

HOMEWORK PROBLEMS #2

2-1 Consider the following directed network model [initiated in 1965 by Dr. Derek John de Solla Price (1922–1983), Professor of Science History at Yale University]:

- (o) Initialization: Start from $m_0 \geq 1$ isolated nodes
- (i) Growth: At every step, add one new node to the existing network, which brings in m ($m \leq m_0$) directed edges, each pointing to one existing node
- (ii) Preferential Attachment: for every node i with in-degree k_i^{in} , the above new node will connect to it according to the following probability:

$$\Pi_i = \frac{k_i^{in} + a}{\sum_j (k_j^{in} + a)} \quad (a > 0 \text{ is a constant parameter})$$

It has been proven that the node-degree distribution of the above Price model is given by $P(k) \sim (k^{in})^{-\gamma}$ with $\gamma = 2 + a/m$.

- (a) What kind of network will this model generate?
- (b) State 3 basic properties of this model.
- (c) The model starts from a set of “isolated nodes”. Will this be a problem technically? If so, how to fix it? If not, why not?
- (d) Can this model generate the Barabási-Albert model? If so, under what condition(s)? If not, why not?

2-2 Consider the following four complex network models (A, B, C, D):

(a) Model A

- 1) Start with a small-sized star-shaped network.
- 2) At every step, add one node to the existing network. This new node is connected to every existing node according to the following preferential attachment probability, where k_i is the degree of node i :

$$\Pi_i = 1 - \frac{k_i}{\sum_j k_j}, \quad i = 1, 2, \dots$$

What kind of network will you obtain? Briefly explain why you think so.

(b) Model B

- o Step 1 (Initialization) Start with a large-sized fully-connected network.
- o Step 2 (Process) For every possible pair of nodes, with probability p ($0 < p < 1$) remove the edge between them. Remove all isolated nodes whenever they appear.
- o Step 3 (End) After every possible pair of nodes has been operated once, and once only, stop.

What kind of network will you obtain? Briefly explain why you think so.

(c) Model C

- o Step 1 (Initialization) Start with a large-sized fully-connected network.
- o Step 2 (Process) Pick up an edge: if removing it does not disconnect the whole network, then remove it; but if removing it will disconnect the network, then do nothing. Continue to pick up another edge from the resultant network and repeat the above possible edge-removal operation.
- o Step 3 (End) After every possible edge has been operated once, and once only, stop.

What kind of network will you obtain? Briefly explain why you think so.

(d) Model D

- o Step 1 (Initialization) Start with a large-sized tree.
- o Step 2 (Preferential Attachment) Pick up every possible pair of nodes from the tree. If this pair of nodes is directly connected already, do nothing; if this pair of nodes is not directly connected, then with a probability proportional to the degree of the larger node, add an edge between them.
- o Step 3 (End) After every possible pair of nodes has been operated once, and once only, stop.

What kind of network will you obtain? Is this resulting network a good model for an Internet-like network? If you think so, state three major advantages of this model; if you don't think so, state three major disadvantages of this model.

2-3 To generate a WS small-world model, starting from a perfect ring graph, during the rewiring step one may follow either of the following two different operation rules (always avoid self-loops and multiple edges):

- (i) Pick up every pair of connected nodes on the ring, once and once only, randomly disconnect them either at the first end or at the last end of the edge, and then do random rewiring to a third node on the ring.
- (ii) Pick up every pair of connected nodes on the ring, once and once only, consistently disconnect the first end (but never disconnect the last end) of the edge, and then do random rewiring to a third node on the ring.

Which operation rule is good and which is bad technically? What is the technical problem with the bad operation rule?