

MA2001**Review Exercises for Chapter 2 Vector Integral Calculus**Line Integrals

1. Evaluate the following line integrals.

(a) $\int_C xy^2 ds$, where C is the semi-circle $x = -\sqrt{25 - y^2}$ from $(0, 5)$ to $(0, -5)$.

Ans: $-\frac{1250}{3}$

(b) $\int_C xy ds$, where C is the portion of the parabola $z = y^2$ in the plane from $x = 2$ from $(2, 1, 1)$ to $(2, 3, 9)$.

Ans: $\frac{1}{6}(37\sqrt{37} - 5\sqrt{5})$

2. A wire is in the shape of a straight line segment from $(1, 0, 1)$ to $(2, -2, 2)$. Find its mass if the density ρ at a point (x, y, z) is $\rho(x, y, z) = x^2 + z^2$.

Ans: $\frac{14\sqrt{6}}{3}$

3. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (-y \sin z)\vec{i} + (x \sin z)\vec{j} + (xy \cos z)\vec{k}$, C is the circle cut from the cylinder $x^2 + y^2 = 9$ by the plane $z = -2$. C is oriented clockwise as viewed above from the positive direction of the z -axis.

Ans: $18\pi \sin 2$

4. Find the work done by the force field $\vec{F}(x, y) = (x^2 + xy)\vec{i} + (y - x^2y)\vec{j}$ in moving a particle along the curve $xy = 1$ from $(1, 1)$ to $(2, \frac{1}{2})$ in the first quadrant.

Ans: $\frac{71}{24} + \ln 2$

5. Let $\vec{F}(x, y, z) = (2xy + az)\vec{i} + bx^2\vec{j} + (x + 2z)\vec{k}$ be a vector field on $\mathbf{R}^3$, where a and b are real constants.

(a) Find the values of a and b such that $\vec{F}$ is conservative.

(b) With the values of a and b obtained in (a), evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the shorter path from $(5, 0, 0)$ to $(0, 0, 5)$ formed by the intersection of the surfaces $x + y + z = 5$ and $x^2 + y^2 + z^2 = 25$.

Ans: (a) $a = 1$ and $b = 1$.

(b) A potential function ϕ is $\phi(x, y, z) = x^2y + xz + z^2 + c$.

And $\int_C \vec{F} \cdot d\vec{r} = \int_C \nabla \phi \cdot d\vec{r} = \phi(0, 0, 5) - \phi(5, 0, 0) = (25 + c) - c = 25$