

Operational Amplifiers

Why do we learn operational amplifiers?

- If an output of a sensor was 1mV, which is the smallest your DMM could measure
- You need to **amplify** the voltage for it to be detectable by the DMM
- Through the amplifier, the signal is now stronger (see Fig below).

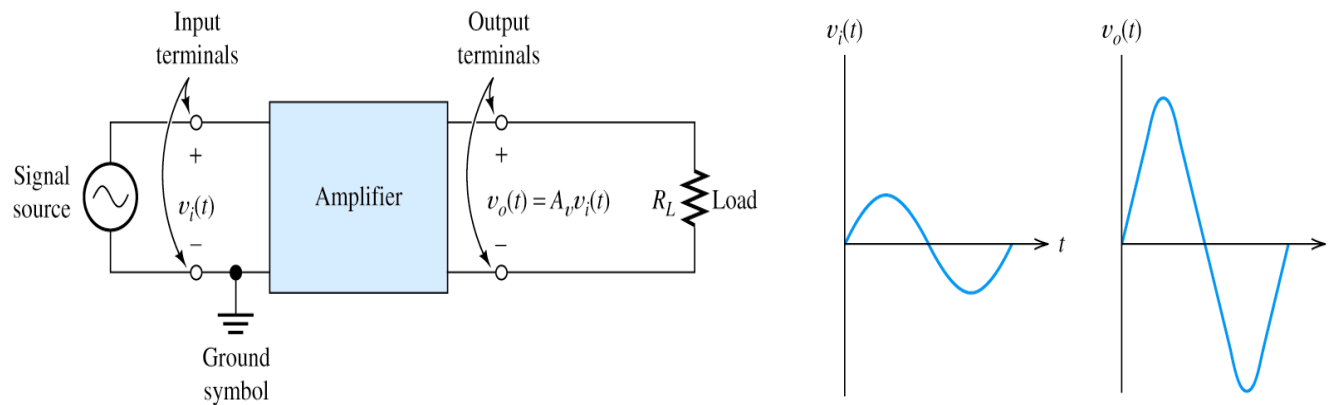


Figure 11.1 Electronic amplifier.

This entire block is dedicated to building circuits for amplifying weak signals.

Outline

I. The Ideal Operational Amplifier (Op Amp)

II. Op Amp Circuits (Part 1)

- Source follower
- Inverting amplifier
- Non-inverting amplifier
- Summing amplifier

III. Op Amp Circuits (Part 2)

- Differential amplifier

IV. Op Amp in Real Life

V. Saturation

VI. Cascaded Amplifiers

Alexander & Sadiku,
“Fundamentals of Electric Circuits”
5th Edition Ch 5

I. The Ideal Operational Amplifier

Terminals

- The ideal op amp has 3 terminals that you should notice. Fig 1 shows the symbol for an op amp. The 3 terminals to notice are:
- Non-inverting input terminal (+)
- Inverting input terminal (-)
- Output terminal (V_{out})

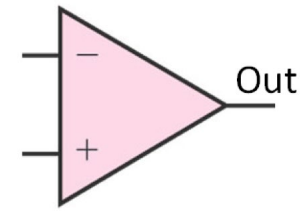


Figure 1: Symbol of an op amp

Open loop gain (A)

The op amp sees the voltage difference across the two inputs and multiplies it by a factor A. This factor A is known as the open loop gain:

$$V_{out} = A(V_{+} - V_{-})$$

Input resistance (R_{in})

A resistance lies between the two inputs.

Output resistance (R_{out})

Refer to the op amp shown in Fig 2. Only when R_{out} is 0, the above equation will hold.

The operational amplifier

3 Characteristics of an ideal op amp

- Infinite open loop gain
- Infinite input resistance
- Zero output resistance

- Input current = 0 (Input resistance ∞)
- $V_{\text{out}} = A(V^+ - V^-)$ (Output resistance = 0)
- $V^+ = V^-$ (A is ∞ , $V_{\text{out}} = A(V^+ - V^-)$)

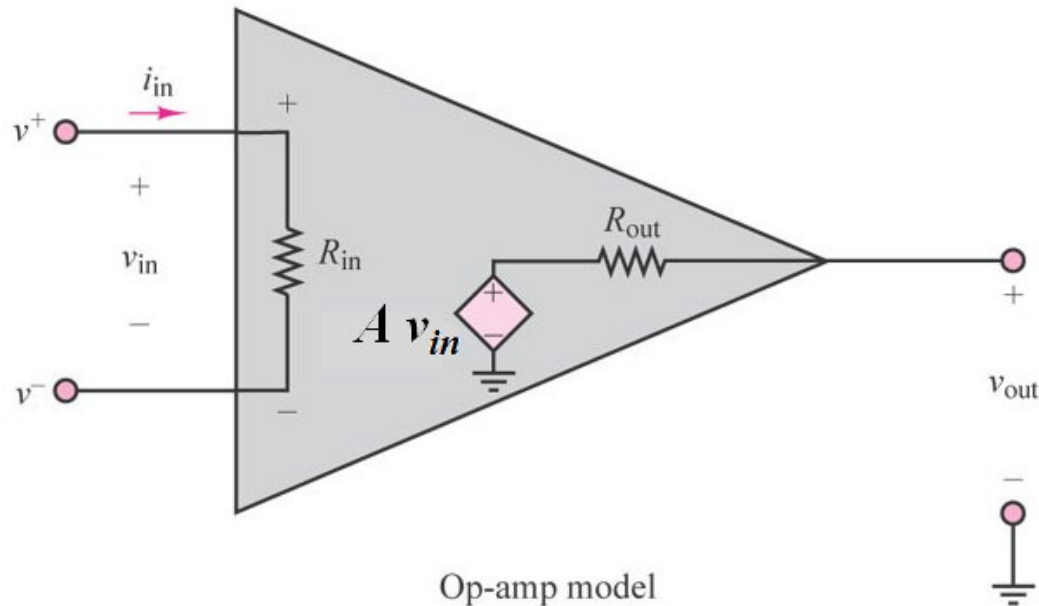
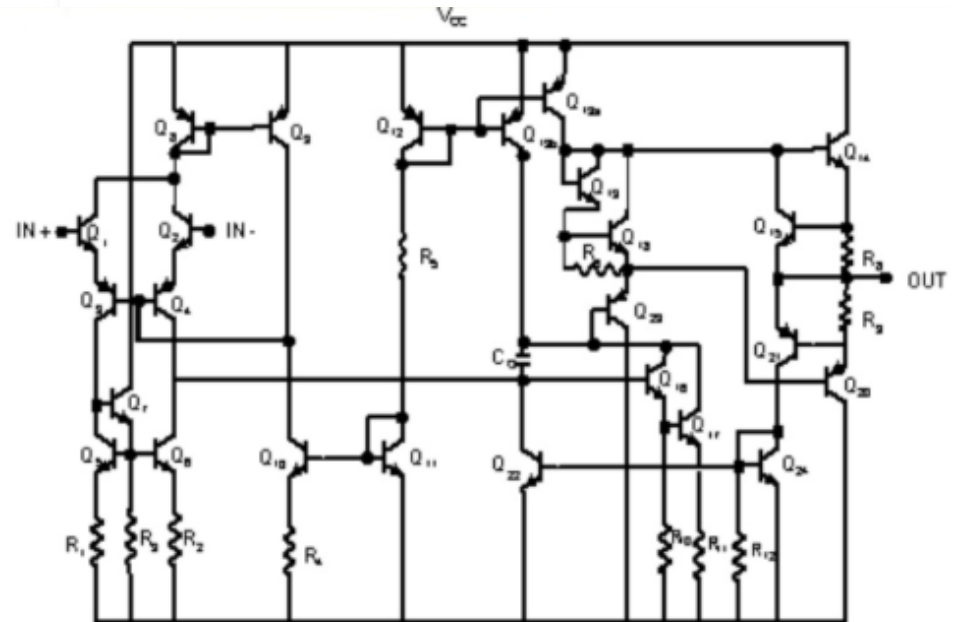
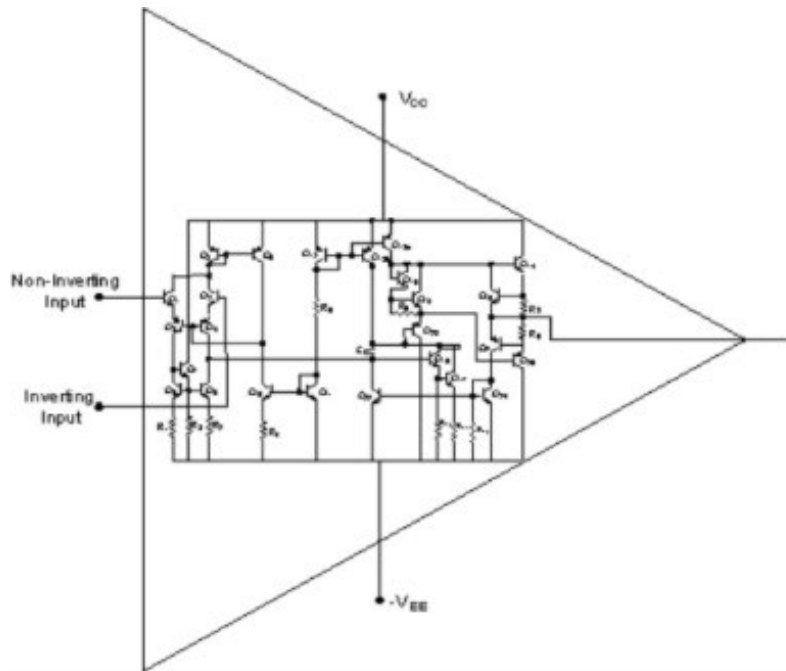


Figure 2: Op amp model

Operational amplifier internal circuit

What is inside the op-amp?



Internal circuit of classic op-amp LM741

We will cover the circuit components that comprise of the internal circuit of the op-amp later.

II. Op amp circuits

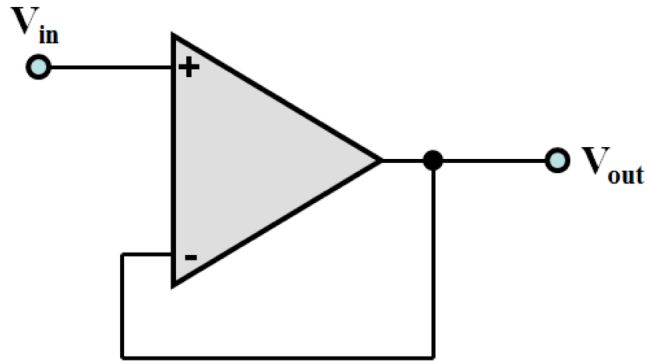
Part 1

- Source follower
- Inverting amplifier
- Non-inverting amplifier
- Summing amplifier

Part 2 (Block B Unit 2)

- Differential amplifier

Source follower



- Closed loop gain (V_{out}/V_{in})

- Gain:
$$\frac{V_{out}}{V_{in}} = 1$$

- Circuit input resistance : Infinite

Figure 3: Source follower

- V_{out} is connected to V_-
 $\rightarrow V_{out} = V_-$
- $V_+ = V_-$ (infinite loop gain with negative feedback)
 $\rightarrow V_{out} = V_+$
- $V_{in} = V_+$
 $\rightarrow V_{out} = V_{in}$

Source follower

- **Negative feedback:** Connect the output to the inverting input of the op amp. The effect is to reduce the voltage difference seen between the non-inverting and inverting input of the op amp
- **Open loop gain (A)** is specific to the op amp and is usually very large such that we can assume it to be infinite
- **Closed loop gain (G)** refers to the ratio of the output to the input specific to the circuit when the circuit is connected in negative feedback. The closed loop gain is always much lower than the open loop gain

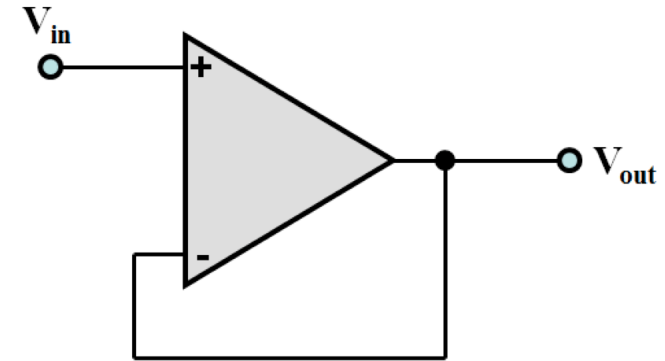


Figure 3: Source follower

Circuit input resistance of source follower

- Definition of circuit input resistance:

$$V_{in} / I_{in}$$

- Here, the input current sees the infinite input resistance of the op amp. So the input current is zero.

→ Circuit input resistance is therefore infinite

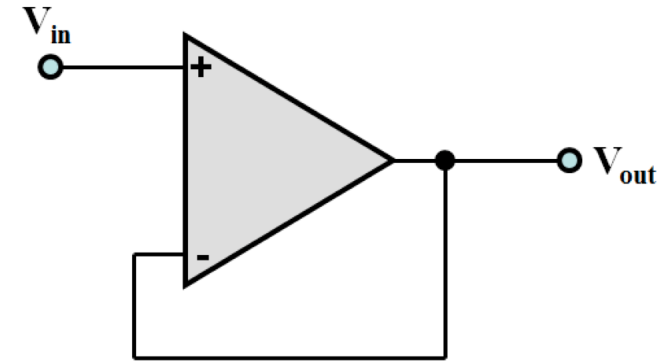


Figure 3: Source follower

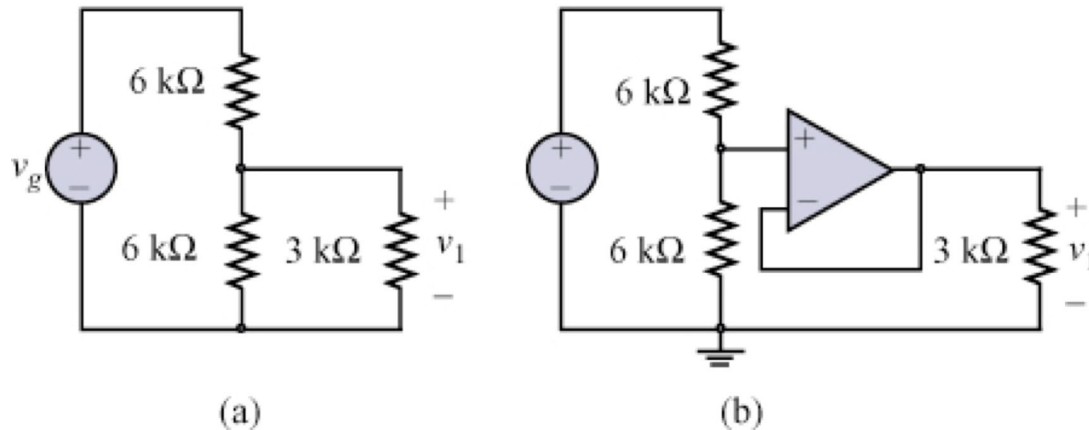
Note: Circuit input resistance is not always equal to the infinite input resistance of the op amp (e.g. the inverting amplifier).

- **Features of source follower:**
 - Large circuit input resistance
 - Small output resistance
 - Gain of close to one

What's source follower good for?

- **Application:** Buffer between a large source resistance and small load resistance. It isolates the input from the output load. See example below.

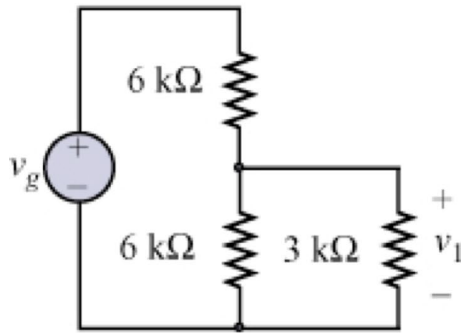
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- Find v_1 for the above 2 circuits where the $3\text{ k}\Omega$ is the output load

What's source follower good for?

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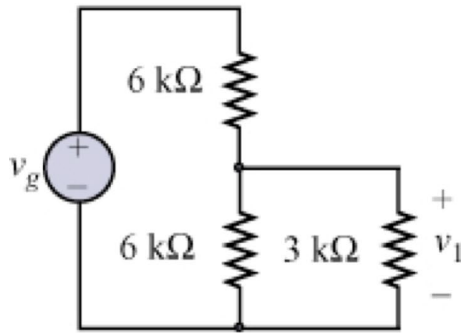


(a)

- **Ideal case:** when the load is infinite, i.e. open circuit
- Without the $3\text{ k}\Omega$ load, the open circuit voltage across the $6\text{ k}\Omega$ resistor = $v_g/2$

What's source follower good for?

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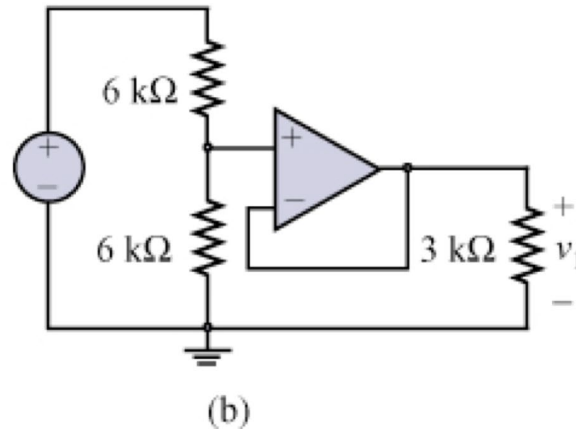


(a)

- For circuit (a), when $3\text{ k}\Omega$ is connect at the output:
 $6\text{ k}\Omega$ and $3\text{ k}\Omega$ are in parallel $= 2\text{ k}\Omega$
So $v_1 = [2/(2+6)]*v_g = v_g/4$
- Note that the output voltage has not dropped by half (it is $v_g/2$ in the ideal case) due to the $3\text{ k}\Omega$ load compared to the no load case.
- **So the $3\text{ k}\Omega$ load actually has an effect on the output. This is not a good thing!**

What's source follower good for?

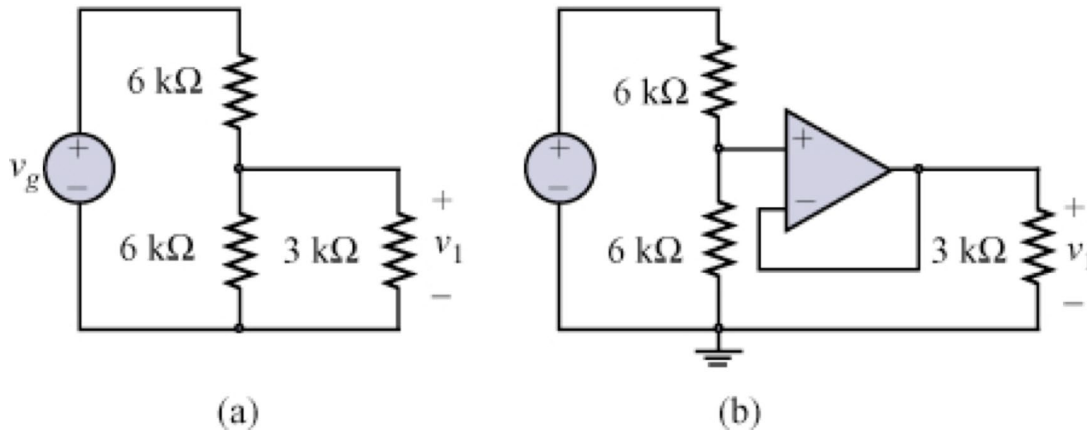
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- The problem of isolating the output load from the input resistance ($6\text{k}\Omega$) can be solved with a source follower like in circuit (b)
- For circuit (b), when $3\text{k}\Omega$ is connected at the output:
- Voltage across $6\text{k}\Omega = v_g/2$ (note that the 2 resistors are **in series since no current is flowing into the op amp inputs**)
- Since the gain is 1, $v_1 = v_g/2$

What's source follower good for?

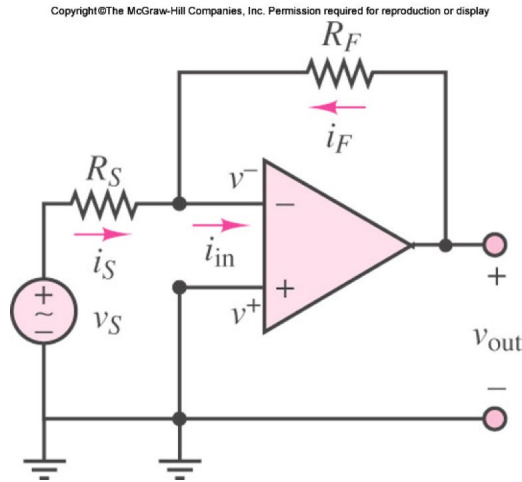
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SUMMARY:

- With the source follower, the $3\text{ k}\Omega$ load has no effect on the circuit on the left hand side
- In other words, the source follower works as a buffer. It isolates the input from the output load

Inverting amplifier



- Closed loop gain (V_{out}/V_{in})
- Gain: $\frac{v_{out}}{v_S} = -\frac{R_F}{R_S}$
- Circuit input resistance : R_S

Figure 4 : Inverting amplifier

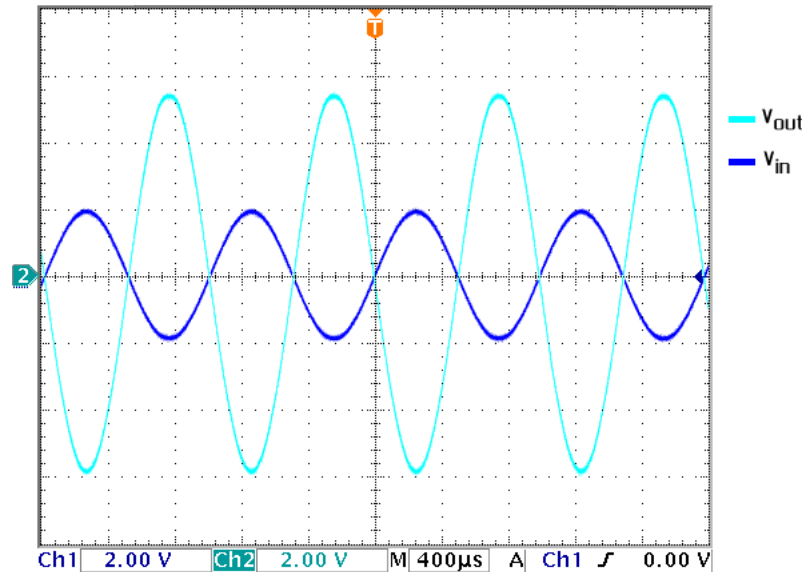
- Note that $v^- = v^+ = 0$
- Apply KCL at inverting input (V^-)

$$\frac{v_S - 0}{R_S} + \frac{v_{out} - 0}{R_F} = 0$$

$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

Comparison of input to output

- What does the minus sign mean?



- The minus sign indicates that the gain is inverting. This means that the direction of the signal is reversed by 180°
- The minus sign **does not** mean that the output is less than the input
- If the modulus of the gain (i.e. $|G|$) is more than 1, that means the output is larger than the input. The value of $|G|$ determines the amplification factor

Circuit input resistance of inverting amplifier

- Definition of circuit input resistance:

$$v_S / i_S$$

- $i_S = (v_S - 0) / R_S$
 $\rightarrow v_S / i_S = R_S$
 $\rightarrow$ Circuit input resistance = R_S

- You can now see that the circuit input resistance is not infinite in the case of the inverting amplifier, unlike the source follower. Circuit input resistance is a circuit-related parameter that depends on the actual connections in the circuit.

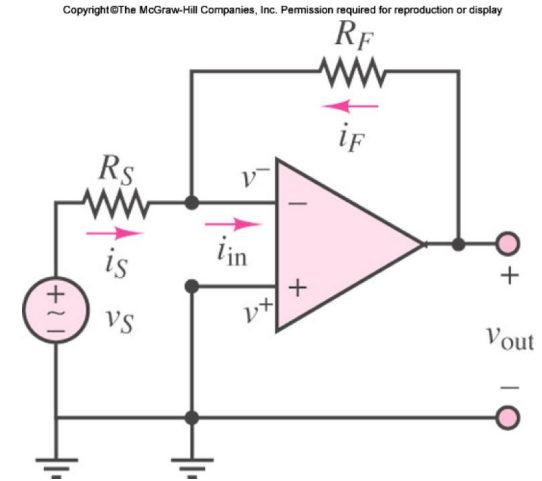


Figure 4 : Inverting amplifier

Circuit input resistance of inverting amplifier

- Since the circuit input resistance ($v_S/i_S = R_S$) is not infinite, the consequence is that any sort of internal resistance from the source (R_a) would have an effect on the output:
- Verify: R_a from the source appears in series with $R_S \rightarrow$ Replace R_S with $R_S + R_a$

$$V_{out} = \frac{-R_F}{R_S + R_a} V_S = \left(\frac{R_S}{R_S + R_a} \right) \left(-\frac{R_F}{R_S} V_S \right)$$

- Hence the output is reduced to a factor of $R_S/(R_S + R_a)$ of the value if $R_a = 0$

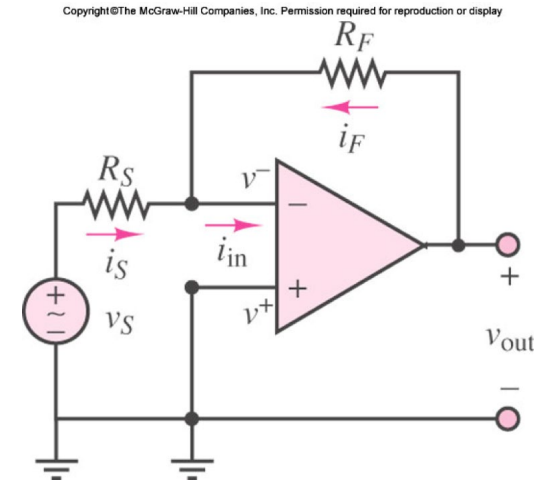
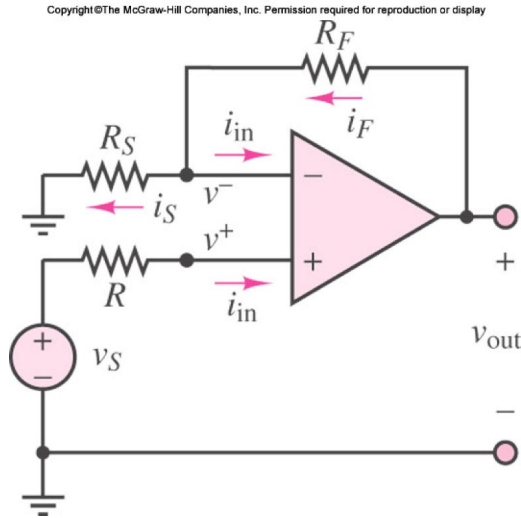


Figure 4 : Inverting amplifier

Non-inverting amplifier



- Closed loop gain (V_{out}/V_{in})
- Gain: $\frac{v_{out}}{v_S} = 1 + \frac{R_F}{R_S}$
- Circuit input resistance : Infinite

Figure 5 : Non-inverting amplifier

Closed loop gain (G)

- Note that $i_{in} = 0$ since it is going into the op amp input
→ $v_S = v^+ = v^-$ (1)
- Note there is also no current running into the inverting input of op amp as well ($i_{in} = 0$ for either inputs)
→ R_S and R_F are thus in series

Non-inverting amplifier

- Apply voltage divider rule:

$$v^- = \frac{R_S}{R_S + R_F} v_{out} \quad (2)$$

- Substitute (1) into (2):

$$v_S = \frac{R_S}{R_S + R_F} v_{out} \quad (3)$$

- After rearranging (3):

$$G = \frac{v_{out}}{v_S} = 1 + \frac{R_F}{R_S}$$

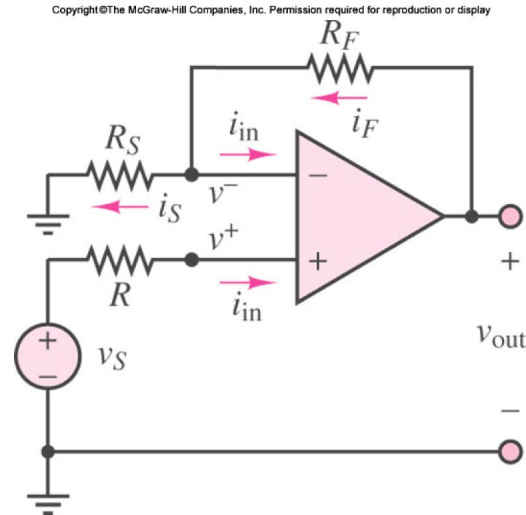


Figure 5 : Non-inverting amplifier

Circuit input resistance of non-inverting amplifier

- As mentioned already, $i_{in} = 0$ since the current from the sources sees the infinite resistance of the op amp
- Hence if $i_{in} = 0$, then:

$$\frac{v_S}{i_{in}} \rightarrow \infty$$

- So just like the source follower, any internal resistance at the source has no effect on the output

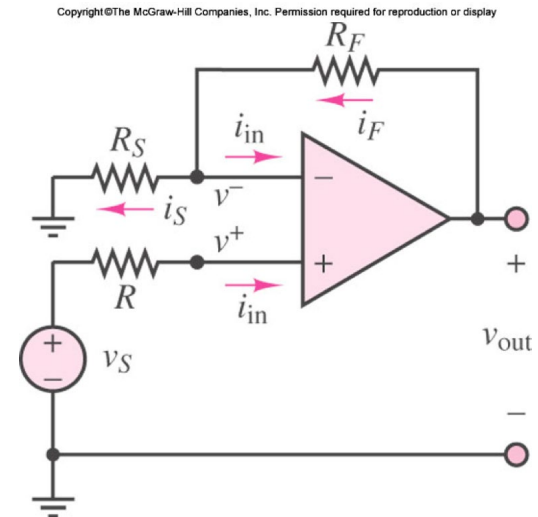
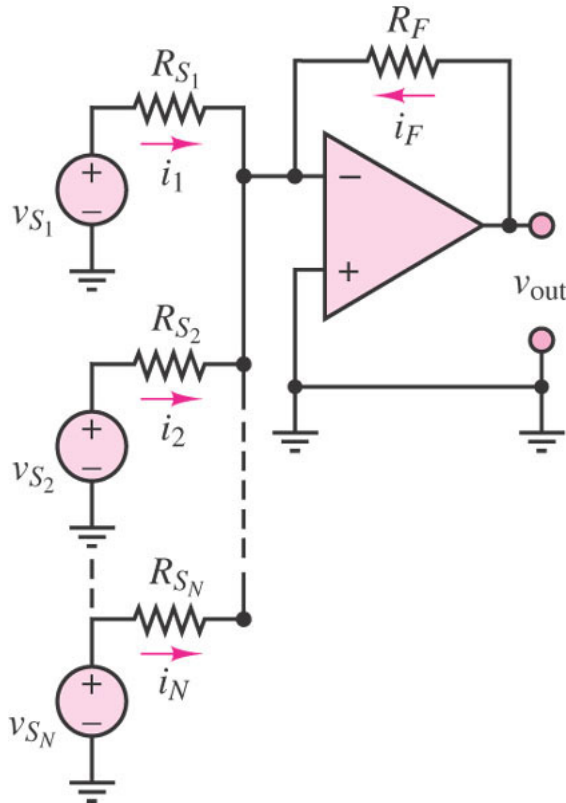


Figure 5 : Non-inverting amplifier

Summing amplifier

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$$v_{out} = - \left(\frac{R_F}{R_{S1}} v_{S1} + \frac{R_F}{R_{S2}} v_{S2} + \dots + \frac{R_F}{R_{SN}} v_{SN} \right)$$

- This circuit is very similar to the inverting amplifier

Figure 6 : Summing amplifier

Summing amplifier

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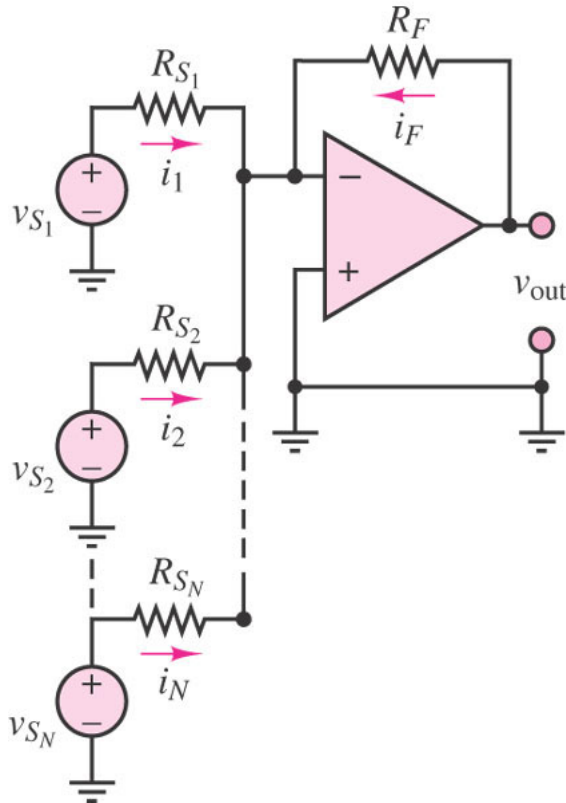


Figure 6 : Summing amplifier

- Note that since $v^+ = 0$, therefore $v^- = 0$
- Therefore:

$$i_1 = \frac{v_{S1}}{R_{S1}}, i_2 = \frac{v_{S2}}{R_{S2}}, \dots, i_N = \frac{v_{SN}}{R_{SN}}$$

- Applying KCL at v^- :

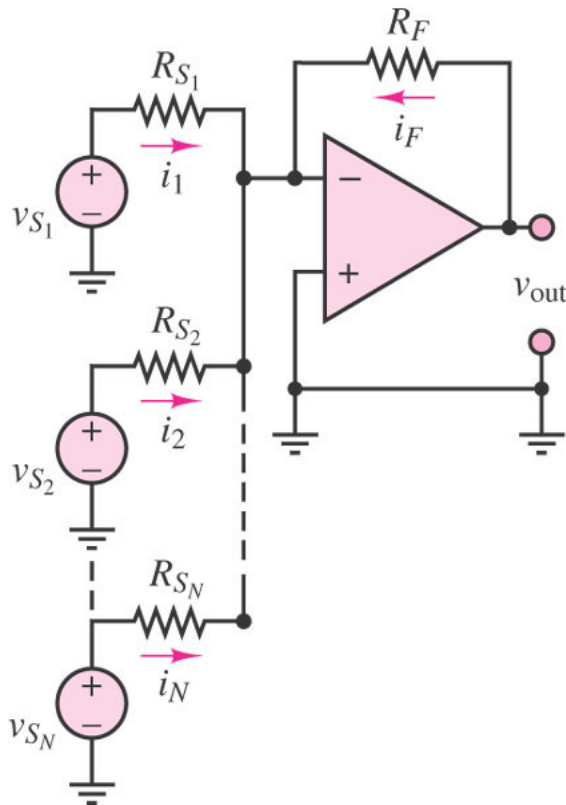
$$i_F + i_1 + i_2 + \dots + i_N = 0$$

- Hence by nodal voltage analysis:

$$\frac{v_{out}}{R_F} + \frac{v_{S1}}{R_{S1}} + \frac{v_{S2}}{R_{S2}} + \dots + \frac{v_{SN}}{R_{SN}} = 0$$

Summing amplifier

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- Rearrange the above equation, we obtain:

$$v_{out} = - \left(\frac{R_F}{R_{S1}} v_{S1} + \frac{R_F}{R_{S2}} v_{S2} + \dots + \frac{R_F}{R_{SN}} v_{SN} \right)$$

Note that as the name suggests, the function of this amplifier circuit is to amplify each input voltage and add them all up at the output. This sum however is **inverted relative to the input** as indicated by the negative sign.

Figure 6 : Summing amplifier

Example I: Inverting Amplifier

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If $v_i = 0.5$ V, calculate:

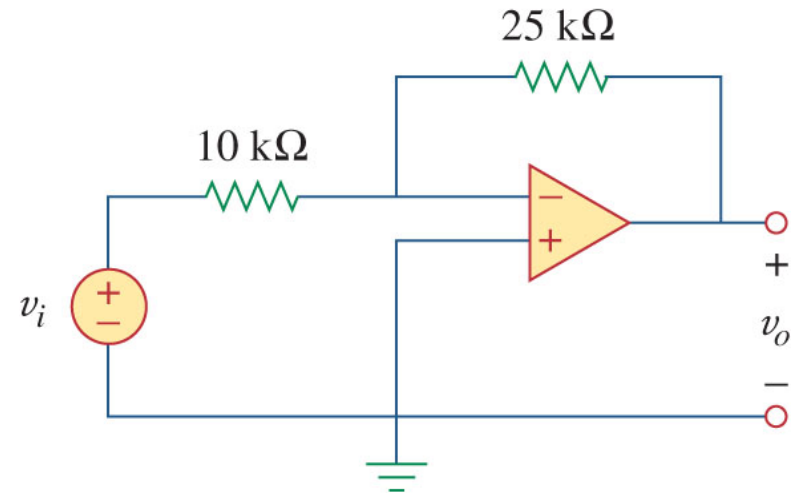
- (a) the output voltage v_o
- (b) the current in the 10-k Ω resistor.

(a) $V^- = V^+ = 0$ V

$$\frac{v_i - V^-}{10000} = \frac{V^- - v_o}{25000} \Rightarrow \frac{v_i}{10} = \frac{-v_o}{25}$$
$$\Rightarrow v_o = -1.25 \text{ V}$$

(b)

$$i_{10k\Omega} = \frac{v_i - 0}{10000} = 50 \mu\text{A}$$



Example II: Non-inverting Amplifier

Calculate the output voltage v_o

On the input side, by voltage divider rule

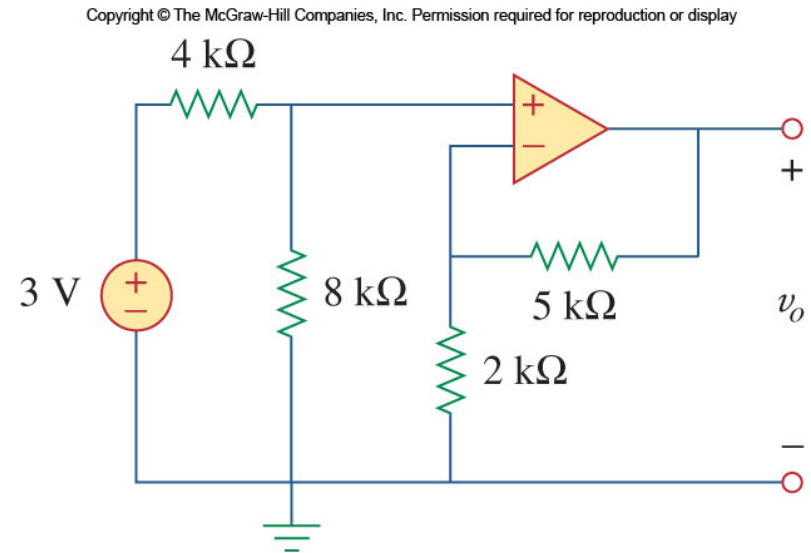
$$V^+ = 3 \times \frac{8}{4 + 8} = 2V$$

$$V^- = V^+ = 2V$$

On the output side, by voltage divider rule

$$v_o \frac{2}{2 + 5} = V^-$$

$$v_o = 2 \times \frac{7}{2} = 7V$$



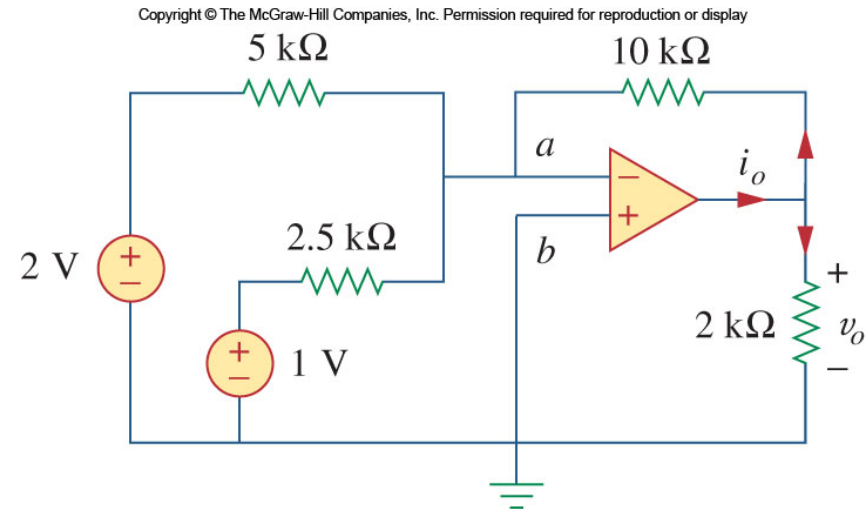
Example III: Summing Amplifier

Calculate v_o and i_o in the op amp circuit.

$$v_a = v_b = 0V$$

$$\begin{aligned}\frac{2 - v_a}{5000} + \frac{1 - v_a}{2500} &= \frac{(v_a - v_o)}{10000} \\ \Rightarrow 4 + 4 &= -v_o \\ \Rightarrow v_o &= -8V\end{aligned}$$

$$i_o = \frac{v_o}{2000} + \frac{v_o - v_a}{10000} = \frac{-8}{2000} + \frac{-8 - 0}{10000} = -0.0048 = -4.8\text{mA}$$



III. Differential amplifier

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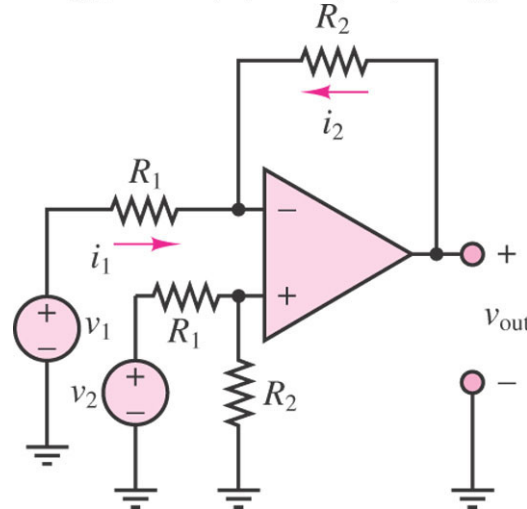


Figure 7 : Differential amplifier

- This circuit is very useful for building instruments that make fine measurements
- A differential amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs

Analysis of differential amplifier I

- Apply KCL at inverting terminal
- Assuming no current into/out of op amp inverting input (v^-)

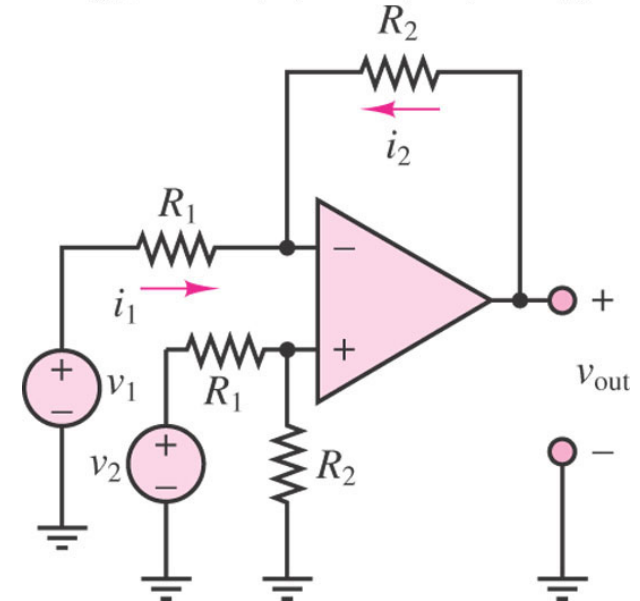
$$i_1 + i_2 = 0$$

$$\rightarrow \frac{v_1 - v^-}{R_1} + \frac{v_{out} - v^-}{R_2} = 0$$

$$\left(\frac{v_1}{R_1} + \frac{v_{out}}{R_2} \right) / \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = v^-$$

$$\left(\frac{R_2}{R_1 + R_2} \right) v_1 + \left(\frac{R_1}{R_1 + R_2} \right) v_{out} = v^- \quad (1)$$

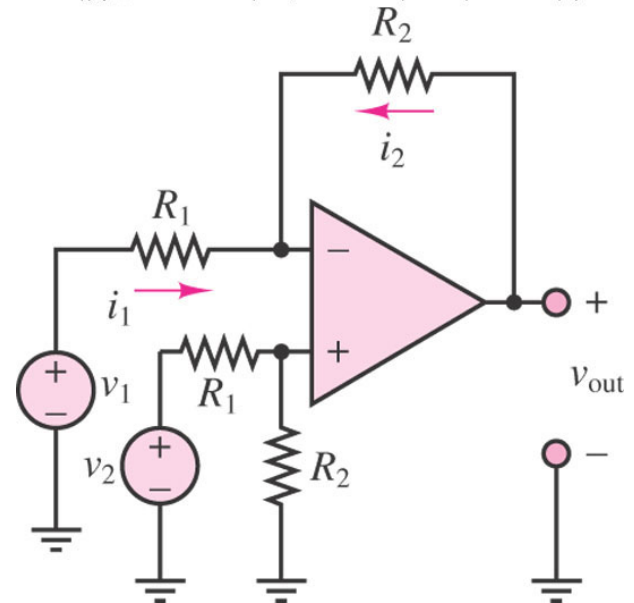
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Analysis of differential amplifier I

- At the non-inverting input (v^+) of op amp:
- Assuming no current into/out of non-inverting input of op amp
 - R_1 and R_2 appear in series
 - Apply voltage divider rule to R_1 and R_2 :

$$v^+ = \left(\frac{R_2}{R_1 + R_2} \right) v_2 \quad (2)$$



Operational amplifier internal circuit

- Finally, assume $v^- = v^+$ due to infinite open loop gain in negative feedback

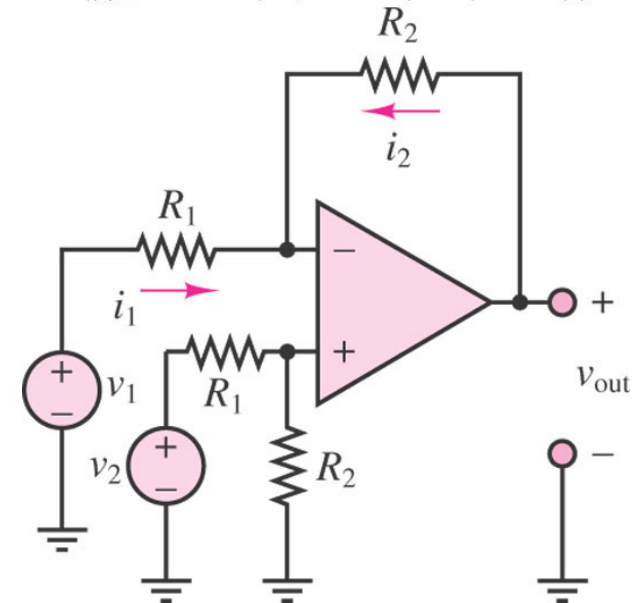
- Combine (1) and (2):

$$\left(\frac{R_2}{R_1 + R_2}\right)v_2 = \left(\frac{R_2}{R_1 + R_2}\right)v_1 + \left(\frac{R_1}{R_1 + R_2}\right)v_{out}$$

$$\left(\frac{R_1}{R_1 + R_2}\right)v_{out} = \left(\frac{R_2}{R_1 + R_2}\right)(v_2 - v_1)$$

$$v_{out} = \left(\frac{R_2}{R_1}\right)(v_2 - v_1)$$

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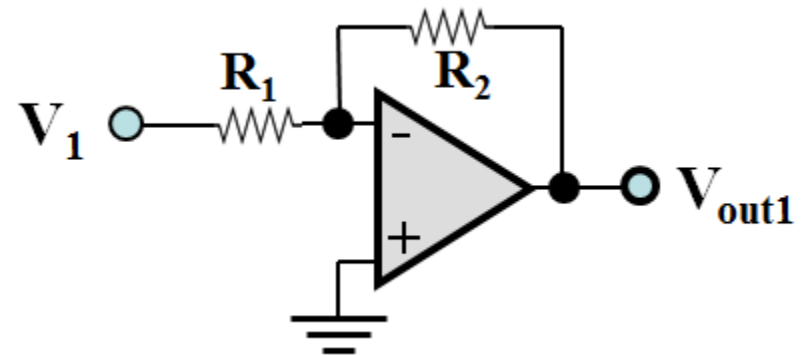
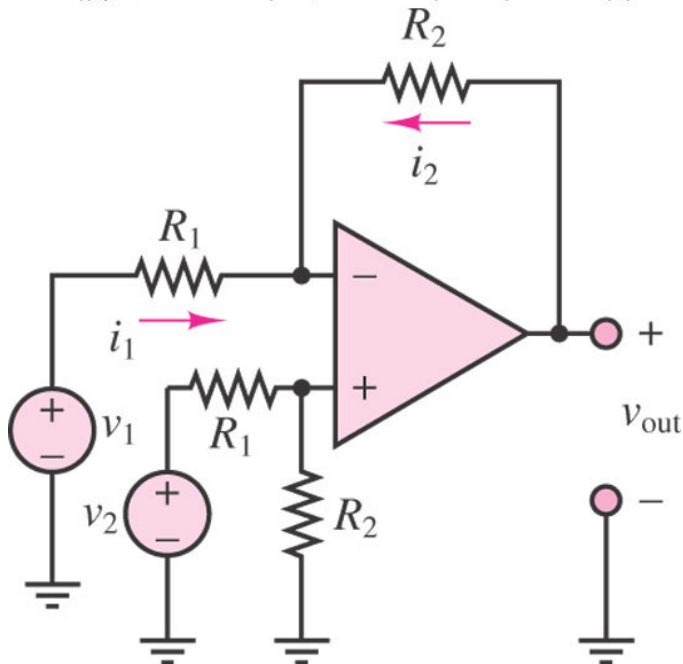
Note that the function of this op amp circuit is to take the difference between two input voltages and amplify them at the output.

Analysis of differential amplifier II

- Using superposition:

Short v_2

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- This is an inverting amp circuit
- Closed loop gain:

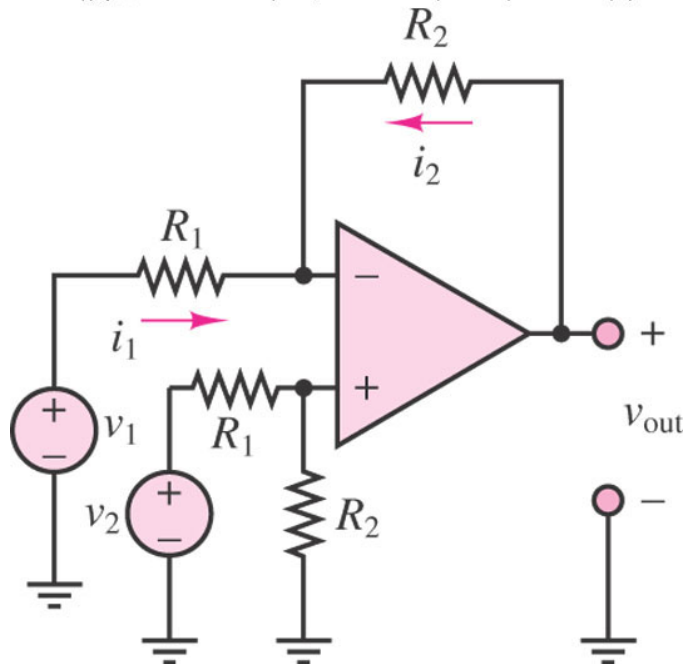
$$V_{out1} = -\frac{R_2}{R_1} V_1$$

Analysis of differential amplifier II

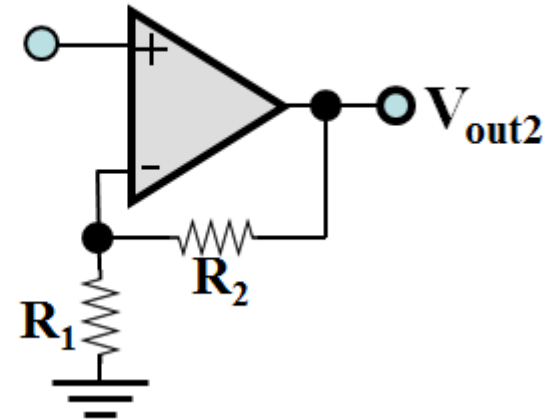
- Using superposition:

Short v_1

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$$\left[\frac{R_2}{R_1 + R_2} \right] V_2$$



- This is a non-inverting amp circuit
- Closed loop gain is $1 + (R_2/R_1)$ with input of $\left[\frac{R_2}{R_1 + R_2} \right] V_2$

$$\begin{aligned} V_{out2} &= \left(1 + \frac{R_2}{R_1} \right) \left(\frac{R_2}{R_1 + R_2} V_2 \right) \\ &= \frac{R_2}{R_1} v_2 \end{aligned}$$

Analysis of differential amplifier II

- Finally, add up the output voltages

$$\rightarrow V_{out} = V_{out1} + V_{out2}$$

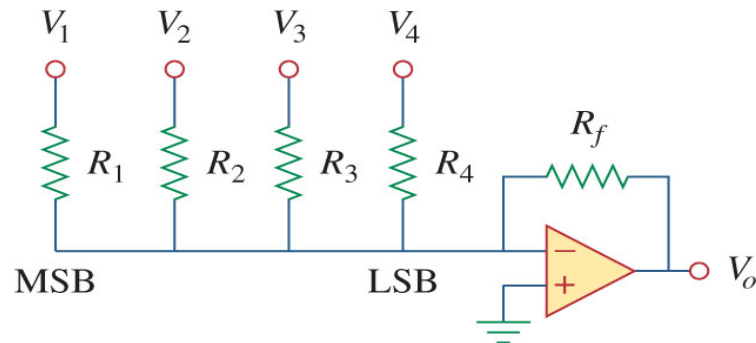
$$V_{out1} = -\frac{R_2}{R_1} v_1$$

$$\begin{aligned} V_{out2} &= \left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_2}{R_1 + R_2} v_2\right) \\ &= \frac{R_2}{R_1} v_2 \end{aligned}$$

$$V_{out} = \frac{R_2}{R_1} (v_2 - v_1)$$

Application: Digital-to-Analog Converter

- The following is circuit used for transforming digital signals into analog form, i.e. digital-to analog converter (DAC)
- The four-bit DAC is realized in the form of *binary weighted ladder* using the circuit below. The bits are weights according to the magnitude of R_f/R_n , where $R_f/R_1 > R_f/R_2 > R_f/R_3 > R_f/R_4$, such that each lesser bit has half the weight of the next higher bit



- Inputs V_1 to V_4 represent the most (MSB) to least significant bit (LSB)
- Inputs V_1 to V_4 can assume only a logic low (0 V) or a logic high (1 V) level

Digital-to-Analog Converter

- The previous circuit is an inverting summing amplifier

$$-V_o = \frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 + \frac{R_f}{R_4} V_4$$

- Let $R_f = 10 \text{ k}\Omega$, $R_1 = 10 \text{ k}\Omega$, $R_2 = 20 \text{ k}\Omega$, $R_3 = 40 \text{ k}\Omega$, $R_4 = 80 \text{ k}\Omega$, the binary inputs of [0000], [0001], [0010], and [1111] will produce analog outputs of 0 V, 0.125 V, 0.25 V, and 1.875 V.

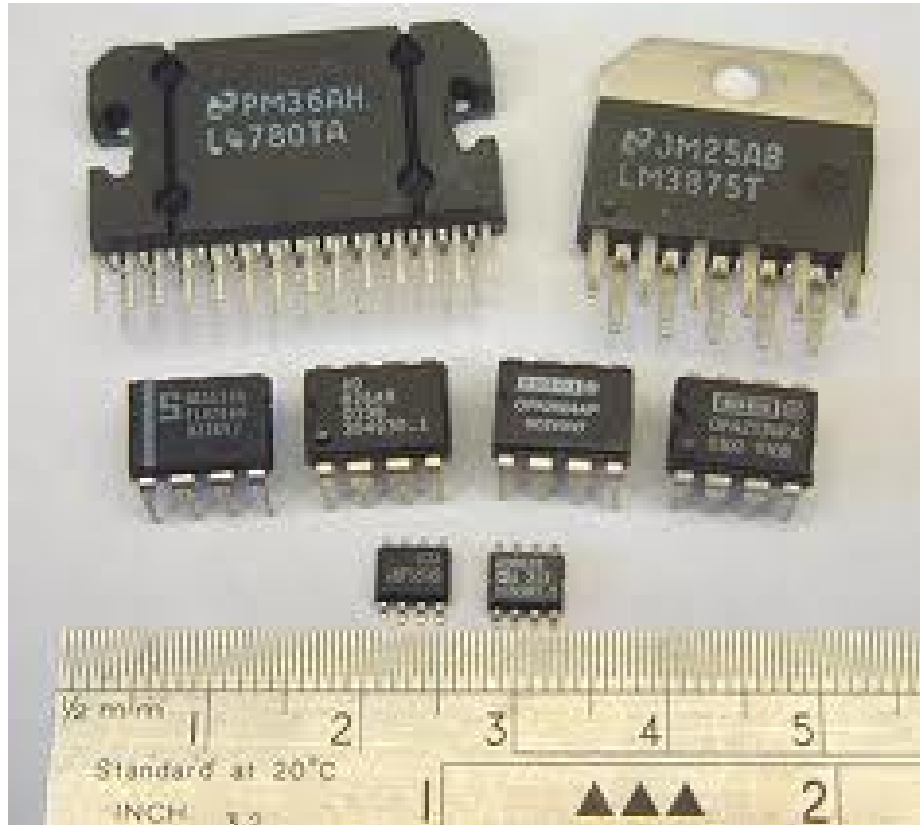
$$[V_1 V_2 V_3 V_4] = [0000] \\ -V_o =$$

$$[V_1 V_2 V_3 V_4] = [0001] \\ -V_o =$$

$$[V_1 V_2 V_3 V_4] = [0010] \\ -V_o =$$

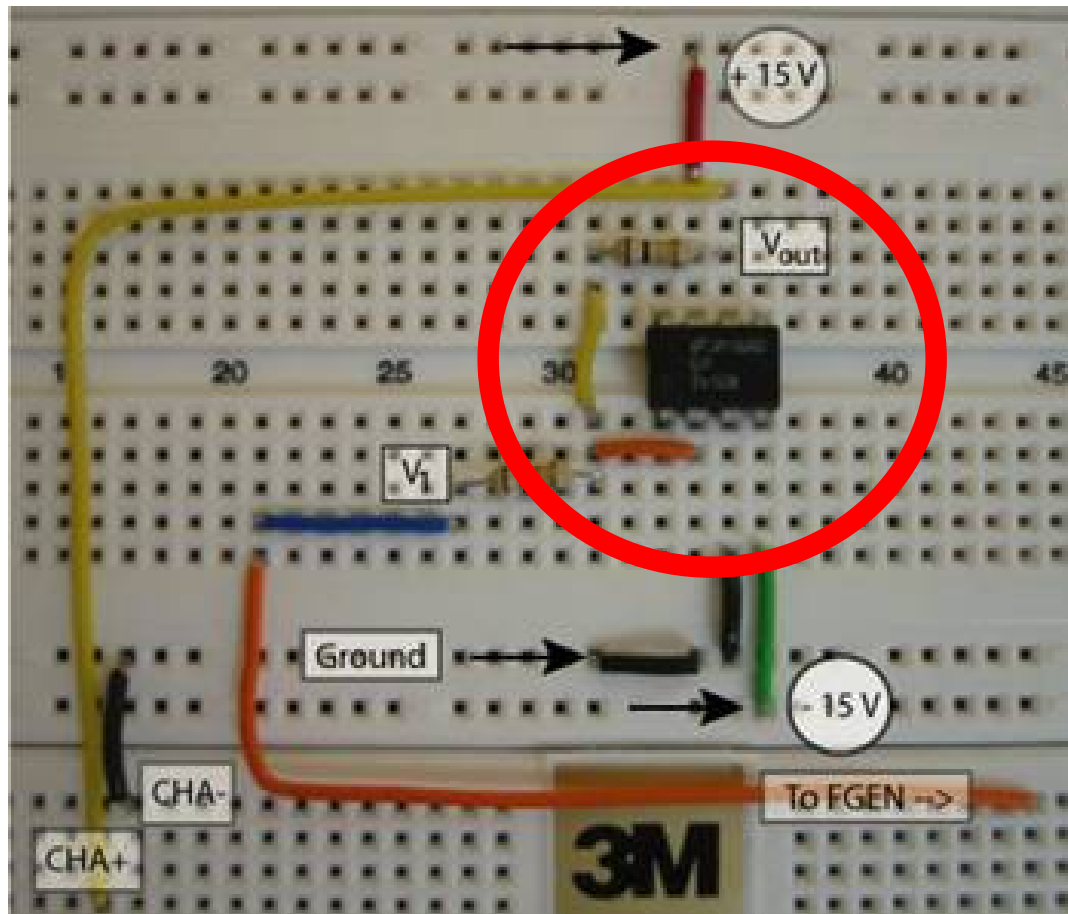
$$[V_1 V_2 V_3 V_4] = [1111] \\ -V_o =$$

Op amp wears different clothes



They come in different shapes and sizes

Op amp connects on breadboard



Op amp specification sheet



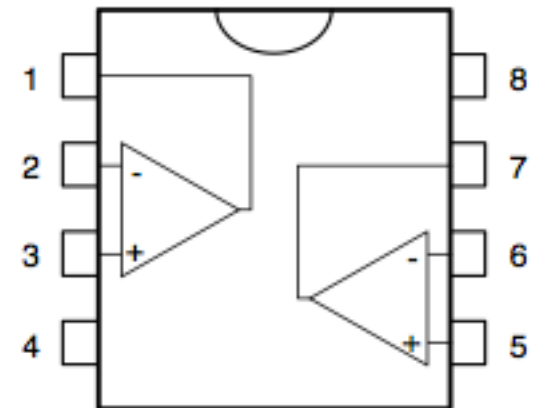
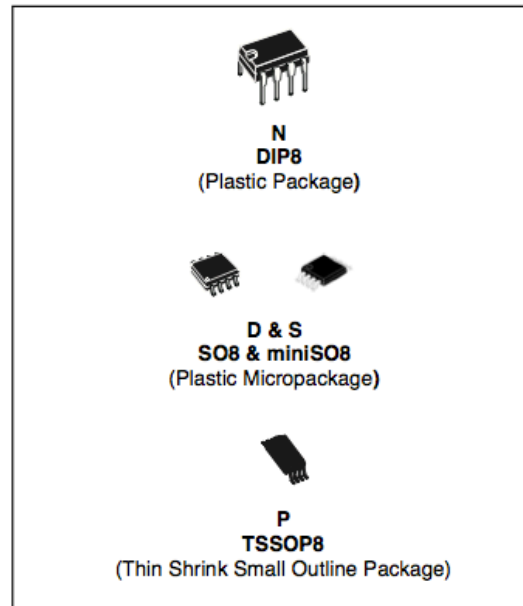
**LM158,A-LM258,A
LM358,A**

LOW POWER DUAL OPERATIONAL AMPLIFIERS

- INTERNALLY FREQUENCY COMPENSATED
- LARGE DC VOLTAGE GAIN: 100dB
- WIDE BANDWIDTH (unity gain): 1.1MHz (temperature compensated)
- VERY LOW SUPPLY CURRENT/OP (500 μ A) ESSENTIALLY INDEPENDENT OF SUPPLY VOLTAGE
- LOW INPUT BIAS CURRENT: 20nA (temperature compensated)
- LOW INPUT OFFSET VOLTAGE: 2mV
- LOW INPUT OFFSET CURRENT: 2nA
- INPUT COMMON-MODE VOLTAGE RANGE INCLUDES GROUND
- DIFFERENTIAL INPUT VOLTAGE RANGE EQUAL TO THE POWER SUPPLY VOLTAGE
- LARGE OUTPUT VOLTAGE SWING 0V TO ($V_{CC} - 1.5V$)

DESCRIPTION

These circuits consist of two independent, high gain, internally frequency compensated which were designed specifically to operate from a single power supply over a wide range of voltages. The low power supply drain is independent of the magnitude of the power supply voltage.



- 1 - Output 1
- 2 - Inverting input
- 3 - Non-inverting input
- 4 - V_{CC}^-
- 5 - Non-inverting input 2
- 6 - Inverting input 2
- 7 - Output 2
- 8 - V_{CC}^+

IV. Op amps in real life

- Op amps are actually electronic chips that are kept in a package. The various terminals of the op amp are then connected to different pins on the package.
- The figure below shows one example:

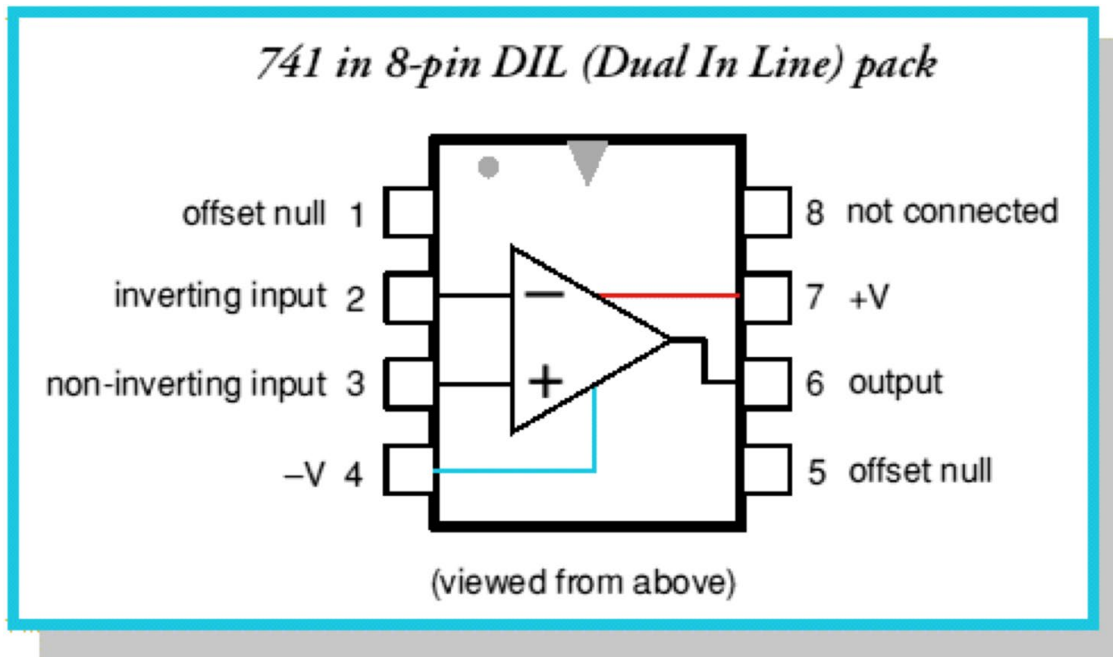


Figure 8: Symbol of an op amp

IV. Op amps in real life

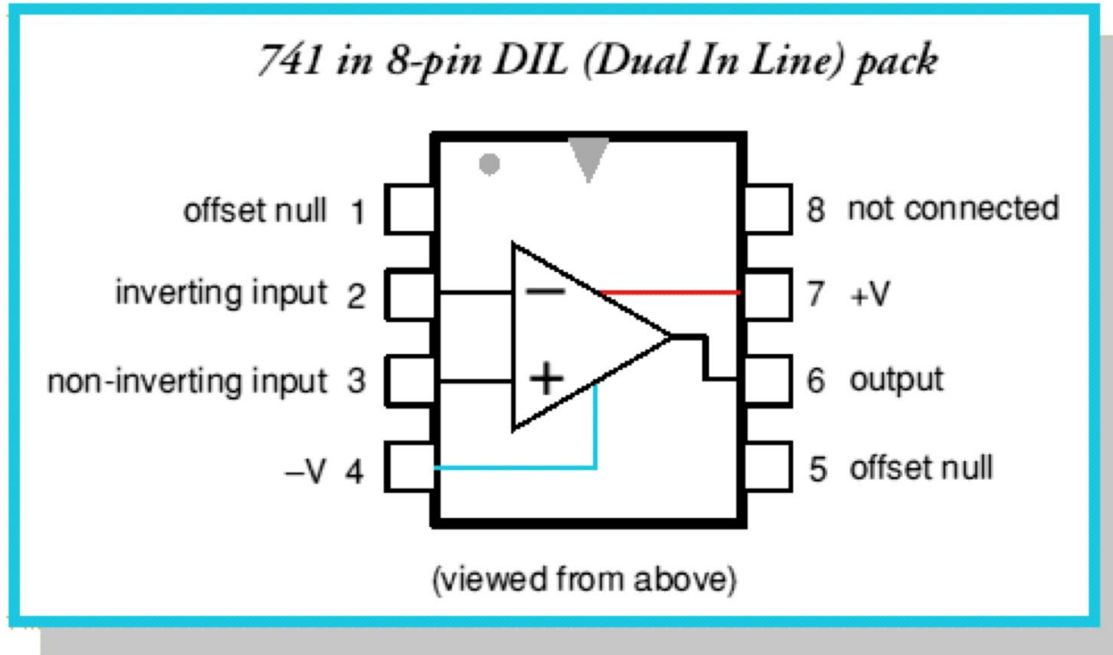


Figure 8: Symbol of an op amp

- One may see that the package has 8 pins, of which only 7 are used (1 pin has no connection – pin 8). To build the circuits mentioned so far, we only need to consider 5 of the pins.
- The pins -V and +V each refer to the negative and the positive supply. For the op amp to work, power needs to be supplied to the -V and +V pins.

Power Supply

- The following figure shows an inverting amplifier with the power supply connections included
- You might realize that we did not consider the power supply pins when deriving expressions for v_{out}

$$G = \frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

- In practice, we need to supply power otherwise the op amp will not work and will not get these expressions of V_{out} .
- So what then is the effect of the power supply pins apart from giving power to the op amp for it work?

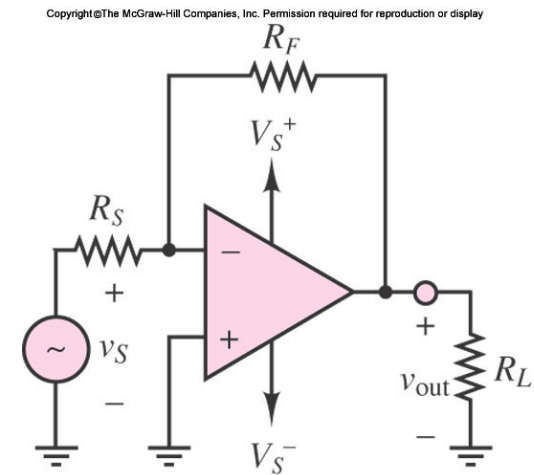


Figure 9: Inverting op amp with power supply

V. Saturation

- From the expression of $V_{\text{out}}/V_{\text{in}}$, you cannot see whether there is a limit on the maximum V_{out} . In a real op amp, V_{out} cannot exceed either of the supply voltages
- Hence if $V_{\text{S}+}$ is 15V and $V_{\text{S}-}$ is -15V, then V_{out} cannot be higher than 15V. It also cannot be lower than -15V

Example on saturation

- You have built an inverting op amp with gain of 10 and you use power supply of 15V to the op amp

Case 1: $V_S = 2V$

If we multiply 2V by -10, V_{out} should be -20V. This is lower than the negative supply. So V_{out} will be -15V. In the labs, you will find that the minimum V_{out} does not even reach V_S^- .

Case 2: $V_S = -2V$

If we multiply -2V by -10, V_{out} should be +20V. This is higher than the positive supply. Thus V_{out} will be +15V. In the labs, you will find that the maximum V_{out} does not even reach V_S^+ .

Example on saturation

Case 3: $V_s = 2 \sin(2000t)$ V

When V_{out} reaches V_{s+} or V_{s-} , the output gets chopped off.

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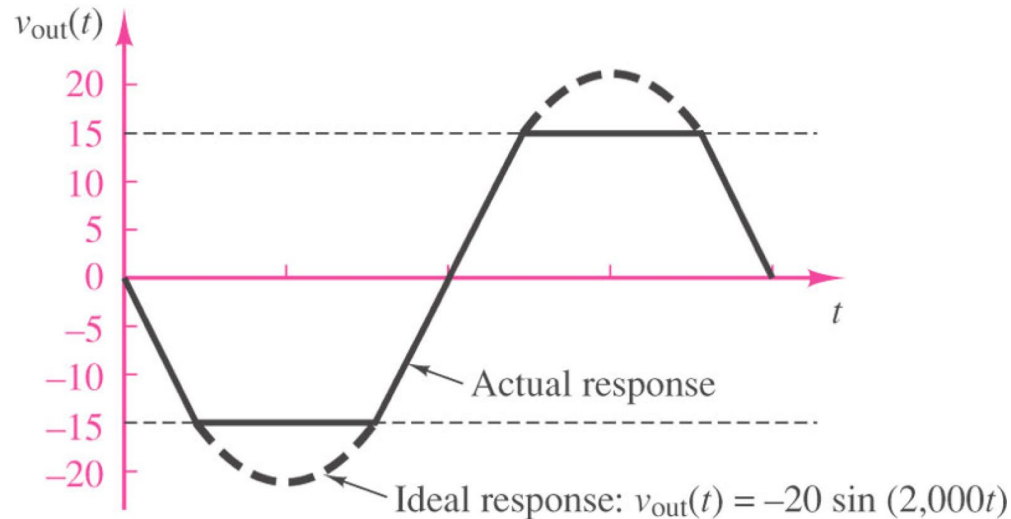


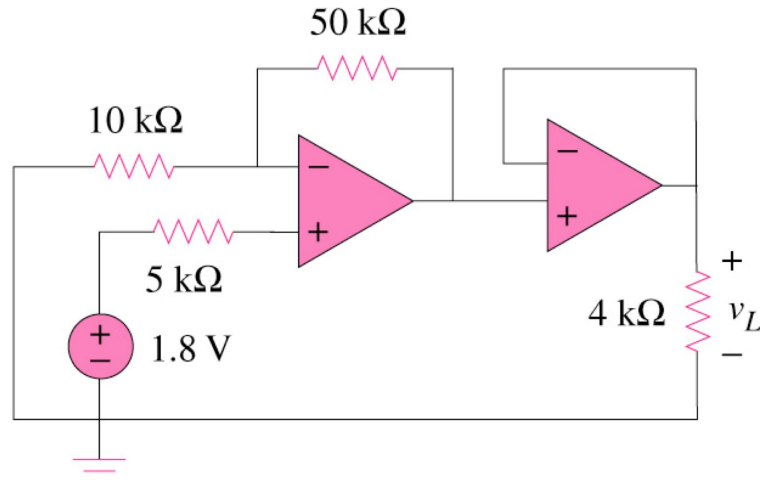
Figure 10. Saturated output

Op amps are only capable of amplifying signals within the range of their supply voltages.

The output cannot exceed the supply voltages while the input is also capped by the supply voltages.

VI. Cascaded amplifiers 1

Identify the types of amplifiers in the circuit below. Find v_L .



First stage: Non-inverting amp
Second stage: Source follower

$$v_L \times \frac{10}{10 + 50} = 1.8 \text{ V}$$
$$v_L = 10.8 \text{ V}$$

Cascaded amplifiers 2

Find v_o and i_o in the circuit.

Solution:

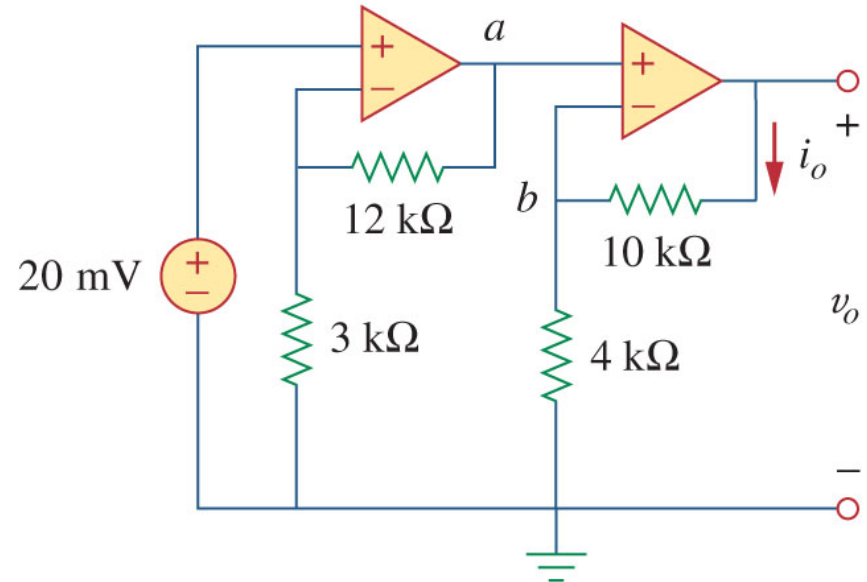
$$V_a \times \frac{3}{12 + 3} = 20 \text{ mV}$$
$$\Rightarrow V_a = 0.1 \text{ V}$$

$$V_b = V_a = 0.1 \text{ V}$$

$$v_o \times \frac{4}{10 + 4} = V_b$$
$$\Rightarrow v_o = 0.35 \text{ V}$$

$$i_o = \frac{v_o}{10000 + 4000} = 25 \mu\text{A}$$

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Summary

- (1) Remember the 3 characteristics of an ideal op amp
- (2) The output voltage is limited by the power supply
- (3) Op amps can be cascaded and combined