

EE3109 Applied Electromagnetics
2022/23 Semester B
Week 3: Tutorial Solution

Q1. Sol'n.

7.1 Write general expressions for the electric and magnetic fields of a 1 GHz sinusoidal plane wave traveling in the $+y$ -direction in a lossless nonmagnetic medium with relative permittivity $\epsilon_r = 9$. The electric field is polarized along the x -direction, its peak value is 6 V/m, and its intensity is 4 V/m at $t = 0$ and $y = 2$ cm.

Solution: For $f = 1$ GHz, $\mu_r = 1$, and $\epsilon_r = 9$,

$$\omega = 2\pi f = 2\pi \times 10^9 \text{ rad/s},$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\lambda_0} \sqrt{\epsilon_r} = \frac{2\pi f}{c} \sqrt{\epsilon_r} = \frac{2\pi \times 10^9}{3 \times 10^8} \sqrt{9} = 20\pi \text{ rad/m},$$

$$\mathbf{E}(y, t) = \hat{\mathbf{x}} 6 \cos(2\pi \times 10^9 t - 20\pi y + \phi_0) \quad (\text{V/m}).$$

At $t = 0$ and $y = 2$ cm, $E = 4$ V/m:

$$4 = 6 \cos(-20\pi \times 2 \times 10^{-2} + \phi_0) = 6 \cos(-0.4\pi + \phi_0).$$

Hence,

$$\phi_0 - 0.4\pi = \cos^{-1}\left(\frac{4}{6}\right) = 0.84 \text{ rad},$$

which gives

$$\phi_0 = 2.1 \text{ rad} = 120.19^\circ$$

and

$$\mathbf{E}(y, t) = \hat{\mathbf{x}} 6 \cos(2\pi \times 10^9 t - 20\pi y + 120.19^\circ) \quad (\text{V/m}).$$

Q1. Add-on:

$$H(y, t) = -\hat{\mathbf{z}} \frac{6}{\eta} \cos(2\pi \times 10^9 t - 20\pi y + 120.19^\circ)$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{9}} = 40\pi \, \Omega$$

$$\Rightarrow H(y, t) = -\hat{\mathbf{z}} 47.7 \cos(2\pi \times 10^9 t - 20\pi y + 120.19^\circ) \text{ [mA/m]}$$

7. A plane wave decays in a low-loss medium at a rate of 10 [Np/km].
- How far does it have to travel for the electric field to decay to 10% of its original value?
 - Assume that the material has $\mu_r = 1$ and $\epsilon_r = 9$. What is the conductivity of the material?

Solution

- (a) We first define the propagation direction as $\hat{z}$ and the $\mathbf{E}$ field direction as $\hat{y}$, such that at a known location z , $E = E_0 e^{-\alpha z} \hat{y}$. At some reference location $z = 0$, $E(z = 0) = E_0$. At $z = \Delta z$, $E(z = \Delta z) = E_0 e^{-\alpha \Delta z} \hat{y}$. We seek for a distance Δz such that

$$0.1 = \frac{E(z = \Delta z)}{E(z = 0)} = e^{-\alpha \Delta z} = e^{-(10) \Delta z}$$

Note that Δz is now in [km]. Crunching the numbers gives $\Delta z = 0.2303 \text{ [km]} = 230.3 \text{ [m]}$.

- (b) For a low-loss wave:

$$\alpha = \frac{\sigma}{2} \sqrt{\frac{\mu}{\epsilon}} \Rightarrow \sigma = \frac{2\alpha}{\sqrt{\frac{\mu}{\epsilon}}} = \frac{2(10 \times 10^{-3})(3)}{377} = 1.592 \times 10^{-4} \text{ [S/m]} = 159.2 \text{ [\mu S/m]} .$$

Q2. Sol'n.

7.2 The magnetic field of a wave propagating through a certain nonmagnetic material is given by

$$\mathbf{H} = \hat{\mathbf{z}} 30 \cos(10^8 t - 0.5y) \quad (\text{mA/m})$$

Find the following:

- (a) The direction of wave propagation.
- (b) The phase velocity.
- (c) The wavelength in the material.
- (d) The relative permittivity of the material.
- (e) The electric field phasor.

Solution:

- (a) Positive y-direction.
- (b) $\omega = 10^8 \text{ rad/s}$, $k = 0.5 \text{ rad/m}$.

$$u_p = \frac{\omega}{k} = \frac{10^8}{0.5} = 2 \times 10^8 \text{ m/s}.$$

(c) $\lambda = 2\pi/k = 2\pi/0.5 = 12.6 \text{ m}$.

(d) $\epsilon_r = \left(\frac{c}{u_p} \right)^2 = \left(\frac{3 \times 10^8}{2 \times 10^8} \right)^2 = 2.25$.

(e) From Eq. (7.39b),

$$\tilde{\mathbf{E}} = -\eta \hat{\mathbf{k}} \times \tilde{\mathbf{H}},$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{120\pi}{1.5} = 251.33 \quad (\Omega),$$

$$\hat{\mathbf{k}} = \hat{\mathbf{y}}, \quad \text{and} \quad \tilde{\mathbf{H}} = \hat{\mathbf{z}} 30 e^{-j0.5y} \times 10^{-3} \quad (\text{A/m}).$$

Hence,

$$\tilde{\mathbf{E}} = -251.33 \hat{\mathbf{y}} \times \hat{\mathbf{z}} 30 e^{-j0.5y} \times 10^{-3} = -\hat{\mathbf{x}} 7.54 e^{-j0.5y} \quad (\text{V/m}),$$

and

$$\mathbf{E}(y, t) = \Re(\tilde{\mathbf{E}} e^{j\omega t}) = -\hat{\mathbf{x}} 7.54 \cos(10^8 t - 0.5y) \quad (\text{V/m}).$$

Q3. Sol'n.

7.3 The electric field phasor of a uniform plane wave is given by $\tilde{\mathbf{E}} = \hat{\mathbf{y}} 10e^{j0.2z}$ (V/m). If the phase velocity of the wave is 1.5×10^8 m/s and the relative permeability of the medium is $\mu_r = 2.4$, find the following:

- (a) The wavelength.
- (b) The frequency f of the wave.
- (c) The relative permittivity of the medium.
- (d) The magnetic field $\mathbf{H}(z, t)$.

Solution:

(a) From $\tilde{\mathbf{E}} = \hat{\mathbf{y}} 10e^{j0.2z}$ (V/m), we deduce that $k = 0.2$ rad/m. Hence,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2} = 10\pi = 31.42 \text{ m.}$$

(b)

$$f = \frac{u_p}{\lambda} = \frac{1.5 \times 10^8}{31.42} = 4.77 \times 10^6 \text{ Hz} = 4.77 \text{ MHz.}$$

(c) From

$$u_p = \frac{c}{\sqrt{\mu_r \epsilon_r}}, \quad \epsilon_r = \frac{1}{\mu_r} \left(\frac{c}{u_p} \right)^2 = \frac{1}{2.4} \left(\frac{3}{1.5} \right)^2 = 1.67.$$

(d)

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \simeq 120\pi \sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi \sqrt{\frac{2.4}{1.67}} = 451.94 \quad (\Omega),$$

$$\tilde{\mathbf{H}} = \frac{1}{\eta} (-\hat{\mathbf{z}}) \times \tilde{\mathbf{E}} = \frac{1}{\eta} (-\hat{\mathbf{z}}) \times \hat{\mathbf{y}} 10e^{j0.2z} = \hat{\mathbf{x}} 22.13e^{j0.2z} \quad (\text{mA/m}),$$

$$\mathbf{H}(z, t) = \hat{\mathbf{x}} 22.13 \cos(\omega t + 0.2z) \quad (\text{mA/m}),$$

with $\omega = 2\pi f = 9.54\pi \times 10^6$ rad/s.

Q4. Sol'n.

7.5 A wave radiated by a source in air is incident upon a soil surface, whereupon a part of the wave is transmitted into the soil medium. If the wavelength of the wave is 60 cm in air and 15 cm in the soil medium, what is the soil's relative permittivity? Assume the soil to be a very low-loss medium.

Solution: From $\lambda = \lambda_0 / \sqrt{\epsilon_r}$,

$$\epsilon_r = \left(\frac{\lambda_0}{\lambda} \right)^2 = \left(\frac{60}{15} \right)^2 = 16.$$

Q5. Sol'n.

7.7 The electric field of a plane wave propagating in a lossless, nonmagnetic, dielectric material with $\epsilon_r = 2.56$ is given by

$$\mathbf{E} = \hat{\mathbf{y}} 5 \cos(6\pi \times 10^9 t - kz) \quad (\text{V/m})$$

Determine:

- (a) f , u_p , λ , k , and η .
- (b) The magnetic field $\mathbf{H}$.

Solution:

(a)

$$\omega = 2\pi f = 6\pi \times 10^9 \text{ rad/s},$$

$$f = 3 \times 10^9 \text{ Hz} = 3 \text{ GHz},$$

$$u_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.56}} = 1.875 \times 10^8 \text{ m/s},$$

$$\lambda = \frac{u_p}{f} = \frac{1.875 \times 10^8}{3 \times 10^9} = 6.24 \text{ cm},$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6.24 \times 10^{-2}} = 100.8 \text{ rad/m},$$

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{377}{\sqrt{2.56}} = \frac{377}{1.6} = 235.62 \text{ } \Omega.$$

(b)

$$\begin{aligned} \mathbf{H} &= -\hat{\mathbf{x}} \frac{5}{\eta} \cos(6\pi \times 10^9 t - kz) \\ &= -\hat{\mathbf{x}} \frac{5}{235.62} \cos(6\pi \times 10^9 t - 100.8z) \\ &= -\hat{\mathbf{x}} 2.12 \times 10^{-2} \cos(6\pi \times 10^9 t - 100.8z) \quad (\text{A/m}). \end{aligned}$$

Q6. Sol'n.

7.11 The magnetic field of a uniform plane wave propagating in a dielectric medium with $\epsilon_r = 36$ is given by

$$\tilde{\mathbf{H}} = 60(\hat{\mathbf{y}} + j\hat{\mathbf{z}})e^{-j\pi x/6} \quad (\text{mA/m}).$$

Specify the modulus and direction of the electric field intensity at the $x = 0$ plane at $t = 0$ and 5 ns.

Solution: From the expression for $\tilde{\mathbf{H}}$, we deduce that the wave is traveling in the $+x$ direction and that

$$k = \frac{\pi}{6} \text{ rad/m.}$$

Hence,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi} \times 6 = 12 \text{ m.}$$

Also,

$$u_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{36}} = 0.5 \times 10^8 \text{ m/s.}$$

Hence,

$$f = \frac{u_p}{\lambda} = \frac{0.5 \times 10^8}{12} = 4.17 \times 10^6 = 4.17 \text{ MHz,}$$

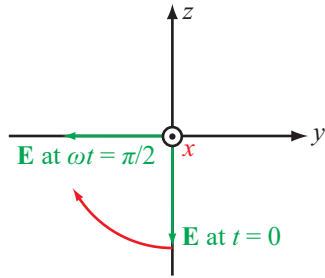
and

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{377}{\sqrt{36}} = 62.83 \text{ } \Omega.$$

From Eq. (7.39b),

$$\begin{aligned} \tilde{\mathbf{E}} &= -\eta \hat{\mathbf{k}} \times \tilde{\mathbf{H}} \\ &= -62.83 \hat{\mathbf{x}} \times [60(\hat{\mathbf{y}} + j\hat{\mathbf{z}})e^{-j\pi x/6}] \times 10^{-3} \\ &= 3.77(-\hat{\mathbf{z}} + \hat{\mathbf{y}}e^{j\pi/2})e^{-j\pi x/6} \quad (\text{V/m}), \\ \mathbf{E}(x, t) &= \Re \{ \tilde{\mathbf{E}} e^{j\omega t} \} \\ &= 3.77 \left[-\hat{\mathbf{y}} \sin \left(\omega t - \frac{\pi x}{6} \right) - \hat{\mathbf{z}} \cos \left(\omega t - \frac{\pi x}{6} \right) \right] \quad (\text{V/m}), \\ |\mathbf{E}| &= [E_y^2 + E_z^2]^{1/2} = 3.77 \text{ V/m.} \end{aligned}$$

This is a LHC wave.



At $x = 0$ and $t = 0$, $\mathbf{E} = -\hat{\mathbf{z}}3.77$ (V/m).

At $x = 0$ and $t = 5$ ns,

$$\omega t = 2\pi \times 4.17 \times 10^6 \times 5 \times 10^{-9} = 0.13 \text{ rad},$$

$$\begin{aligned}\mathbf{E} &= -3.77(\hat{\mathbf{y}} \sin 0.13 + \hat{\mathbf{z}} \cos 0.13) \\ &= -3.77(\hat{\mathbf{y}}0.13 + \hat{\mathbf{z}}0.99) \quad (\text{V/m}).\end{aligned}$$



Q8. Sol'n.

7.17 For each of the following combinations of parameters, determine if the material is a low-loss dielectric, a quasi-conductor, or a good conductor, and then calculate α , β , λ , u_p , and η_c :

- (a) Glass with $\mu_r = 1$, $\epsilon_r = 5$, and $\sigma = 10^{-12}$ S/m at 10 GHz.
- (b) Animal tissue with $\mu_r = 1$, $\epsilon_r = 12$, and $\sigma = 0.3$ S/m at 100 MHz.
- (c) Wood with $\mu_r = 1$, $\epsilon_r = 3$, and $\sigma = 10^{-4}$ S/m at 1 kHz.

Solution: Using equations given in Table 7-1:

	Case (a)	Case (b)	Case (c)
$\sigma/\omega\epsilon$	3.6×10^{-13}	4.5	600
Type	low-loss dielectric	quasi-conductor	good conductor
α	8.42×10^{-11} Np/m	9.75 Np/m	6.3×10^{-4} Np/m
β	468.3 rad/m	12.16 rad/m	6.3×10^{-4} rad/m
λ	1.34 cm	51.69 cm	10 km
u_p	1.34×10^8 m/s	0.52×10^8 m/s	0.1×10^8 m/s
η_c	$\simeq 168.5 \Omega$	$39.54 + j31.72 \Omega$	$6.28(1 + j) \Omega$

Q9. Sol'n.

7.19 In a medium characterized by $\epsilon_r = 9$, $\mu_r = 1$, and $\sigma = 0.1$ S/m, determine the phase angle by which the magnetic field leads the electric field at 100 MHz.

Solution: The phase angle by which the magnetic field leads the electric field is $-\theta_\eta$ where θ_η is the phase angle of η_c .

$$\frac{\sigma}{\omega\epsilon} = \frac{0.1 \times 36\pi}{2\pi \times 10^8 \times 10^{-9} \times 9} = 2.$$

Hence, quasi-conductor.

$$\begin{aligned}\eta_c &= \sqrt{\frac{\mu}{\epsilon'}} \left(1 - j\frac{\epsilon''}{\epsilon'}\right)^{-1/2} = \frac{120\pi}{\sqrt{\epsilon_r}} \left(1 - j\frac{\sigma}{\omega\epsilon_0\epsilon_r}\right)^{-1/2} \\ &= 125.67(1 - j2)^{-1/2} = 71.49 + j44.18 = 84.04 \angle 31.72^\circ.\end{aligned}$$

Therefore $\theta_\eta = 31.72^\circ$.

Since $\mathbf{H} = (1/\eta_c)\hat{\mathbf{k}} \times \mathbf{E}$, $\mathbf{H}$ leads $\mathbf{E}$ by $-\theta_\eta$, or by -31.72° . In other words, $\mathbf{H}$ lags $\mathbf{E}$ by 31.72° .

Q10. Sol'n.

7.20 Ignoring reflection at the air-water boundary, if the amplitude of a 1 GHz incident wave in air is 20 V/m at the water surface, at what depth will it be down to 1 μ V/m? Water has $\mu_r = 1$ and at 1 GHz, $\epsilon_r = 80$ and $\sigma = 1$ S/m.

Solution: For water at 1 GHz,

$$\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2\pi f\epsilon_r\epsilon_0} = \frac{1}{2\pi \times 10^9 \times 80 \times 8.85 \times 10^{-12}} \\ = 0.225.$$

Hence, it is neither a low-loss nor a conducting medium. Using Eq. (7.66a):

$$\alpha = \omega \left\{ \frac{\mu\epsilon'}{2} \left[\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'} \right)^2} - 1 \right] \right\}^{1/2} \\ = \omega \left\{ \frac{\mu_r\epsilon_r'}{2c^2} \left[\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'} \right)^2} - 1 \right] \right\}^{1/2} \\ = 2\pi \times 10^9 \left\{ \frac{80}{2(3 \times 10^8)^2} \left[\sqrt{1 + (0.225)^2} - 1 \right] \right\}^{1/2} \\ = 20.94 \text{ Np/m.}$$

The depth d at which E is down to 1 μ V/m is obtained from

$$10^{-6} = 20e^{-\alpha d} = 20e^{-20.94d},$$

which leads to

$$\ln \left(\frac{10^{-6}}{20} \right) = -20.94d,$$

or

$$d = \frac{-16.81}{20.94} = 0.803 \text{ m} = 80.3\text{cm.}$$

Q11. Sol'n.

7.25 The electric field of a plane wave propagating in a nonmagnetic medium is given by

$$\mathbf{E} = \hat{\mathbf{z}} 25 e^{-30x} \cos(2\pi \times 10^9 t - 40x) \quad (\text{V/m})$$

Obtain the corresponding expression for $\mathbf{H}$.

Solution: From the given expression for $\mathbf{E}$,

$$\omega = 2\pi \times 10^9 \quad (\text{rad/s}),$$

$$\alpha = 30 \quad (\text{Np/m}),$$

$$\beta = 40 \quad (\text{rad/m}).$$

From (7.65a) and (7.65b),

$$\alpha^2 - \beta^2 = -\omega^2 \mu \epsilon' = -\omega^2 \mu_0 \epsilon_0 \epsilon_r' = -\frac{\omega^2}{c^2} \epsilon_r',$$

$$2\alpha\beta = \omega^2 \mu \epsilon'' = \frac{\omega^2}{c^2} \epsilon_r''.$$

Using the above values for ω , α , and β , we obtain the following:

$$\epsilon_r' = 1.6,$$

$$\epsilon_r'' = 5.47.$$

$$\begin{aligned} \eta_c &= \sqrt{\frac{\mu}{\epsilon'}} \left(1 - j \frac{\epsilon_r''}{\epsilon_r'} \right)^{-1/2} \\ &= \frac{\eta_0}{\sqrt{\epsilon_r'}} \left(1 - j \frac{\epsilon_r''}{\epsilon_r'} \right)^{-1/2} = \frac{377}{\sqrt{1.6}} \left(1 - j \frac{5.47}{1.6} \right)^{-1/2} = 157.9 e^{j36.85^\circ} \quad (\Omega). \end{aligned}$$

$$\tilde{\mathbf{E}} = \hat{\mathbf{z}} 25 e^{-30x} e^{-j40x},$$

$$\tilde{\mathbf{H}} = \frac{1}{\eta_c} \hat{\mathbf{k}} \times \tilde{\mathbf{E}} = \frac{1}{157.9 e^{j36.85^\circ}} \hat{\mathbf{x}} \times \hat{\mathbf{z}} 25 e^{-30x} e^{-j40x} = -\hat{\mathbf{y}} 0.16 e^{-30x} e^{-40x} e^{-j36.85^\circ},$$

$$\mathbf{H} = \Re\{\tilde{\mathbf{H}} e^{j\omega t}\} = -\hat{\mathbf{y}} 0.16 e^{-30x} \cos(2\pi \times 10^9 t - 40x - 36.85^\circ) \quad (\text{A/m}).$$
