

Tutorial 7
Linear Programming Part 1**Question 1**

The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is \$5 per unit on special risk insurance and \$2 per unit on mortgages.

Management wishes to establish sales quotas for the new product lines to maximum total expected profit. The work requirements are as follows:

Department	Work-Hours per Unit		Work-Hours Available
	Special Risk	Mortgage	
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

Questions:

- Identify the decisions to be made, the constraints on these decisions, and the overall measure of performance for the decisions in mathematical expressions.
- Solve this model by using Excel Solver.

Solutions:

Let S = units of special risk insurance
 M = units of mortgages

Maximize $P = \$5S + \$2M$

Subject to $3S + 2M \leq 2400$
 $M \leq 800$
 $2S \leq 1200$
and $S \geq 0, M \geq 0$

CB2203 Data-Driven Business Modeling

Question 2

International City Trust (ICT) invests in short-term trade credits, corporate bonds, gold stocks, and construction loans.

ICT has \$5 million available for immediate investment and wishes to achieve the following:

1. Maximize interest earned on investments made over the next six months, and
2. Satisfy diversification requirements as set by the board of directors:
 - Board specifies at least 55% of funds invested must be in gold stocks and construction loans
 - No less than 15% be invested in trade credit

The interest earned and maximum investment amount are as follow:

Investment	Interest earned (%)	Maximum investment (\$ millions)
Trade credit	7	1.0
Corporate bonds	11	2.5
Gold stock	19	1.5
Construction loans	15	1.8

Formulate this problem and determine the best investment portfolio.

Solutions:

Decision variables:

X_1 =dollars invested in trade credit

X_2 =dollars invested in corporate bonds

X_3 =dollars invested in gold stock

X_4 =dollars invested in construction loans

Objective: Maximize investment interest dollars earned

$$= 0.07 X_1 + 0.11 X_2 + 0.19 X_3 + 0.15 X_4$$

Subject to

$$X_1 \leq 1$$

$$X_2 \leq 2.5$$

$$X_3 \leq 1.5$$

$$X_4 \leq 1.8$$

$$X_3 + X_4 \geq 0.55 (X_1 + X_2 + X_3 + X_4)$$

$$X_1 \geq 0.15 (X_1 + X_2 + X_3 + X_4)$$

$$X_1 + X_2 + X_3 + X_4 \leq 5$$

$$X_1, X_2, X_3, X_4 \geq 0$$

Question 3

The Electrocomp Corporation manufactures two electrical products: air conditioners and larger fans. The assembly process for each is similar in that both require a certain amount of wiring and drilling. Each air conditioner takes 3 hours of wiring and 2 hours of drilling. Each fan must go through 2 hours of wiring and 1 hour of drilling. During the next production period, 240 hours of wiring time are available and up to 140 hours of drilling time may be used. Each air conditioner sold yields a profit of \$25. Each fan assembled may be sold for a \$15 profit. Formulate and solve this LP production mix situation to find the best combination of air conditioners and fans that yields the highest profit. Use the corner point graphical approach.

Solutions:

Let X_1 = number of air conditioners to be produced

X_2 = number of large fans produced

Max. Profit, $Z = \$25X_1 + \$15X_2$

Subject to

$$3X_1 + 2X_2 \leq 240 \quad (\text{available wiring})$$

$$2X_1 + X_2 \leq 140 \quad (\text{available drilling})$$

$$X_1, X_2 \geq 0 \quad (\text{non-negativity})$$

Compare Corner Points

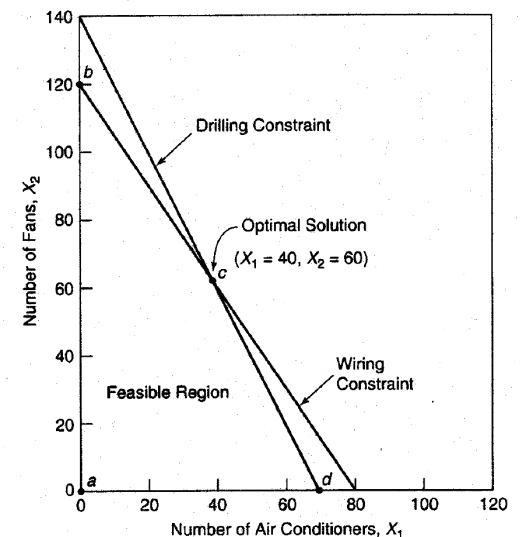
Point a:	$X_1 = 0, X_2 = 0$	$Z = \$25 \times 0 + \$15 \times 0 = \$0$
Point b:	$X_1 = 0, X_2 = 120$	$Z = \$25 \times 0 + \$15 \times 120 = \$1,800$
Point c:	$X_1 = 40, X_2 = 60$	$Z = \$25 \times 40 + \$15 \times 60 = \$1,900^*$
Point d:	$X_1 = 70, X_2 = 0$	$Z = \$25 \times 70 + \$15 \times 0 = \$1,750$

- **Point b** (wiring constraint)
 $3X_1 + 2X_2 = 240$
 when $X_1 = 0, X_2 = 120$; when $X_2 = 0, X_1 = 80$ $\therefore$ Point b: $X_1 = 0, X_2 = 120$
- **Point c** (the intersection of wiring constraint and drilling constraint)
 $3X_1 + 2X_2 = 240$ --- (1)
 $2X_1 + X_2 = 140$ --- (2)
 $(2) \times 2, \quad 4X_1 + 2X_2 = 280$ --- (3)
 $(3) - (1), \quad X_1 = 40$
 Put $X_1 = 40$ into (2), $X_2 = 60$ $\therefore$ Point c: $X_1 = 40, X_2 = 60$
- **Point d** (drilling constraint)
 $2X_1 + X_2 = 140$
 when $X_1 = 0, X_2 = 140$; when $X_2 = 0, X_1 = 70$ $\therefore$ Point d: $X_1 = 70, X_2 = 0$

Optimal Solution

The optimal solution is to produce 40 air conditioners and 60 fans during each production period. Profit will be \$1,900.

7-14.



Question 4

The Outdoor Furniture Corporation manufactures two products, benches and picnic tables, for use in yards and parks. The firm has two main resources: its carpenters (labor force) and a supply of redwood for use in the furniture. During the next production cycle, 1,200 hours of labor are available under a union agreement. The firm also has a stock of 3,500 feet of good-quality redwood. Each bench that Outdoor Furniture produces requires 4 labor hours and 10 feet of redwood; each picnic table takes 6 labor hours and 35 feet of redwood. Completed benches will yield a profit of \$9 each, and tables will result in a profit of \$20 each. How many benches and tables should Outdoor Furniture produce to obtain the largest possible profit? Use the graphical LP approach.

Solutions:

Let B = number of benches to be produced

T = number of picnic tables to be produced

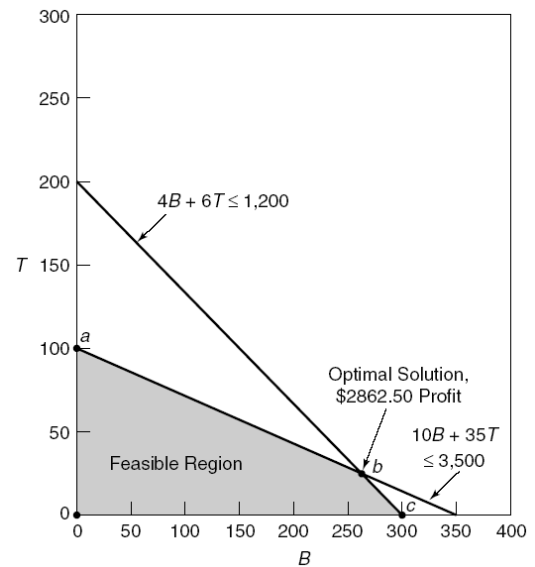
Max. Profit, $Z = \$9B + \$20T$

Subject to

$$4B + 6T \leq 1200 \quad (\text{available labor hours})$$

$$10B + 35T \leq 3500 \quad (\text{available stock})$$

$$B, T \geq 0 \quad (\text{non-negativity})$$



Compare Corner Points

Point a: $B = 0, T = 100$ $Z = \$9 \times 0 + \$20 \times 100 = \$2,000$

Point b: $B = 262.5, T = 25$ $Z = \$9 \times 262.5 + \$20 \times 25 = \$2,862.5^*$

Point c: $B = 300, T = 0$ $Z = \$9 \times 300 + \$20 \times 0 = \$2,700$

- **Point a** (available stock)

$$10B + 35T = 3500$$

when $B = 0, T = 100$; when $T = 0, B = 350$; $\therefore$ Point a: $B = 0, T = 100$

- **Point b** (intersection of stock and labor hours constraint)

$$4B + 6T = 1200 \quad \text{--- (1)}$$

$$10B + 35T = 3500 \quad \text{--- (2)}$$

$$(1) \times 2.5, \quad 10B + 15T = 3000 \quad \text{--- (3)}$$

$$(2) - (3), \quad 20T = 500, T = 25$$

Put $T = 25$ into (1), $B = 262.5$

$\therefore$ Point b: $B = 262.5, T = 25$

- **Point c** (available labor hours)

$$4B + 6T = 1200$$

when $B = 0, T = 200$; when $T = 0, B = 300$ $\therefore$ Point c: $B = 300, T = 0$

Optimal Solution

The optimal solution is to produce 262.5 benches and 25 tables. Profit will be \$2862.5.