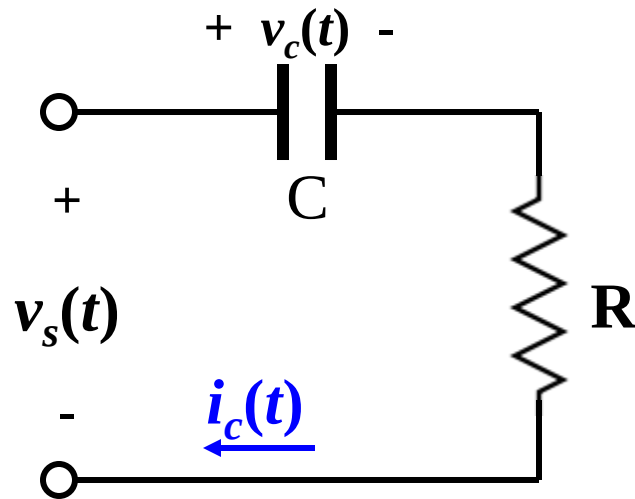


Solving with sinusoids is complex

Solving with complex numbers makes things simpler



Define voltages as sinusoids

$$v_s = V_s \cos \omega t$$

$$v_c = V_c \cos(\omega t + \theta)$$

I-V relation for C

$$\begin{aligned} i_c &= C \frac{dv_c(t)}{dt} \\ &= -\omega C V_c \sin(\omega t + \theta) \end{aligned}$$

Apply KVL around loop

$$v_s(t) = v_c(t) + i_c R$$

SOLVE for V_c :

$$V_s \cos \omega t = V_c \cos(\omega t + \theta) - \omega C R V_c \sin(\omega t + \theta)$$

Block C Unit 2 Outline

- Examples on analyzing AC circuits
 - > See Supplementary Notes on Phasors and Complex Numbers for information on methods
 - > Make sure you familiarize yourself on working with phasors and calculating impedance by completing Problem Set 5
- Frequency response (effect of frequency)
 - > Low pass filter
 - > High pass filter
 - > Resonators and resonance

G. Rizzoni, “Fundamentals of EE” Section 4.4

G. Rizzoni, “Fundamentals of EE” Sections 6.1-6.2

Analysis methodology

1. Any sinusoidal signal may be mathematically represented in one of two ways: a **time-domain form** and a **frequency-domain (or phasor) form**

Note the $j\omega$ in the notation $\mathbf{V}(j\omega)$, indicating the $e^{j\omega t}$ dependence of the phasor. In the remainder of this chapter, bold uppercase quantities indicate phasor voltages or currents.

2. A phasor is a complex number, expressed in polar form, consisting of a *magnitude* equal to the peak amplitude of the sinusoidal signal and a *phase angle* equal to the phase shift of the sinusoidal signal *referenced to a cosine signal*.

3. When one is using phasor notation, it is important to note the specific frequency ω of the sinusoidal signal, since this is not explicitly apparent in the phasor expression.

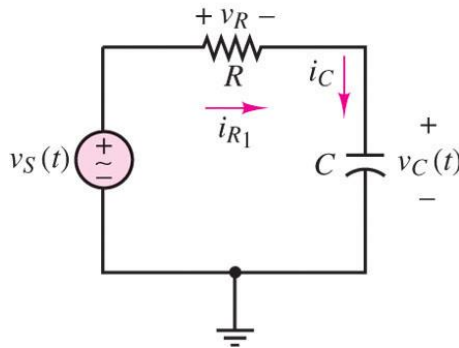
A very basic example to start

Find $V_C(t)$ in terms of R and C given that $V_S(t) = V_S \cos(\omega t)$

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.

A circuit containing energy-storage elements is described by a differential equation. The differential equation describing the series RC circuit shown is

$$\frac{di_C}{dt} + \frac{1}{RC} i_C = \frac{dv_S}{dt}$$



First find the respective impedances in the circuit,

Capacitor: $1/j\omega C$

Resistor: R

Next transform sinusoids to phasors,

Voltage source: V_S

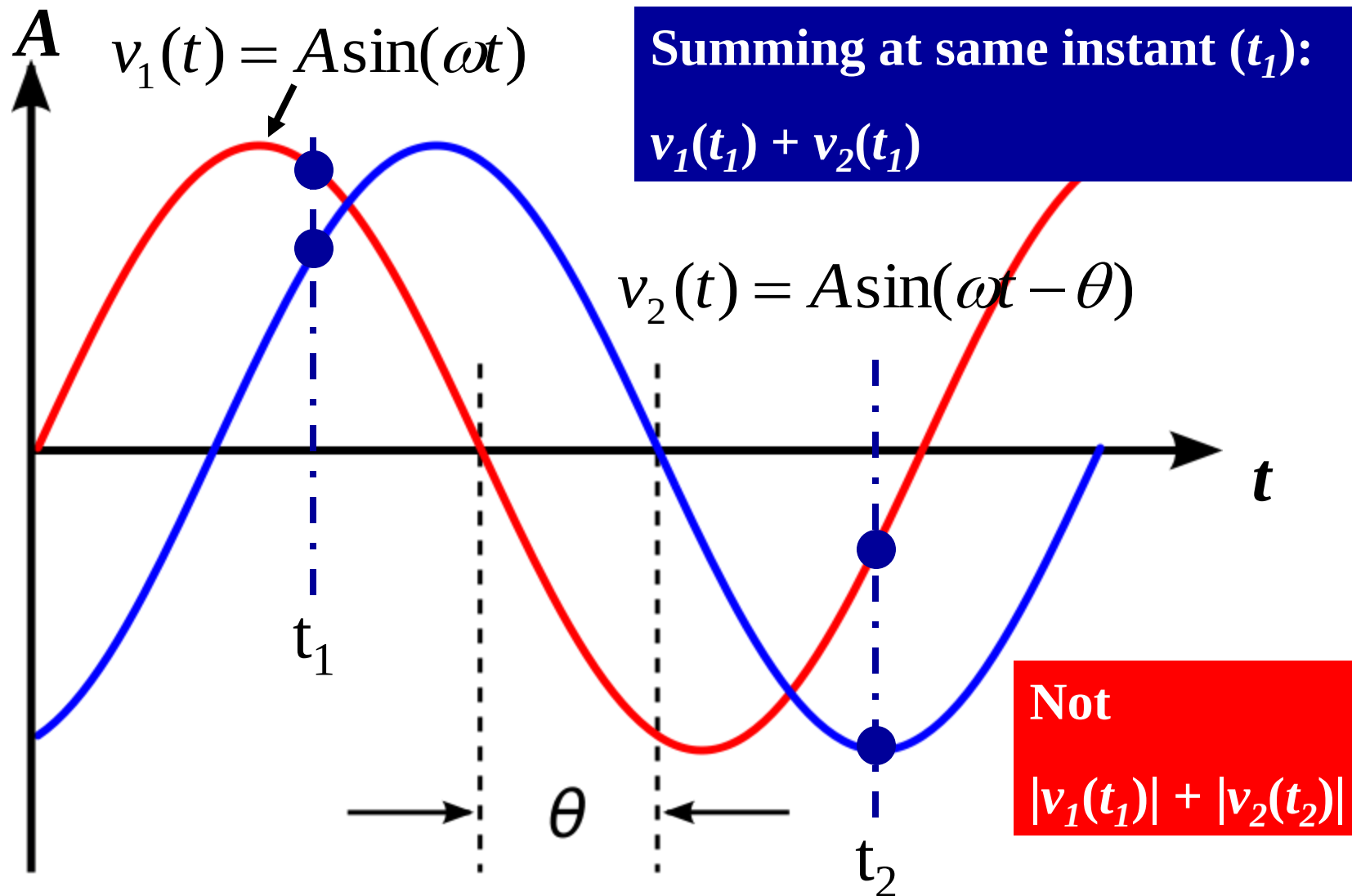
Apply voltage divider rule for R and C in series,

$$V_C = (1/j\omega C) / [R + (1/j\omega C)] V_S$$

$$V_C = R / [1 + j\omega RC] V_S$$

$$V_C = A V_S \cos(\omega t + \theta); \text{ where } A = R / \sqrt{1 + (\omega RC)^2} \text{ and } \theta = -\tan^{-1}(\omega RC)$$

Caution when summing I or V



Phasor techniques: Example 1

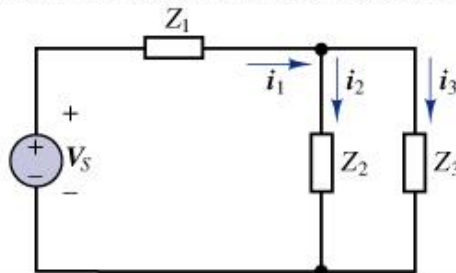
Problem 4.50

Determine $i_3(t)$ in the following circuit for $\omega = 377$ rad/s if:

$$i_1(t) = 141.4 \cos(\omega t + 2.356) \text{ mA}$$

$$i_2(t) = 50 \sin(\omega t - 0.927) \text{ mA}$$

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



We can simply use KCL to find i_3 since i_1 and i_2 are both already known. But to do so, we must first convert the sinusoids into phasor form and then to complex numbers to add or subtract the currents.

$$I_1 = 141.4 \cos(2.356) + j141.4 \sin(2.356) = -100 + j100 \text{ mA}$$

Since i_2 is a sine function, we should convert it to a cosine function first to get the correct phase.

$$i_2(t) = 50 \cos[\omega t - 0.927 - \pi/2] = 50 \cos(\omega t - 2.498)$$

$$I_2 = 50 \cos(2.498) - j50 \sin(2.498) = -40 - j30 \text{ mA}$$

$$I_3 = I_1 - I_2$$

$$= -60 + j130$$

$$= 143.1 \cos(\omega t + 2) \text{ mA}$$

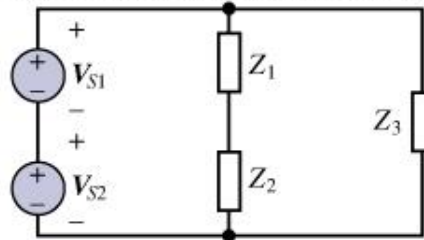
Phasor techniques: Example 2

Problem 4.51

Determine the current through Z_3 if $V_{S1} = V_{S2} = 170 \cos(377t) \text{ V}$;

$$Z_1 = 5.9 \angle 0.122 \text{ k}\Omega; \quad Z_2 = 2.3 \angle 0 \Omega; \quad Z_3 = 17 \angle 0.192 \Omega$$

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



Note that the two voltage sources are in phase, so we can just add up their magnitudes.

$$V_T = V_{S1} + V_{S2} = 340 \cos(377t) \text{ V}$$

This is the voltage across Z_3 , therefore we can find the current through Z_3 :

$$I_3 = \frac{340 \angle 0}{17 \angle 0.192}$$

$$= 20 \angle -0.192 \text{ mA}$$

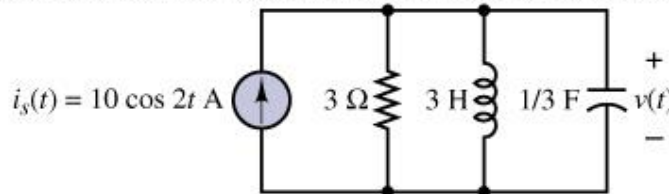
$$i_3(t) = 20 \cos(\omega t - 0.192) \text{ mA}$$

Phasor techniques: Example 3

Problem 4.55

Find the voltage across the capacitor

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



The resistor, inductor and capacitor are all in parallel with the current source. So we can find the effective impedance seen by the current source and use this to find the voltage.

$$\begin{aligned} Y &= \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \\ &= \frac{1}{3} + \frac{1}{j(2)(3)} + j(2)(1/3) \\ &= (1/3) + j(1/2) \\ &= 0.601 \angle 0.9827 \text{ S} \end{aligned}$$

$$Z = 1/Y = 1.664 \angle -0.9827 \Omega$$

$$v(t) = 16.64 \cos(2t - 0.983) \text{ V}$$

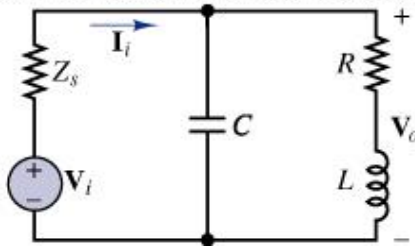
Phasor techniques: Example 4

Problem 4.52

Determine the frequency so that I_i and V_o in the following circuit are in phase.

$$Z_S = 13000 + j3\omega, R = 120\Omega, L = 19\text{mH}, C = 220\text{pF}$$

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



When I_i and V_o are in phase, the admittance and the impedance seen across V_o must have a phase equal to zero.

$$Y = j\omega C + \frac{1}{R + j\omega L} = j\omega C + \frac{R - j\omega L}{R^2 + (\omega L)^2}$$

If the phase of Y is zero, then its value must be purely real

⇒ Imaginary part must be equal to zero

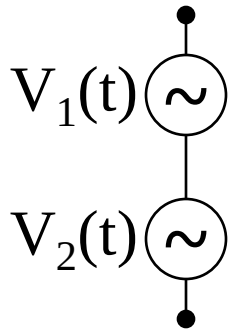
$$\omega C - \frac{\omega L}{R^2 + (\omega L)^2} = 0$$

$$\omega = \frac{1}{L} \sqrt{\frac{L}{C} - R^2}$$

We find in this case that $L/C \gg R$, hence

$$\omega \approx \frac{1}{\sqrt{LC}} = 489.1 \text{ krad/s}$$

Multiple sources: different phase



Given that:

$$V_1(t) = 15\cos(377t + \pi/4) \text{ and } V_2(t) = 15\cos(377t + \pi/12)$$

Find the total voltage across the two sources

The first and most important thing to note here is that the 2 voltage sources are **not in phase**. Hence $V_1 + V_2$ must be derived by vector addition. To add them as vectors, we can convert the sinusoids to complex numbers.

$$V_1 = 15\cos(\pi/4) + j15\sin(\pi/4); V_2 = 15\cos(\pi/12) + j15\sin(\pi/12)$$

$$\text{Therefore, } V_1 + V_2 = 15[\cos(\pi/4) + \cos(\pi/12)] + j15[\sin(\pi/4) + \sin(\pi/12)]$$

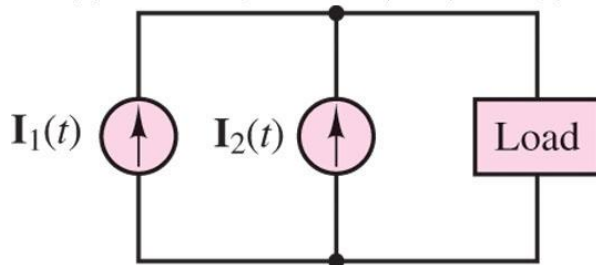
$$V_1 + V_2 = 25.095 + j14.489 = 38.33\cos(\omega t + 0.857) \text{ V}$$

Multiple sources: different frequency

Given that $i_1(t) = A_1 \cos(\omega_1 t)$ and $i_2(t) = A_2 \cos(\omega_2 t)$

Find the current through the load

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



In this case, we can think about each source to be independent of the other. Hence we can revert to the use of superposition.

Note that if we blindly follow the same procedure used in the previous case, we would arrive at the wrong answer. This is because the frequencies of the sources are now different. The phasor form only describes the magnitude and phase but not the frequency of the component: so take note of this whenever there is a change in frequency.

Current through load

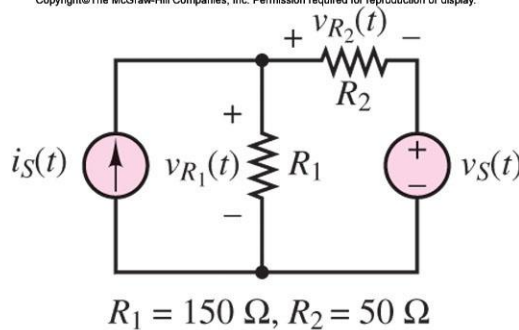
$$= i_1 + i_2 = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)$$

Two frequency components

Note that the phase relationship between the two sources is not consequential in this case

AC superposition example 1 (Pt 1)

Copyright©The McGraw-Hill Companies, Inc. Permission required for reproduction or display.

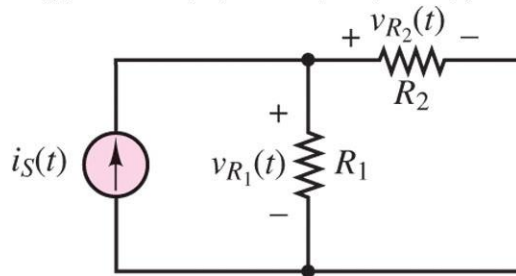


$$I_s(t) = 0.5\cos[2\pi(100t)] \text{ A}; V_s(t) = 20\cos[2\pi(1000t)] \text{ V}$$

Find $V_{R1}(t)$ and $V_{R2}(t)$

Note first that the frequencies of the two sources are different, so we can apply superposition.

Copyright©The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



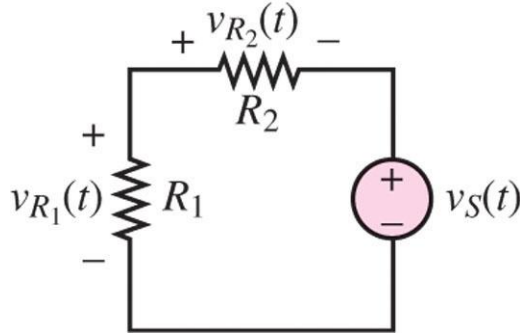
Short-circuit the voltage source and find V_{R1} and V_{R2} due only to i_S ,

We find that R_1 and R_2 are in parallel and therefore:

$$V_{R1} = V_{R2} = I_S * (R_1 || R_2) = 18.75\cos[2\pi(100t)] \text{ V}$$

AC superposition example 1 (Pt 2)

Copyright©The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



Open-circuit the current source and find V_{R1} and V_{R2} due only to V_S (20V)

We can then apply voltage divider rule:

$$V_{R1} = [R_1/(R_1+R_2)]V_S;$$

$$V_{R2} = [R_2/(R_1+R_2)]V_S$$

$$\Rightarrow V_{R1} = 0.75V_S; V_{R2} = 0.25V_S$$

$$V_{R1} = 15\cos[2\pi(1000t)] \text{ V};$$

$$V_{R2} = 5\cos[2\pi(1000t)] \text{ V}$$

Finally we sum up the voltage components at each frequency:

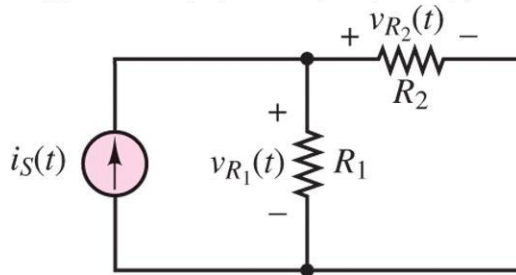
$$V_{R1} = 18.75\cos[2\pi(100t)] + 15\cos[2\pi(1000t)] \text{ V};$$

$$V_{R2} = 18.75\cos[2\pi(100t)] + 5\cos[2\pi(1000t)] \text{ V}$$

Most Common Mistake #1

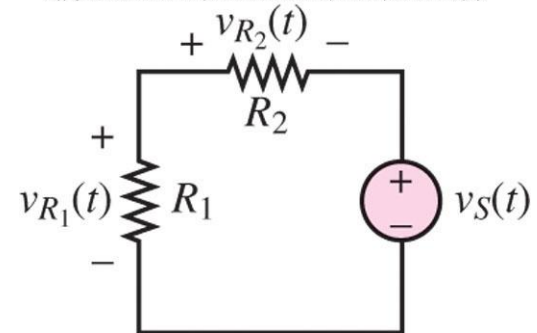
Adding up voltages or currents, forgetting that their frequencies are different

Copyright©The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



$$V_{R1} = V_{R2} = 18.75 \cos[2\pi(100t)] \text{ V}$$

Copyright©The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



$$V_{R1} = 15 \cos[2\pi(1000t)] \text{ V};$$

$$V_{R2} = 5 \cos[2\pi(1000t)] \text{ V}$$

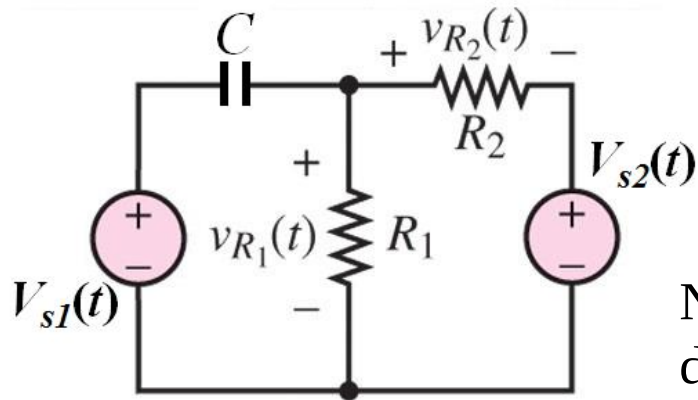
Critical step done WRONG

$$V_{R1} = (18.75 + 15) \text{ V}$$

$$V_{R2} = (18.75 + 5) \text{ V}$$

Tell me what is the frequency?

AC superposition example 2 (Pt 1)



$$V_{s1} = 5 \cos(5000t), V_{s2} = 10V$$

$$R_1 = 1k\Omega, R_2 = 1k\Omega, C = 0.4\mu F$$

Note that the frequencies of the two sources are different. This time there is also a capacitor in the circuit. At different frequencies, the impedance of the capacitor will also be different.

For example:

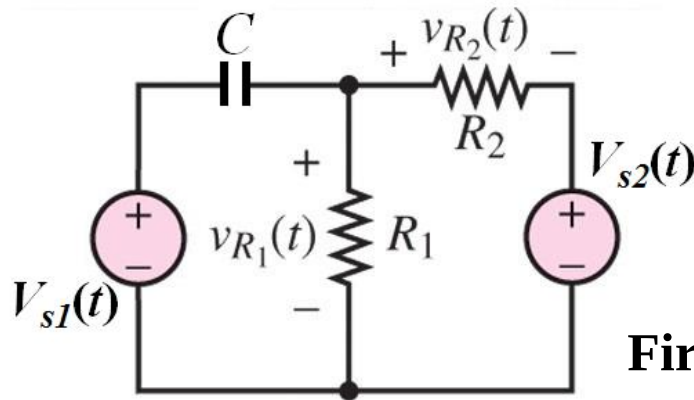
At DC (i.e. 0 rad/s), the capacitor has an infinite impedance

At 5000 rad/s, the capacitor will have impedance given by $1/j\omega C$

TRICK: Analyze the circuit one source and one frequency at a time

⇒ Superposition

AC superposition example 2 (Pt 2)



Find the current in R_1 given:

$$V_{s1} = 5 \cos(5000t), V_{s2} = 10V$$

$$R_1 = 1k\Omega, R_2 = 1k\Omega, C = 0.4\mu F$$

First: Analyze with V_{s2} at DC

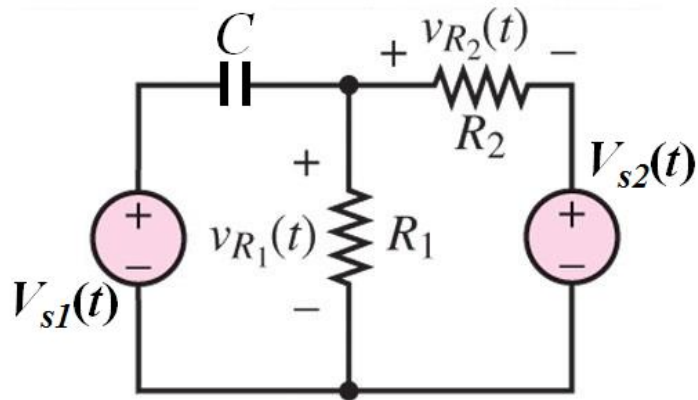
$\Rightarrow$ At DC, V_{s1} has no effect on the rest of the circuit

$\Rightarrow$ It's like it does not exist (i.e. short circuit)

$$\begin{aligned} V_{R1} &= \frac{R_1}{R_1 + R_2} V_{s2} \\ &= 5V \end{aligned}$$

$$I_{s2} = 5mA$$

AC superposition example 2 (Pt 3)



Next: Analyze with V_{s1} at 5000 rad/s

$\Rightarrow$ Now @ 5000 rad/s, V_{s2} has no effect on the rest of the circuit

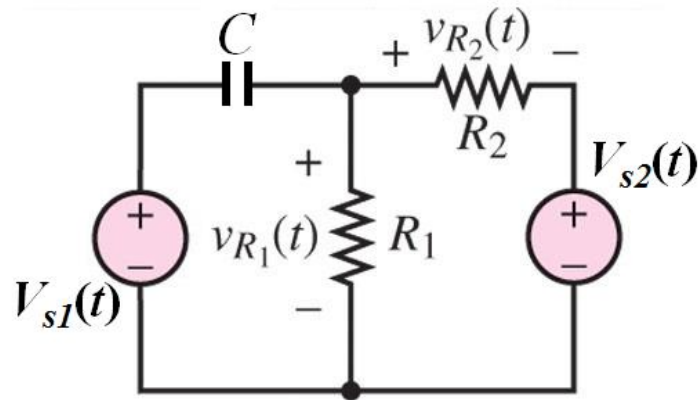
$\Rightarrow$ It's like it does not exist (i.e. short circuit)

$$\begin{aligned} V_{R1} &= \left(\frac{R_1 \parallel R_2}{(R_1 \parallel R_2) + 1/j\omega C} \right) V_{s1} \\ &= \left(\frac{500}{500 - j500} \right) 5 \\ &= \frac{5}{\sqrt{2}} \angle -\pi/4 \text{ V} \end{aligned}$$

$$I_{s1} = 3.536 \cos(5000t - \pi/4) \text{ mA}$$

$$I_{R1} = 3.536 \cos(5000t - \pi/4) + 5 \text{ mA}$$

AC superposition example 3 (Pt 1)



$$V_{s1} = 5 \cos(5000t), V_{s2} = 2 \cos(2500t) \text{ V}$$

$$R_1 = 1 \text{ k}\Omega, R_2 = 1 \text{ k}\Omega, C = 0.4 \mu\text{F}$$

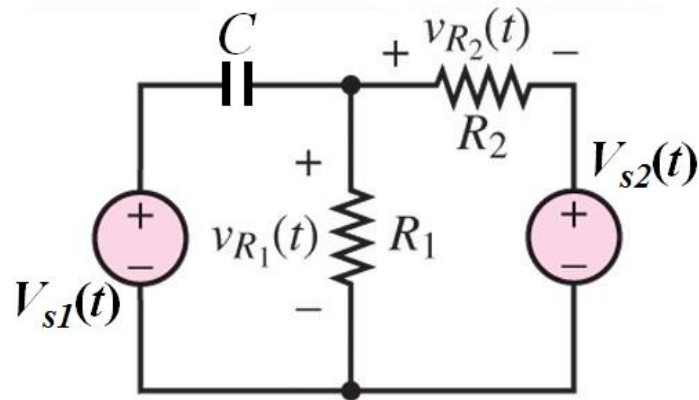
First: Analyze with V_{s1} at 5000 rad/s

$\Rightarrow$ @ 5000 rad/s, V_{s2} has no effect on the rest of the circuit

$$\begin{aligned} V_{R1} &= \left(\frac{R_1 \parallel R_2}{(R_1 \parallel R_2) + 1/j\omega C} \right) V_{s1} \\ &= \left(\frac{500}{500 - j500} \right) 5 \\ &= \frac{5}{\sqrt{2}} \angle -\pi/4 \text{ V} \end{aligned}$$

$$I_{s1} = 3.536 \cos(5000t - \pi/4) \text{ mA}$$

AC superposition example 3 (Pt 2)



$$V_{s1} = 5 \cos(5000t), V_{s2} = 2 \cos(2500t) \text{ V}$$

$$R_1 = 1 \text{ k}\Omega, R_2 = 1 \text{ k}\Omega, C = 0.4 \mu\text{F}$$

Next: Analyze with V_{s2} at 2500 rad/s

$\Rightarrow$ @ 2500 rad/s, V_{s1} has no effect on the rest of the circuit

$$X_{RC} = R_1 \parallel C = \frac{R_1 / j\omega C}{R_1 + 1 / j\omega C}$$

$$= \frac{R_1}{1 + j\omega C R_1} = \frac{1000}{1 + j}$$

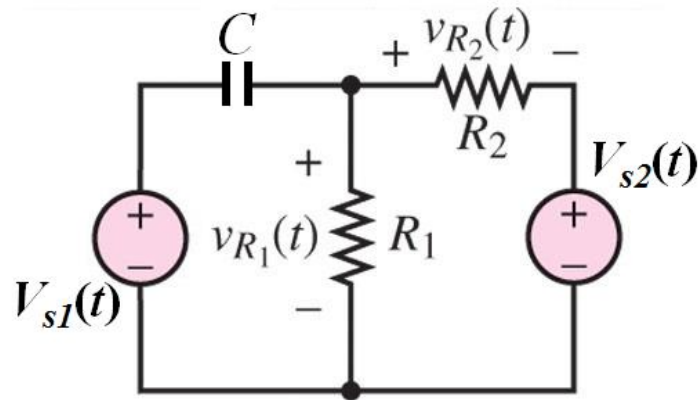
$$V_{R1} = \frac{R_1}{R_1 + X_{RC}} V_{s2} = \left(\frac{1000}{1000 + 1000 / (1 + j)} \right)^2$$

$$= 2 \left(\frac{1 + j}{2 + j} \right) = 1.2 + j0.4$$

$$= 1.265 \angle 0.322 \text{ V}$$

$$I_{s2} = 1.265 \cos(2500t + 0.322) \text{ mA}$$

AC superposition example 3 (Pt 3)



$$V_{s1} = 5 \cos(5000t), V_{s2} = 2 \cos(2500t) \text{ V}$$

$$R_1 = 1 \text{ k}\Omega, R_2 = 1 \text{ k}\Omega, C = 0.4 \mu\text{F}$$

$$I_{s1} = 3.536 \angle -\pi/4 \text{ mA}$$

$$I_{s1} = 3.536 \cos(5000t - \pi/4) \text{ mA}$$

$$I_{s2} = 1.265 \angle 0.322 \text{ mA}$$

$$I_{s2} = 1.265 \cos(2500t + 0.322) \text{ mA}$$

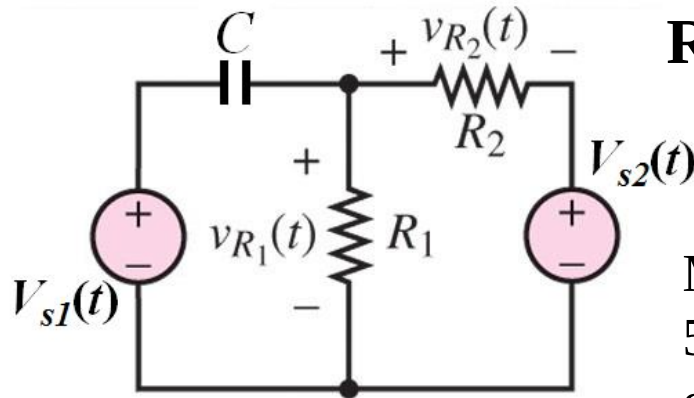
$$I_{R1} = 3.536 \cos(5000t - \pi/4) \text{ mA} + 1.265 \cos(2500t + 0.322) \text{ mA}$$

Most Common Mistake #2

Forgetting to recalculate impedance when frequency has changed

$$V_{s1} = 5 \cos(5000t), V_{s2} = 2 \cos(2500t) \text{ V}$$

$$R_1 = 1 \text{ k}\Omega, R_2 = 1 \text{ k}\Omega, C = 0.4 \mu\text{F}$$



Mistake: After calculating X_C for V_{s1} using 5000 rad/s, you use the same X_C again for V_{s2} even though frequency is now 2500 rad/s

$$\begin{aligned} X_{RC} &= R_1 \parallel C = \frac{R_1 / j\omega C}{R_1 + 1 / j\omega C} \\ &= \frac{R_1}{1 + j\omega C R_1} = \frac{1000}{1 + 2j} \end{aligned}$$

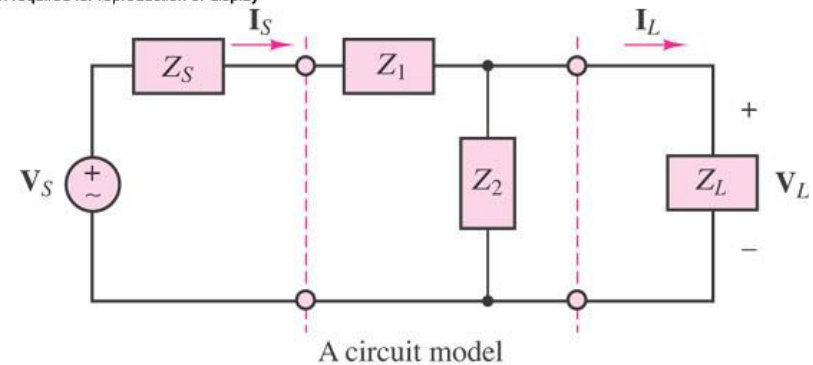
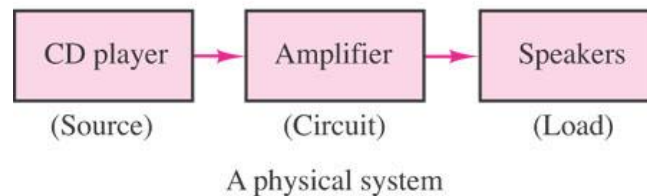
A red arrow points to the $2j$ term in the denominator of the final expression, indicating that this value was incorrectly calculated using the 5000 rad/s frequency instead of the 2500 rad/s frequency.

Frequency response

- Changing the frequency affects the currents and voltages in a circuit
- This is due to changes in the impedances of the various components in a circuit
- This affects the working frequency range of a particular device or circuit
- Hence it is important to find out the frequency response of a circuit
- The frequency response of a circuit is a measure of the variation of a load-related voltage or current in relation to the input frequency
- We typically express this in terms of variation in output voltage over the source:

$$H_V(j\omega) = \frac{V_L(j\omega)}{V_S(j\omega)}$$

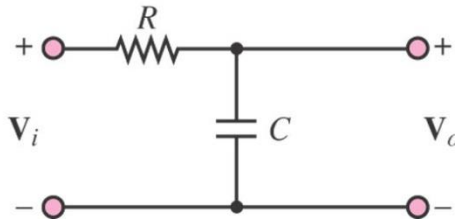
Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display



Low pass filter

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display

RC low-pass filter. The circuit preserves lower frequencies while attenuating the frequencies above the cutoff frequency $\omega_0 = 1/RC$. The voltages V_i and V_o are the filter input and output voltages, respectively.



Let us consider the response of the output V_o in relation to the input V_i . We keep the amplitude of V_i constant but vary its frequency ω .

By voltage divider rule:

$$\begin{aligned}\frac{V_o}{V_i}(j\omega) &= \frac{1/j\omega C}{R + 1/j\omega C} \\ &= \frac{1}{1 + j\omega CR} \\ &= \frac{1}{\sqrt{1 + (\omega CR)^2}} \angle -\tan^{-1}(\omega CR)\end{aligned}$$

Observations:

When ω approaches zero, V_o/V_i approaches 1 and phase is close to zero

When ω becomes large, V_o/V_i approaches zero and phase is close to $-\pi/2$

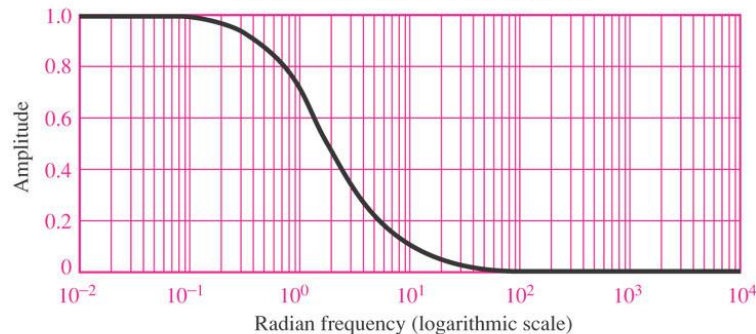
Allows lower frequency signal to pass and filters off higher frequency signals

Response of low pass filter

Allows lower frequency signals to pass and filters off higher frequency signals

Copyright ©The McGraw-Hill Companies, Inc. Permission required for reproduction or display

Magnitude response of RC low-pass filter



$$\begin{aligned}\frac{V_o}{V_i}(j\omega) &= \frac{1}{1 + j\omega CR} \\ &= \frac{1}{1 + j\omega/\omega_c}\end{aligned}$$

Note that when $\omega = \omega_c = 1/RC$, the imaginary and real parts of the denominator will be equal.

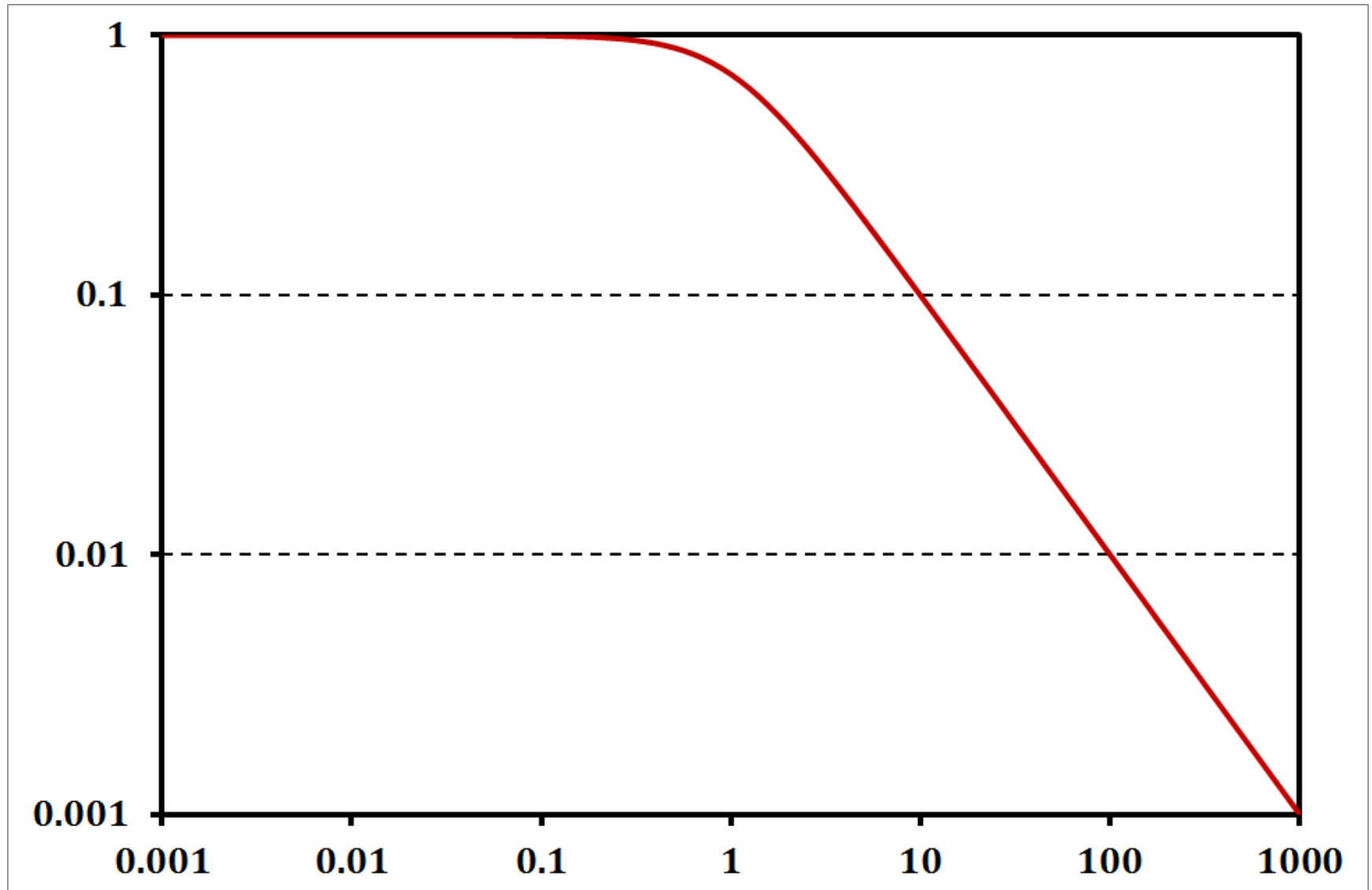
$$\Rightarrow \text{Magnitude} = 1/\sqrt{2}$$

$$\Rightarrow \text{Phase} = -\pi/4 \text{ (or } -45^\circ)$$

This frequency is called the cutoff frequency (ω_c) of the filter, and gives an indication of the filtering characteristics of the circuit.

Cut off frequency is when the ratio drops to $1/\sqrt{2}$ of the maximum

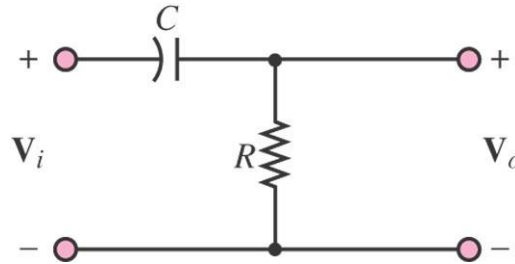
Log-Log Plot (Low Pass)



High pass filter

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display

RC high-pass filter. The circuit preserves higher frequencies while attenuating the frequencies below the cutoff frequency $\omega_0 = 1/RC$.



Let us consider the response of the output V_o in relation to the input V_i . We keep the amplitude of V_i constant but vary its frequency ω .

By voltage divider rule:

$$\begin{aligned}\frac{V_o}{V_i}(j\omega) &= \frac{R}{R + 1/j\omega C} \\ &= \frac{1}{1 + 1/j\omega CR} \\ &= \frac{1}{\sqrt{1 + (1/\omega CR)^2}} \angle \tan^{-1}(\omega CR)\end{aligned}$$

Observations:

When ω approaches zero, V_o/V_i approaches zero and phase is close to $\pi/2$

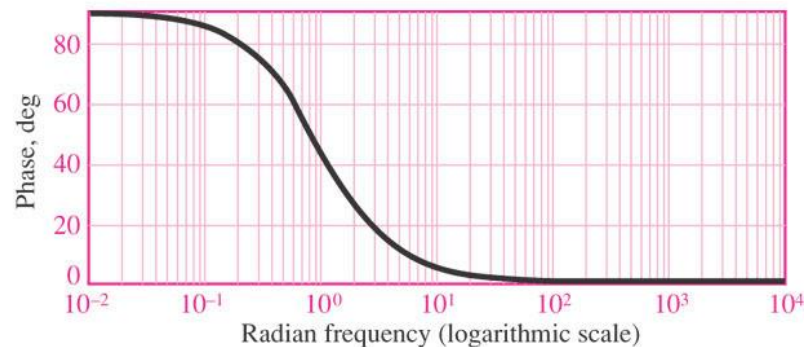
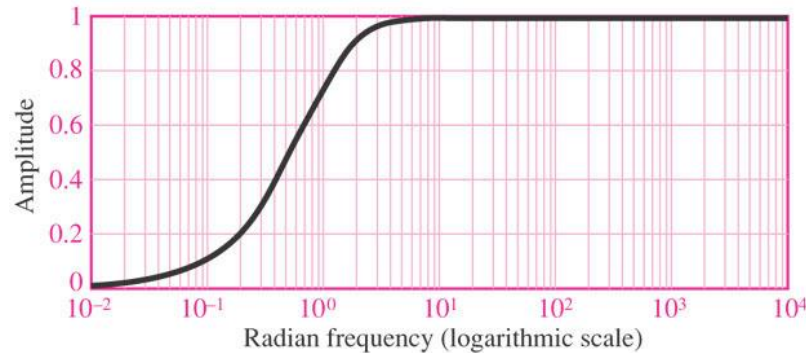
When ω becomes large, V_o/V_i approaches 1 and phase is close to 0

Allows higher frequency signals to pass and filters off lower frequency signals

Response of high pass filter

Allows higher frequency signals to pass and filters off lower frequency signals

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display



$$\frac{V_o}{V_i}(j\omega) = \frac{1}{1 + \omega_c / j\omega}$$

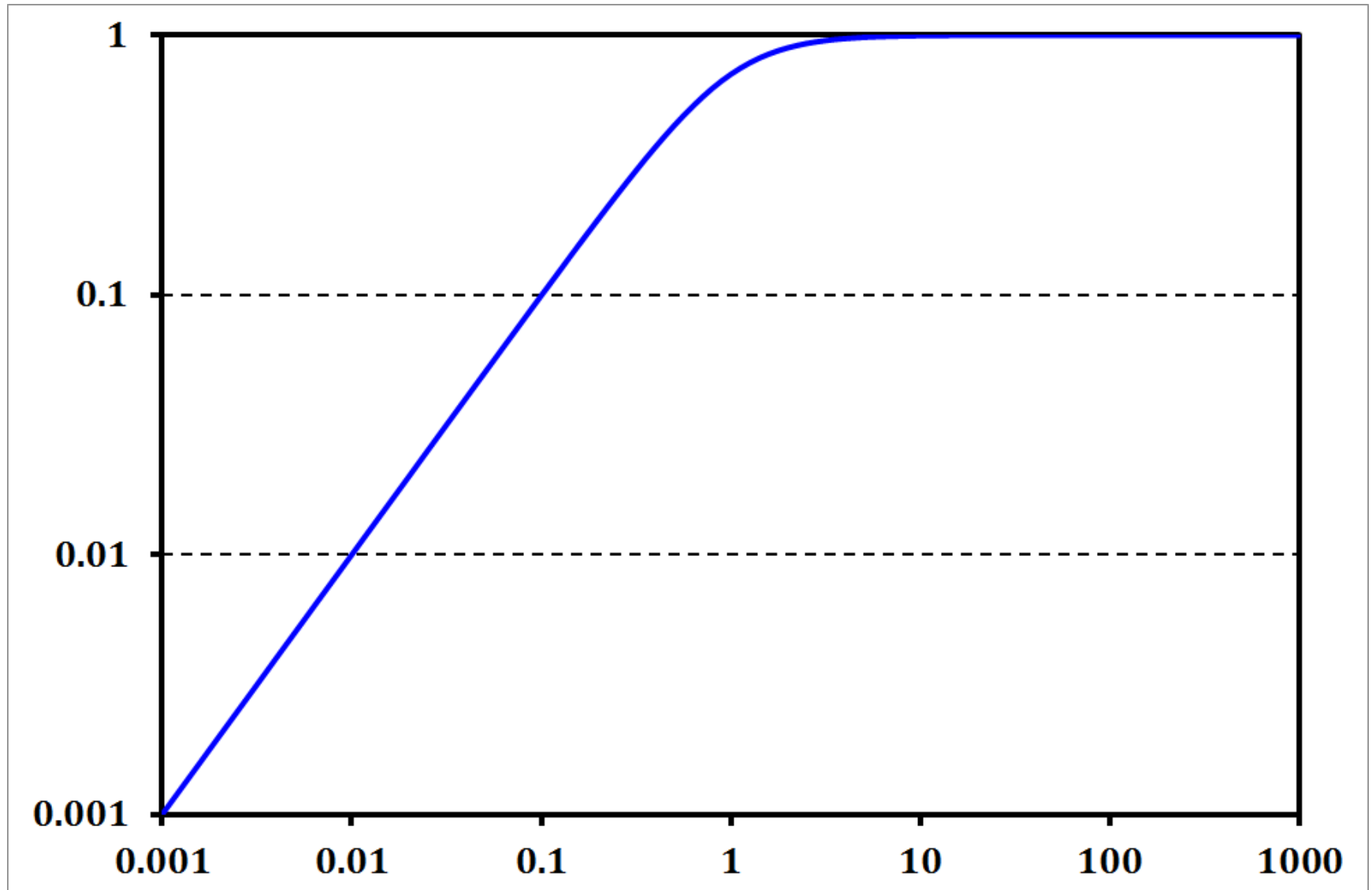
Note that when $\omega = 1/RC$, the imaginary and real parts of the denominator will be equal.

Magnitude = $1/\sqrt{2}$

Phase = $\pi/4$ (or 45°)

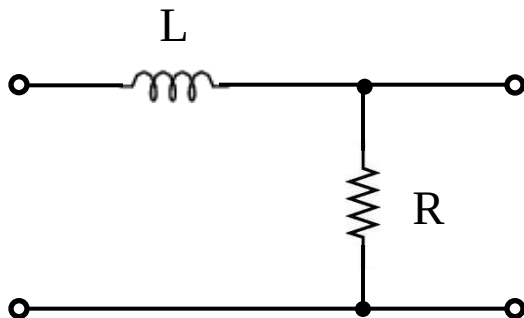
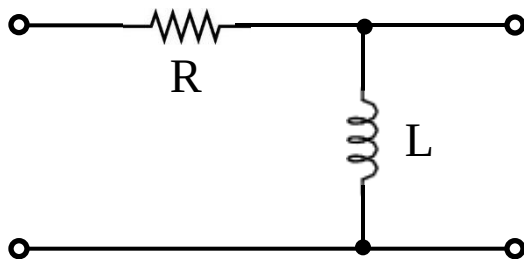
This frequency is called the cutoff frequency (ω_0) of the filter, and gives an indication of the filtering characteristics of the circuit.

Log-Log Plot (High Pass)



Other examples on filters

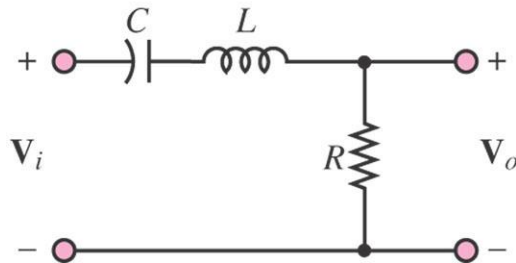
Determine the frequency response characteristics (low pass or high pass) for the following filter circuits.



Bandpass filter

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display

RLC bandpass filter. The circuit preserves frequencies within a band.



Let us consider the response of the output V_o in relation to the input V_i . We keep the amplitude of V_i constant but vary its frequency ω .

By voltage divider rule:

$$\begin{aligned}\frac{V_o}{V_i}(j\omega) &= \frac{R}{R + j\omega L + 1/j\omega C} \\ &= \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)}\end{aligned}$$

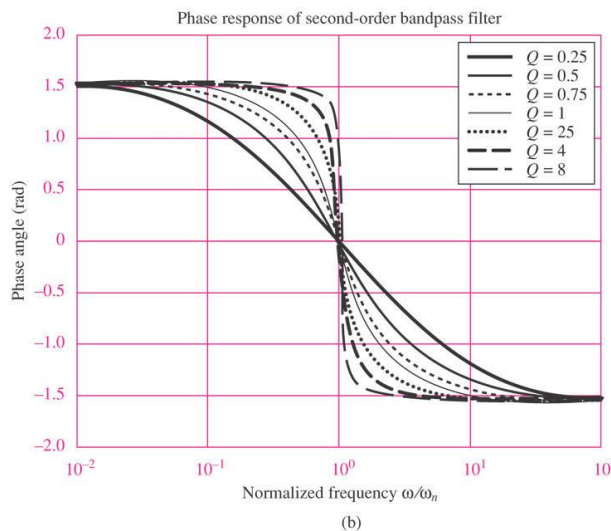
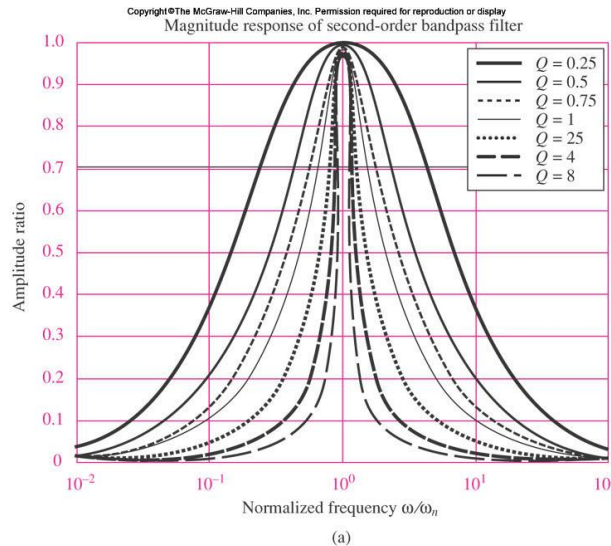
We can see that the output will be maximum when the imaginary part of the denominator is zero:

$$(\omega L/R) - [1/(\omega RC)] = 0$$

$$\omega^2 = 1/(LC)$$

When this happens, the impedances of the capacitor and inductor are equal and opposite. This is known as resonance.

Response of band pass filter



The shape of the band pass is governed by the quality factor defined as:

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

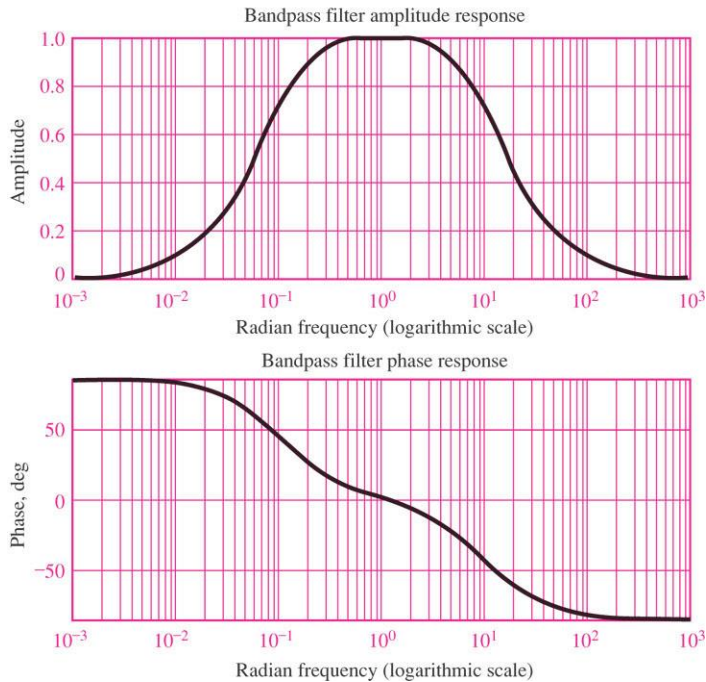
It is also defined using the half-power bandwidth $\Delta\omega$

$$Q = \frac{\omega_0}{\Delta\omega}$$

The larger the value of Q , the sharper the band pass so one can have a narrow band or broad band filter

Half power bandwidth

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display



Half power bandwidth: The frequency span whereby the power is half of the maximum at resonance.

A half in power corresponds to a scale factor of $1/\sqrt{2}$ for the voltage frequency response.

This occurs when the real and imaginary parts are of the same magnitude with each other.

$$\left(\frac{\omega L}{R} - \frac{1}{\omega RC} \right)^2 = 1$$

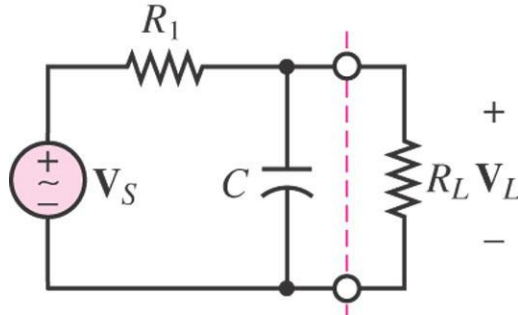
$$\frac{V_o}{V_i}(j\omega) = \frac{1}{1 + j \left(\frac{\omega L}{R} - \frac{1}{\omega RC} \right)}$$

Frequency response example 1

Compute the frequency response $H_V(j\omega)$ for the following circuit:

$$R_1 = 1\text{k}\Omega; C = 10\mu\text{F}; R_L = 10\text{k}\Omega$$

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display



First find the combined impedance of the capacitor and resistor in parallel:

$$Z_{RC} = \frac{R_L}{(1 + j\omega R_L C)} = \frac{10}{(1 + j0.1\omega)} \text{ k}\Omega$$

Now apply voltage divider rule:

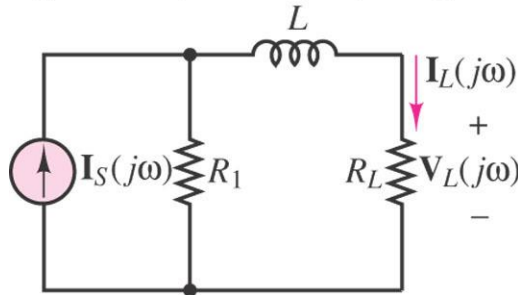
$$\begin{aligned} \frac{V_L(j\omega)}{V_S(j\omega)} &= \frac{Z_{RC}}{Z_{RC} + R_1} = \frac{10/(1 + j0.1\omega)}{[10/(1 + j0.1\omega)] + 1} \\ &= \frac{10}{11 + j0.1\omega} = \frac{100}{110 + j\omega} \\ &= \frac{100}{\sqrt{110^2 + \omega^2}} \angle -\tan^{-1}\left(\frac{\omega}{110}\right) \end{aligned}$$

Frequency response example 2

Compute the frequency response $H_Z(j\omega)$ for the following circuit:

$$R_1 = 1\text{k}\Omega; L = 2\text{mH}; R_L = 4\text{k}\Omega$$

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display



First find the combined impedance of the inductor and resistor in parallel:

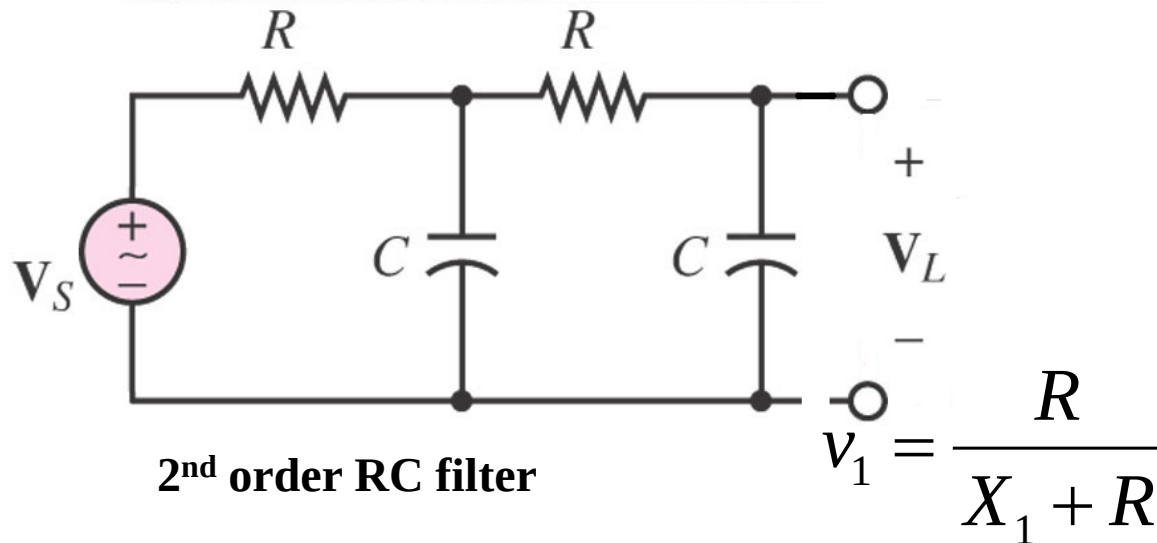
$$Z_{RL} = R_L + j\omega L = 4000 + j(2 \times 10^{-3})\omega \Omega$$

Now apply current divider rule:

$$\begin{aligned} \frac{V_L(j\omega)}{I_S(j\omega)} &= \frac{I_L R_L}{I_S} = \frac{R_1 R_L}{R_1 + Z_{RL}} \\ &= \frac{(1000)(4000)}{1000 + 4000 + j(2 \times 10^{-3})\omega} = \frac{4 \times 10^6}{5000 + j(2 \times 10^{-3})\omega} \\ &= \frac{800}{1 + j(4 \times 10^{-7})\omega} \Omega \end{aligned}$$

RC Ladders 1

We can cascade the filters to make the cut off effect even greater



Summary

Working with phasors allows us to:

- Simplify the process of analysis for AC circuits
 - > Mistake #1: Adding up I or V components of different frequencies as if the difference does not matter
 - > Mistake #2:
- Analyze AC circuits in the frequency domain
 - > Now we can see the frequency response