

Tutorial 10

Magnetic Field due to Current

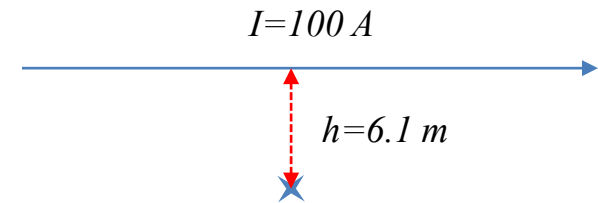
Practice Questions:

Chapter 29 - Magnetic Fields Due to Currents

Questions: *1, *7, *23, *56, *57, *68

*1

•1 A surveyor is using a magnetic compass 6.1 m below a power line in which there is a steady current of 100 A. (a) What is the magnetic field at the site of the compass due to the power line? (b) Will this field interfere seriously with the compass reading? The horizontal component of Earth's magnetic field at the site is $20 \mu\text{T}$.



(a) The magnitude of the magnetic field due to the current in the wire, at a point a distance r from the wire, is given by

$$B = \frac{\mu_0 i}{2\pi r}.$$

With $r = 6.10 \text{ m}$, we have

$$B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(100 \text{ A})}{2\pi(6.10 \text{ m})} = 3.3 \times 10^{-6} \text{ T} = 3.3 \mu\text{T}.$$

(b) This is about one-sixth the magnitude of the Earth's field. It will affect the compass reading.

*7

•7 In Fig. 29-38, two circular arcs have radii $a = 13.5$ cm and $b = 10.7$ cm, subtend angle $\theta = 74.0^\circ$, carry current $i = 0.411$ A, and share the same center of curvature P . What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at P ?

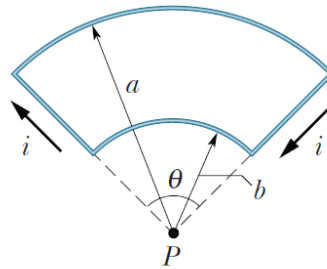


Fig. 29-38 Problem 7.

(a) Recalling the *straight sections* discussion in Sample Problem — “Magnetic field at the center of a circular arc of current,” we see that the current in the straight segments collinear with P *do not contribute to the field at that point*. Using Eq. 29-9 (with $\phi = \theta$) and the right-hand rule, we find that the current in the semicircular arc of radius b contributes $\mu_0 i \theta / 4\pi b$ (out of the page) to the field at P .

Also, the current in the large radius arc contributes $\mu_0 i \theta / 4\pi a$ (into the page) to the field there. Thus, the net field at P is

$$B = \frac{\mu_0 i \theta}{4} \left(\frac{1}{b} - \frac{1}{a} \right) = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.411 \text{ A})(74^\circ \cdot \pi/180^\circ)}{4\pi} \left(\frac{1}{0.107 \text{ m}} - \frac{1}{0.135 \text{ m}} \right)$$

$$= 1.02 \times 10^{-7} \text{ T}.$$

(b) The direction is out of the page.

*23

••23 ILW Figure 29-50 shows a snapshot of a proton moving at velocity $\vec{v} = (-200 \text{ m/s})\hat{j}$ toward a long straight wire with current $i = 350 \text{ mA}$. At the instant shown, the proton's distance from the wire is $d = 2.89 \text{ cm}$. In unit-vector notation, what is the magnetic force on the proton due to the current?

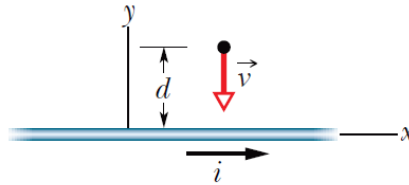


Fig. 29-50 Problem 23.

We assume the current flows in the $+x$ direction and the particle is at some distance d in the $+y$ direction (away from the wire). Then, the magnetic field at the location of a proton with charge q is $\vec{B} = (\mu_0 i / 2\pi d)\hat{k}$. Thus,

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{\mu_0 i q}{2\pi d} (\vec{v} \times \hat{k}).$$

In this situation, $\vec{v} = v(-\hat{j})$ (where v is the speed and is a positive value), and $q > 0$. Thus,

$$\begin{aligned} \vec{F} &= \frac{\mu_0 i q v}{2\pi d} ((-\hat{j}) \times \hat{k}) = -\frac{\mu_0 i q v}{2\pi d} \hat{i} = -\frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.350 \text{ A})(1.60 \times 10^{-19} \text{ C})(200 \text{ m/s})}{2\pi(0.0289 \text{ m})} \hat{i} \\ &= (-7.75 \times 10^{-23} \text{ N})\hat{i}. \end{aligned}$$

*56

•56 Figure 29-71 shows an arrangement known as a Helmholtz coil. It consists of two circular coaxial coils, each of 200 turns and radius $R = 25.0$ cm, separated by a distance $s = R$. The two coils carry equal currents $i = 12.2$ mA in the same direction. Find the magnitude of the net magnetic field at P , midway between the coils.

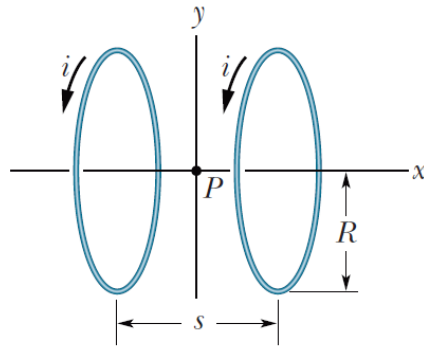


Fig. 29-71 Problems 56 and 90.

We use Eq. 29-26 and note that the contributions to $\vec{B}_P$ from the two coils are the same. Thus,

$$B_P = \frac{2\mu_0 i R^2 N}{2[R^2 + (R/2)^2]^{3/2}} = \frac{8\mu_0 N i}{5\sqrt{5}R} = \frac{8(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(200)(0.0122 \text{ A})}{5\sqrt{5}(0.25 \text{ m})} = 8.78 \times 10^{-6} \text{ T}.$$

$\vec{B}_P$ is in the positive x direction.

*57

•57 **SSM** A student makes a short electromagnet by winding 300 turns of wire around a wooden cylinder of diameter $d = 5.0$ cm. The coil is connected to a battery producing a current of 4.0 A in the wire. (a) What is the magnitude of the magnetic dipole moment of this device? (b) At what axial distance $z \gg d$ will the magnetic field have the magnitude $5.0 \mu\text{T}$ (approximately one-tenth that of Earth's magnetic field)?

(a) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. We use $A = \pi R^2$, where R is the radius. Thus,

$$\mu = Ni\pi R^2 = (300)(4.0\text{ A})\pi(0.025\text{ m})^2 = 2.4\text{ A} \cdot \text{m}^2.$$

(b) The magnetic field on the axis of a magnetic dipole, a distance z away, is given by Eq. 29-27:

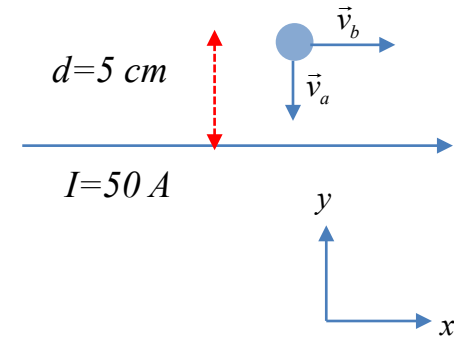
$$B = \frac{\mu_0}{2\pi} \frac{\mu}{z^3}.$$

We solve for z :

$$z = \left(\frac{\mu_0}{2\pi} \frac{\mu}{B} \right)^{1/3} = \left(\frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.36 \text{ A} \cdot \text{m}^2)}{2\pi(5.0 \times 10^{-6} \text{ T})} \right)^{1/3} = 46 \text{ cm}.$$

*68

68 A long straight wire carries a current of 50 A. An electron, traveling at 1.0×10^7 m/s, is 5.0 cm from the wire. What is the magnitude of the magnetic force on the electron if the electron velocity is directed (a) toward the wire, (b) parallel to the wire in the direction of the current, and (c) perpendicular to the two directions defined by (a) and (b)?



We take the current ($i = 50$ A) to flow in the $+x$ direction, and the electron to be at a point P , which is $r = 0.050$ m above the wire (where “up” is the $+y$ direction). Thus, the field produced by the current points in the $+z$ direction at P . Then, combining Eq. 29-4 with Eq. 28-2, we obtain

$$\vec{F}_e = (-e\mu_0 i / 2\pi r) (\vec{v} \times \hat{k}).$$

(a) The electron is moving down: $\vec{v} = -v\hat{j}$ (where $v = 1.0 \times 10^7$ m/s is the speed) so

$$\vec{F}_e = \frac{-e\mu_0 i v}{2\pi r} (-\hat{i}) = (3.2 \times 10^{-16} \text{ N}) \hat{i}$$

(b) In this case, the electron is in the same direction as the current: $\vec{v} = v\hat{i}$ so

$$\vec{F}_e = \frac{-e\mu_0 i v}{2\pi r} (-\hat{j}) = (3.2 \times 10^{-16} \text{ N}) \hat{j} \quad \text{or} \quad |\vec{F}_e| = 3.2 \times 10^{-16} \text{ N}$$

(c) Now, $\vec{v} = \pm v\hat{k}$ so $\vec{F}_e \propto \hat{k} \times \hat{k} = 0$.