation

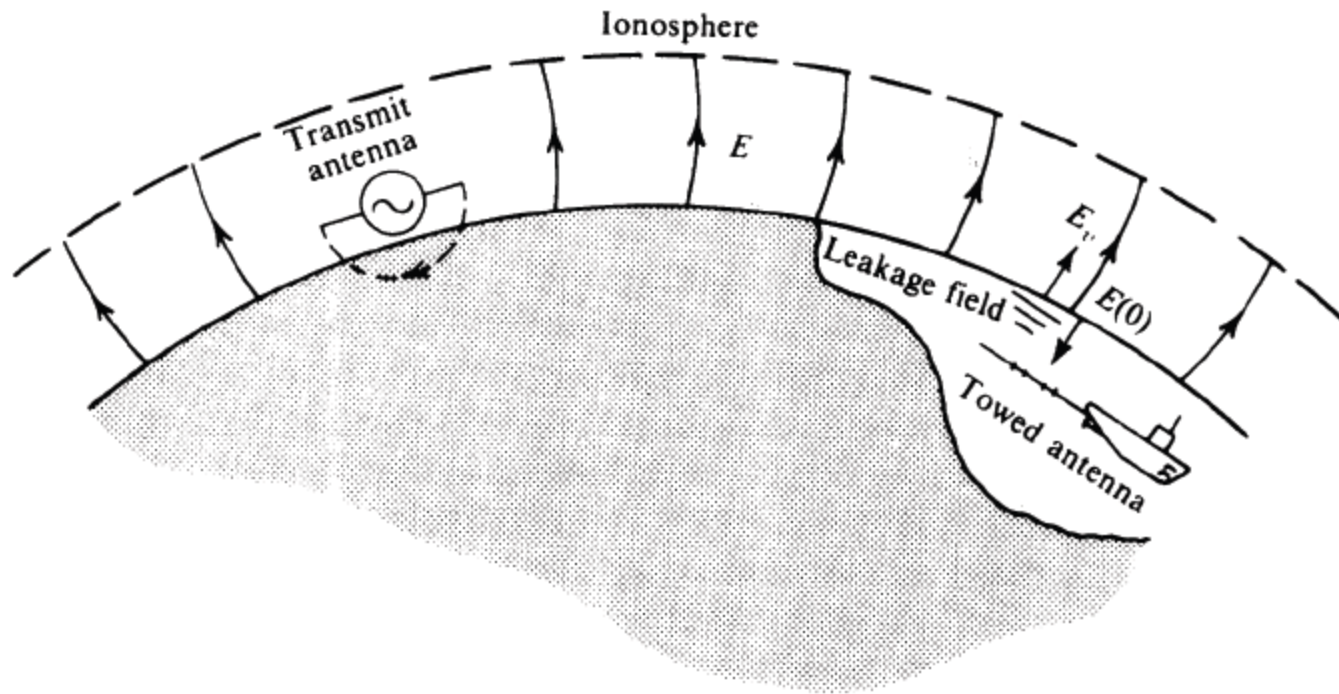


Figure 1.7.1 ELF propagation. (After Bernstein et al., "Long Range Communication at Extremely Low Frequencies," *Proc. IEEE* 62(3), 1974, pp. 292–312.)

Extremely Low Frequency Propagation

The propagation coefficient γ is made up of two components, the attenuation coefficient α , and the phase shift coefficient β . These depend on the conductivity, permittivity, and permeability of the medium and on the frequency of the wave. The ratio $\sigma/\omega\epsilon$ is known as the loss tangent of the medium, and for $\sigma/\omega\epsilon \gg 1$, the attenuation coefficient is numerically equal to the phase shift coefficient, each being given by

$$\alpha = \beta = \sqrt{\frac{\omega\sigma\mu}{2}} \quad (1.7.2)$$

The permeability μ may be assumed to be equal to $4\pi \times 10^{-7} \text{H/m}$.

Extremely Low Frequency Propagation

EXAMPLE 1.7.1

Sea water has an average conductivity of 4 S/m and a relative permittivity of 80. Calculate the attenuation coefficient in dB/m for a signal of (a) 100 Hz and (b) 1 MHz.

SOLUTION (a) First it is necessary to evaluate the ratio $\sigma/\omega\epsilon$. At 100 Hz this is

$$\begin{aligned}\frac{\sigma}{\omega\epsilon} &= \frac{4}{2\pi \times 100 \times 80 \times 8.854 \times 10^{-12}} \\ &= 8.9 \times 10^7\end{aligned}$$

Since this is very much greater than unity, Eq. (1.7.2) may be used to calculate the attenuation coefficient as

$$\begin{aligned}\alpha &= \sqrt{\frac{2\pi \times 100 \times 4 \times 4\pi \times 10^{-7}}{2}} \\ &= 3.974 \times 10^{-2} \text{ N/m}\end{aligned}$$

Extremely Low Frequency Propagation

Using the conversion factor $1\text{N} = 8.686\text{ dB}$, we have

$$[\alpha] = 3.974 \times 10 \times 8.686 = \mathbf{0.345\text{ dB/m}}$$

(b) At $f = 1\text{ MHz}$, the loss tangent is $8.9 \times 10^7 \times 100/10^6 = 8.9 \times 10^3$, and since this is also very much greater than unity, Eq. (1.7.2) can be used in this case also. Thus

$$\alpha = 3.974 \times 10^{-2} \times \sqrt{\frac{10^6}{100}} = 3.974\text{ N/m}$$

and

$$[\alpha] = 3.974 \times 8.686 = \mathbf{34.5\text{ dB/m}}$$

Extremely Low Frequency Propagation

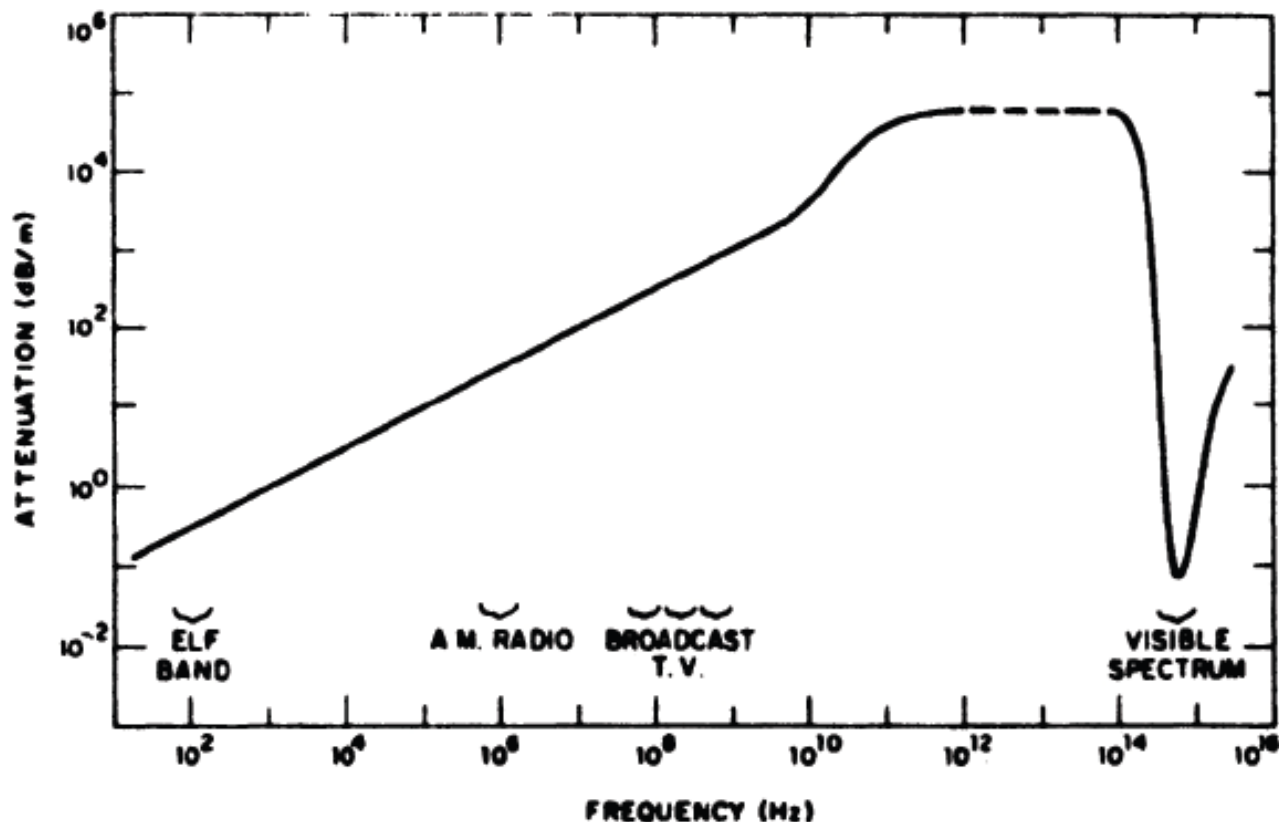


Figure 1.7.2 Attenuation of a plain electromagnetic wave in sea water as a function of frequency. (From Michael L. Burrows, *ELF Communications Antennas*, Peter Peregrine Ltd., 1978, Institute of Electrical Engineers, with permission.)

Summary of Radio-wave Propagation

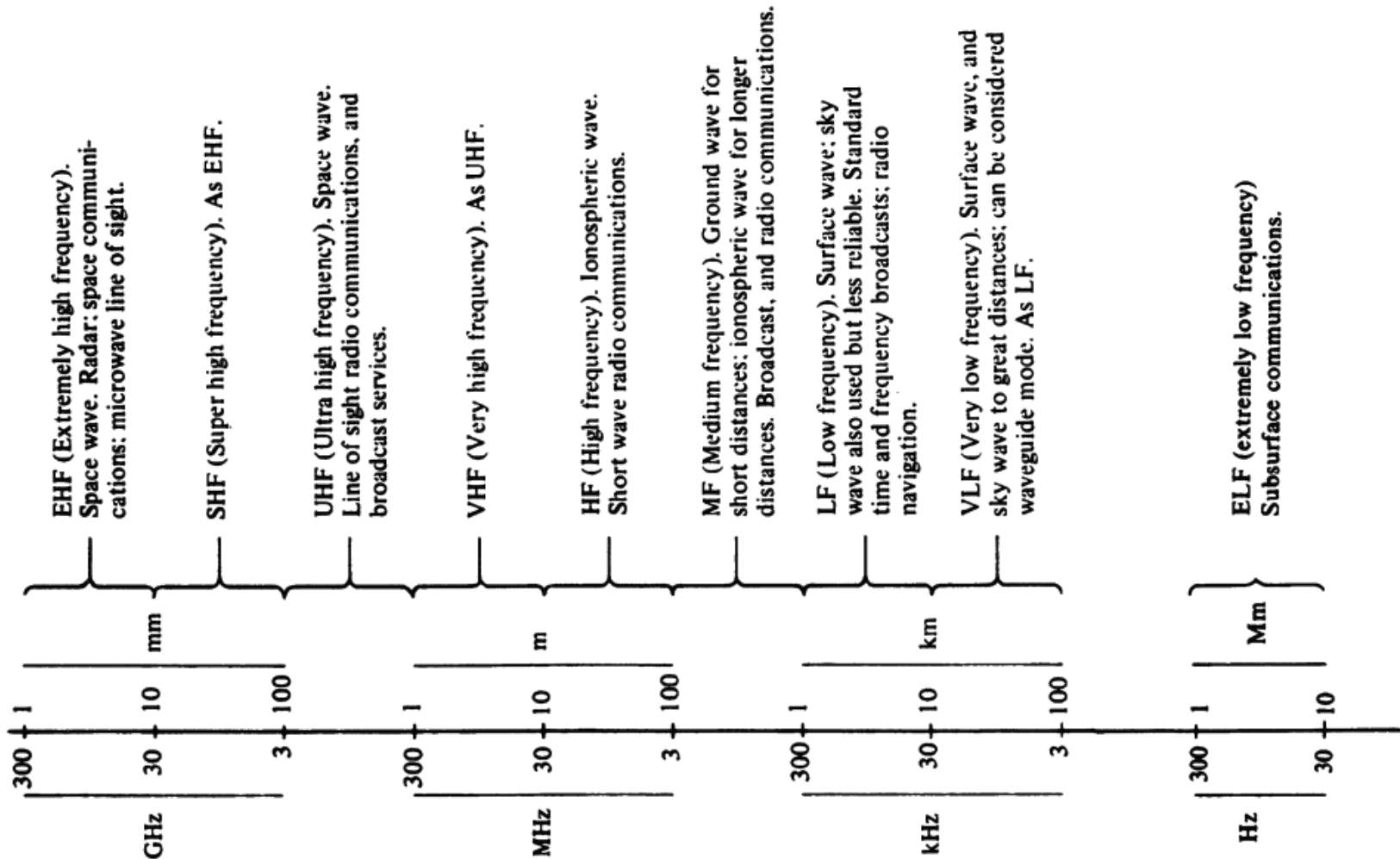


Figure 1.8.1 Classification of the various radio bands.