

EE3210 Signals and Systems

Semester B 2023-2024

Assignment 2

Due Date: 3 Apr. 2024

1. Compute the discrete-time Fourier transform of the discrete signal $x[n] = \delta[n-1] + \delta[n+1]$. Plot the magnitude of the frequency spectrum in one period.
2. For the known frequency spectrum $X(j\Omega) = \cos(3\Omega + 1)$, compute the signal $x(t)$.
3. Assume that the maximum frequency of the continuous-time signal $x(t)$ is 100 Hz. Compute the minimum sampling period (Nyquist sampling period) for the signal $x(t/2)$.
4. Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency Ω_1 and Fourier coefficients a_k . Given that $x_2(t) = x_1(1-t) + x_1(t-1)$.
 - (a) How is the fundamental frequency Ω_2 of $x_2(t)$ related to Ω_1 ?
 - (b) Use the Fourier Series properties to find the relationship between the Fourier series coefficients b_k of $x_2(t)$ and the coefficients a_k .
5. Consider an LTI system with the impulse response $h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}$. The input signal of the system is $x(t) = \cos(6t + \frac{\pi}{2})$.
 - (a) Determine the system frequency response $H(j\Omega)$.
 - (b) Compute the Fourier transform of the input signal $X(j\Omega)$.
 - (c) Determine the system output signal.
6. Consider a continuous-time ideal lowpass filter whose frequency response is
$$H(j\Omega) = \begin{cases} 1, & |\Omega| \leq 100 \\ 0, & |\Omega| > 100 \end{cases}$$
When the input to this filter is a signal $x(t)$ with fundamental period $T = \pi/6$ and Fourier series coefficients a_k , it is found that the filter output $y(t) = x(t)$. Find the values of k for $a_k = 0$.
7. Watch the short video of the 2013 Shaw Prize winner for mathematics, Prof. David Donoho (start at 14:50): <https://www.youtube.com/watch?v=5wv4grOMglU>
 - (a) Briefly describe what is a denoising system, including the system input, output and function, as well as the principle to achieve denoising. Use your own words in no more than 100 words.
 - (b) Suppose you are given the following observed continuous-time signal $x(t)$:

$$x(t) = \cos(100\pi t) + n(t)$$

where $n(t)$ is the unwanted noise. With the use of an appropriate transform you have learned in this course, briefly describe, in theory, how can you extract $\cos(100\pi t)$ from $x(t)$? Also, how can you achieve compression for $\cos(100\pi t)$?