

Lecture 11

Induction and Inductance

Lecture 10 Review

- The Biot-Savart Law in electromagnetism describes the magnetic field generated by an electric current.
- The law is experimentally deduced, it describes the magnitude of the field dB produced at point P at distance r by a current length element $i ds$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \hat{r}}{r^2}$$

- Using the Biot-Savart law, we can determine the net magnetic field due to a variety of current configurations:
 - **B due to a long straight wire**

$$B = \frac{\mu_0 i}{2\pi R}$$



Lecture 10 Review

- B due to the circular current arc of angle ϕ

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

- B at center of a full circle of current.

$$B = \frac{\mu_0 i (2\pi)}{4\pi R} = \frac{\mu_0 i}{2R}$$

- When we place two current carrying wires next to each other, each wire will encounter a force with the same magnitude that is caused by the magnetic field generated by the other wire

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}.$$

Lecture 10 Review

- Ampere's Law states that The line integral of the $\mathbf{B}$ field around any closed loop is proportional to the electric current passing through the area enclosed by the loop.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

- Using the Ampere's Law and with the knowledge of the current, one can determine the magnetic field around current carrying structures such as cylinder, solenoids and toroids.
- A solenoid is just a coil or wire, however the $\mathbf{B}$ field in the current carrying coil is magnified by the number of turns, for the ideal solenoid,

$$B = \mu_0 in \quad (\text{ideal solenoid}).$$

Lecture 10 Review

- A current carrying coil can be treated as a magnetic dipole that produces a dipole magnetic field similar to the electric field from an electric dipole.
- The magnetic field as a function of z , along the axial distance is

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}$$

- One can also use the moment μ to define the strength of the magnetic dipole, where for a N current loop

$$\mu = NiA = Ni\pi R^2$$

- Thus the B field due to the magnetic dipole is $\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$



Lecture Outline

- **Chapter 30**, D Halliday, R Resnick, and J Walker, “Fundamentals of Physics” 9th Edition, Wiley (2005).
- Induction and Inductance
 - Faraday’s Law of Induction
 - Lenz’s Law
 - Induction and Energy Transfers
 - Inductors and Inductance
 - Self-Induction
 - Induced Electric Fields

Inductance: The First Experiment

The magnet's motion creates a current in the loop.

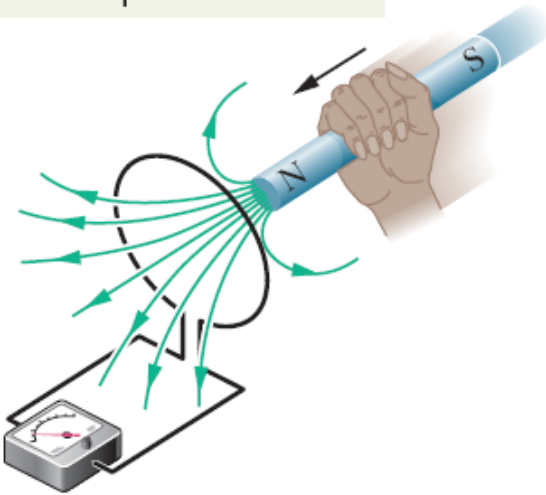


Fig. 30-1 An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.

1. A current appears only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases.

2. Faster motion produces a greater current.

3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current.

Moving the south pole toward or away from the loop also causes currents, but in the reversed directions.

Inductance: The First Experiment

The magnet's motion creates a current in the loop.

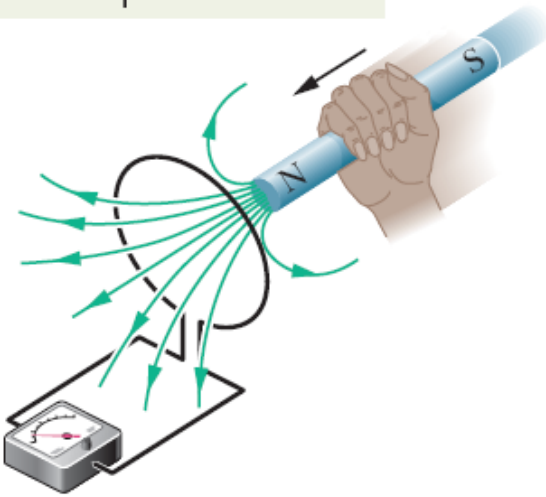


Fig. 30-1 An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.

1. The current thus produced in the loop is called *induced current*.
2. The work done per unit charge to produce that current (to move the conduction electron that constitute the current) is called an *induced emf*. *The same emf as in Lecture 8.*
3. *emf* – electromotive force and have the unit of volt (V)
4. The process of producing the current and *emf* is called *induction*.

Inductance: The Second Experiment

In the second experiment we use the apparatus of Fig. 30-2, with the two conducting loops close to each other but not touching.

If we close switch S , to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—an induced current—in the left-hand loop.

If we then open the switch, another sudden and brief induced current appears in the left hand loop, but in the opposite direction.

We get an induced current (and thus an induced *emf*) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large).

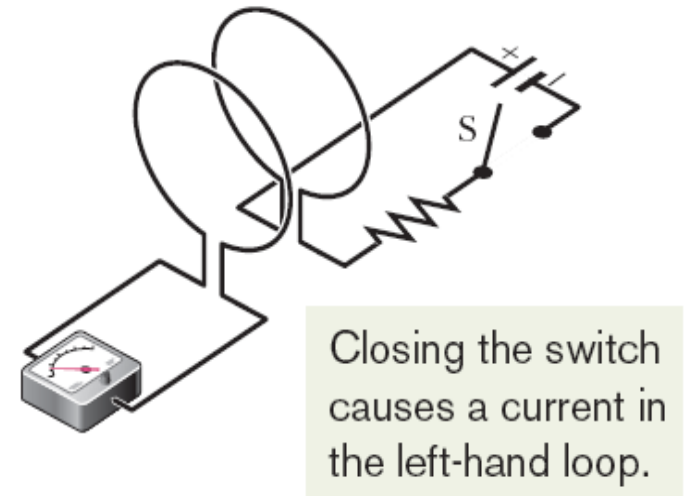


Fig. 30-2 An ammeter registers a current in the left-hand wire loop just as switch S is closed (to turn on the current in the right-hand wire loop) or opened (to turn off the current in the right-hand loop). No motion of the coils is involved.

Faraday's Law of Induction:



The magnitude of the emf $\mathcal{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

The *magnetic flux* is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \quad (\text{magnetic flux through area } A).$$

If the loop lies in a plane and the magnetic field is perpendicular to the plane of the loop, and if the magnetic field is constant, then

$$\Phi_B = BA \quad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform}).$$

The SI unit for magnetic flux is the tesla–square meter, which is called the *weber* (abbreviated *Wb*):

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2.$$

$$\mathcal{E} = - \frac{d\Phi_B}{dt} \quad (\text{Faraday's law}),$$

Faraday's Law of Induction:

If we change the magnetic flux through a coil of N turns, an induced emf appears in every turn and the total emf induced in the coil is the sum of these individual induced $emfs$. If the coil is tightly wound (closely packed), so the same magnetic flux Φ_B passes through all the turns, the total emf induced in the coil is

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} \quad (30-5)$$

Here are the general means by which we can change the magnetic flux through a coil:

- 1) Change the magnitude $\vec{B}$ of the magnetic field within the coil.
- 2) Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field).
- 3) Change the angle between the direction of the magnetic field $\vec{B}$ and the plane of the coil (for example, by rotating the coil so that field $\vec{B}$ is first perpendicular to the plane of the coil and then along that plane).

The long solenoid S shown (in cross section) in Fig. 30-3 has 220 turns/cm and carries a current $i = 1.5$ A; its diameter D is 3.2 cm. At its center we place a 130-turn closely packed coil C of diameter $d = 2.1$ cm. The current in the solenoid is reduced to zero at a steady rate in 25 ms. What is the magnitude of the emf that is induced in coil C while the current in the solenoid is changing?

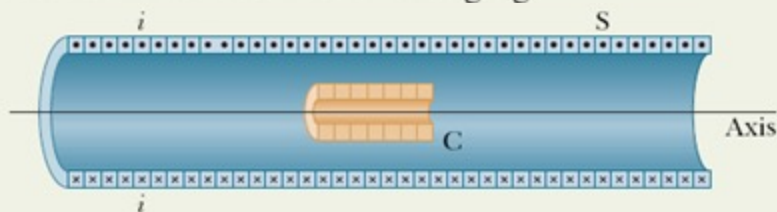


Fig. 30-3 A coil C is located inside a solenoid S, which carries current i .

KEY IDEAS

1. Because it is located in the interior of the solenoid, coil C lies within the magnetic field produced by current i in the solenoid; thus, there is a magnetic flux Φ_B through coil C.
2. Because current i decreases, flux Φ_B also decreases.
3. As Φ_B decreases, emf $\mathcal{E}$ is induced in coil C.
4. The flux through each turn of coil C depends on the area A and orientation of that turn in the solenoid's magnetic field $\vec{B}$. Because $\vec{B}$ is uniform and directed perpendicular to area A , the flux is given by Eq. 30-2 ($\Phi_B = BA$).
5. The magnitude B of the magnetic field in the interior of a solenoid depends on the solenoid's current i and its number n of turns per unit length, according to Eq. 29-23 ($B = \mu_0 in$).

Calculations: Because coil C consists of more than one turn, we apply Faraday's law in the form of Eq. 30-5 ($\mathcal{E} = -N d\Phi_B/dt$), where the number of turns N is 130 and $d\Phi_B/dt$ is the rate at which the flux changes.

Because the current in the solenoid decreases at a steady rate, flux Φ_B also decreases at a steady rate, and so we can write $d\Phi_B/dt$ as $\Delta\Phi_B/\Delta t$. Then, to evaluate $\Delta\Phi_B$, we need the final and initial flux values. The final flux $\Phi_{B,f}$ is zero because the final current in the solenoid is zero. To find the initial flux $\Phi_{B,i}$, we note that area A is $\frac{1}{4}\pi d^2$ ($= 3.464 \times 10^{-4} \text{ m}^2$) and the number n is 220 turns/cm, or 22 000 turns/m. Substituting Eq. 29-23 into Eq. 30-2 then leads to

$$\begin{aligned}\Phi_{B,i} &= BA = (\mu_0 in)A \\ &= (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.5 \text{ A})(22\,000 \text{ turns/m}) \\ &\quad \times (3.464 \times 10^{-4} \text{ m}^2) \\ &= 1.44 \times 10^{-5} \text{ Wb}.\end{aligned}$$

Now we can write

$$\begin{aligned}\frac{d\Phi_B}{dt} &= \frac{\Delta\Phi_B}{\Delta t} = \frac{\Phi_{B,f} - \Phi_{B,i}}{\Delta t} \\ &= \frac{(0 - 1.44 \times 10^{-5} \text{ Wb})}{25 \times 10^{-3} \text{ s}} \\ &= -5.76 \times 10^{-4} \text{ Wb/s} = -5.76 \times 10^{-4} \text{ V}.\end{aligned}$$

We are interested only in magnitudes; so we ignore the minus signs here and in Eq. 30-5, writing

$$\begin{aligned}\mathcal{E} &= N \frac{d\Phi_B}{dt} = (130 \text{ turns})(5.76 \times 10^{-4} \text{ V}) \\ &= 7.5 \times 10^{-2} \text{ V} = 75 \text{ mV}.\end{aligned}\quad (\text{Answer})$$

30.4: Lenz's Law:



An induced current has a direction such that the magnetic field due to *the current* opposes the change in the magnetic flux that induces the current.

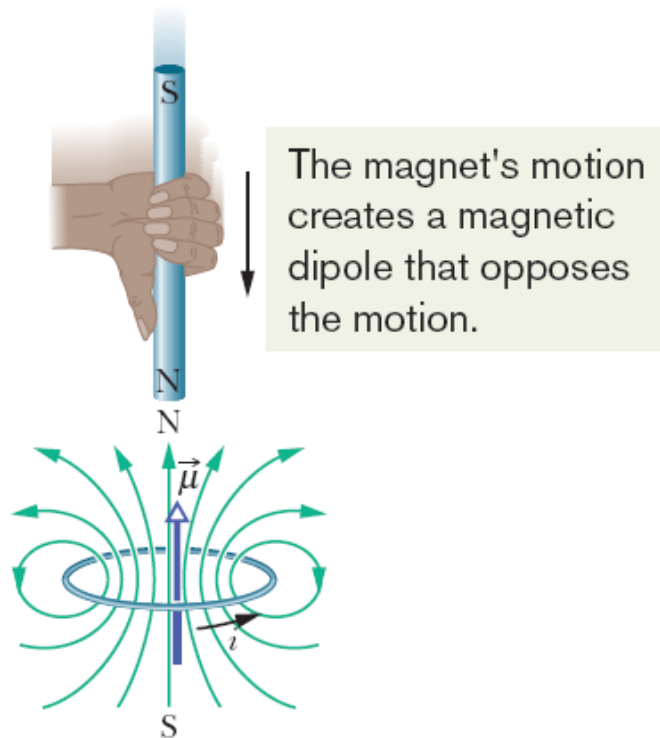


Fig. 30-4 Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment $\vec{\mu}$ oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

Opposition to Pole Movement. The approach of the magnet's north pole in Fig. 30-4 increases the magnetic flux through the loop, inducing a current in the loop. To oppose the magnetic flux increase being caused by the approaching magnet, the loop's north pole (and the magnetic moment μ) must face toward the approaching north pole so as to repel it. The current induced in the loop must be counterclockwise in Fig. 30-4. If we next pull the magnet away from the loop, a current will again be induced in the loop. Now, the loop will have a south pole facing the retreating north pole of the magnet, so as to oppose the retreat. Thus, the induced current will be clockwise.

Example, Induced emf and current due to a changing uniform B field:

Figure 30-6 shows a conducting loop consisting of a half-circle of radius $r = 0.20$ m and three straight sections. The half-circle lies in a uniform magnetic field $\vec{B}$ that is directed out of the page; the field magnitude is given by $B = 4.0t^2 + 2.0t + 3.0$, with B in teslas and t in seconds. An ideal battery with emf $\mathcal{E}_{\text{bat}} = 2.0$ V is connected to the loop. The resistance of the loop is 2.0Ω .

(a) What are the magnitude and direction of the emf $\mathcal{E}_{\text{ind}}$ induced around the loop by field $\vec{B}$ at $t = 10$ s?

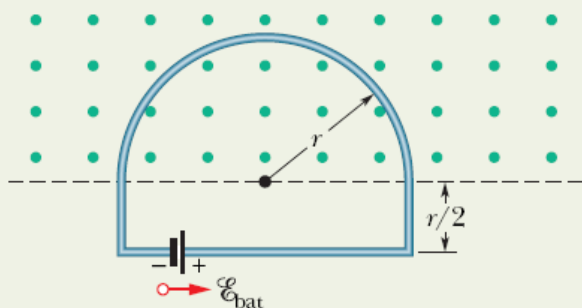


Fig. 30-6 A battery is connected to a conducting loop that includes a half-circle of radius r lying in a uniform magnetic field. The field is directed out of the page; its magnitude is changing.

$$\mathcal{E}_{\text{ind}} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A \frac{dB}{dt}.$$

Because the flux penetrates the loop only within the half-circle, the area A in this equation is $\frac{1}{2}\pi r^2$. Substituting this and the given expression for B yields

$$\begin{aligned}\mathcal{E}_{\text{ind}} &= A \frac{dB}{dt} = \frac{\pi r^2}{2} \frac{d}{dt} (4.0t^2 + 2.0t + 3.0) \\ &= \frac{\pi r^2}{2} (8.0t + 2.0).\end{aligned}$$

At $t = 10$ s, then,

$$\begin{aligned}\mathcal{E}_{\text{ind}} &= \frac{\pi (0.20 \text{ m})^2}{2} [8.0(10) + 2.0] \\ &= 5.152 \text{ V} \approx 5.2 \text{ V}.\end{aligned}\quad (\text{Answer})$$

Direction: To find the direction of $\mathcal{E}_{\text{ind}}$, we first note that in Fig. 30-6 the flux through the loop is out of the page and increasing. Because the induced field B_{ind} (due to the induced current) must oppose that increase, it must be *into* the page. Using the curled–straight right-hand rule (Fig. 30-5c), we find that the induced current is clockwise around the loop, and thus so is the induced emf $\mathcal{E}_{\text{ind}}$.

(b) What is the current in the loop at $t = 10$ s?

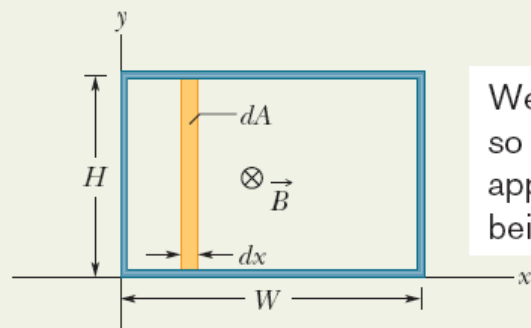
Calculation: The induced emf $\mathcal{E}_{\text{ind}}$ tends to drive a current clockwise around the loop; the battery's emf $\mathcal{E}_{\text{bat}}$ tends to drive a current counterclockwise. Because $\mathcal{E}_{\text{ind}}$ is greater than $\mathcal{E}_{\text{bat}}$, the net emf $\mathcal{E}_{\text{net}}$ is clockwise, and thus so is the current. To find the current at $t = 10$ s, we use Eq. 27-2 ($i = \mathcal{E}/R$):

$$\begin{aligned}i &= \frac{\mathcal{E}_{\text{net}}}{R} = \frac{\mathcal{E}_{\text{ind}} - \mathcal{E}_{\text{bat}}}{R} \\ &= \frac{5.152 \text{ V} - 2.0 \text{ V}}{2.0 \Omega} = 1.58 \text{ A} \approx 1.6 \text{ A}.\end{aligned}\quad (\text{Answer})$$

Example, Induced emf and current due to a changing nonuniform B field:

Figure 30-7 shows a rectangular loop of wire immersed in a nonuniform and varying magnetic field $\vec{B}$ that is perpendicular to and directed into the page. The field's magnitude is given by $B = 4t^2x^2$, with B in teslas, t in seconds,

If the field varies with position, we must integrate to get the flux through the loop.



We start with a strip so thin that we can approximate the field as being uniform within it.

Fig. 30-7 A closed conducting loop, of width W and height H , lies in a nonuniform, varying magnetic field that points directly into the page. To apply Faraday's law, we use the vertical strip of height H , width dx , and area dA .

Calculations: In Fig. 30-7, $\vec{B}$ is perpendicular to the plane of the loop (and hence parallel to the differential area vector $d\vec{A}$); so the dot product in Eq. 30-1 gives $B dA$. Because the magnetic field varies with the coordinate x but not with the coordinate y , we can take the differential area dA to be the area of a vertical strip of height H and width dx (as shown in Fig. 30-7). Then $dA = H dx$, and the flux through the loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B dA = \int BH dx = \int 4t^2x^2H dx.$$

Treating t as a constant for this integration and inserting the integration limits $x = 0$ and $x = 3.0$ m, we obtain

$$\Phi_B = 4t^2H \int_0^{3.0} x^2 dx = 4t^2H \left[\frac{x^3}{3} \right]_0^{3.0} = 72t^2,$$

where we have substituted $H = 2.0$ m and Φ_B is in webers. Now we can use Faraday's law to find the magnitude of $\mathcal{E}$ at any time t :

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(72t^2)}{dt} = 144t,$$

in which $\mathcal{E}$ is in volts. At $t = 0.10$ s,

$$\mathcal{E} = (144 \text{ V/s})(0.10 \text{ s}) \approx 14 \text{ V.} \quad (\text{Answer})$$

The flux of $\vec{B}$ through the loop is into the page in Fig. 30-7 and is increasing in magnitude because B is increasing in magnitude with time. By Lenz's law, the field B_{ind} of the induced current opposes this increase and so is directed out of the page. The curled-straight right-hand rule in Fig. 30-5a then tells us that the induced current is counterclockwise around the loop, and thus so is the induced emf $\mathcal{E}$.

30.5: Induction and Energy Transfers:

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL \frac{dx}{dt} = BLv,$$

The induced current is therefore

$$i = \frac{BLv}{R}.$$

The net deflecting force is:

$$\begin{aligned} F &= F_1 = iLB \sin 90^\circ = iLB. \\ &= \frac{B^2 L^2 v}{R}. \end{aligned}$$

The power is therefore $P = Fv = \frac{B^2 L^2 v^2}{R}$, which is exactly equal to $i^2 R$.

The work done (mechanical energy) is converted to thermal energy.

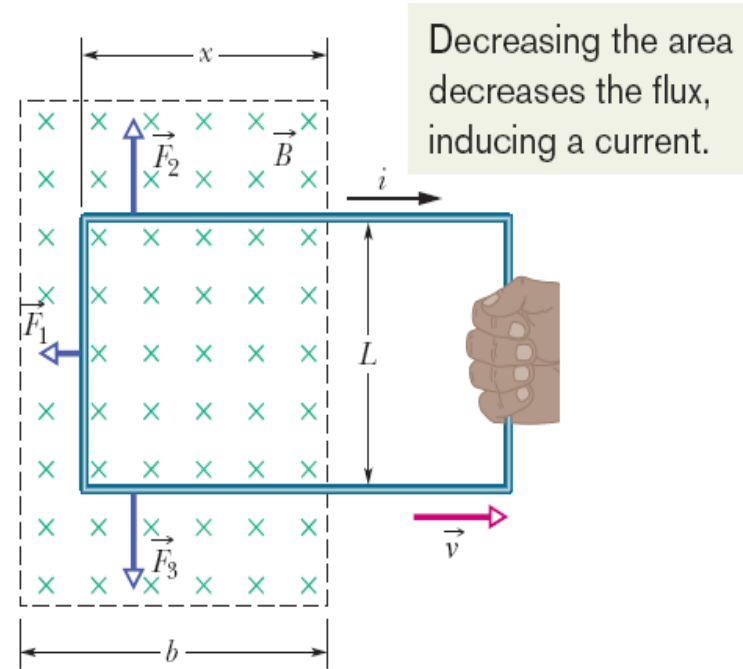


Fig. 30-8 You pull a closed conducting loop out of a magnetic field at constant velocity $\vec{v}$. While the loop is moving, a clockwise current i is induced in the loop, and the loop segments still within the magnetic field experience forces $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$.

30.5: Induction and Energy Transfers: Eddy Currents

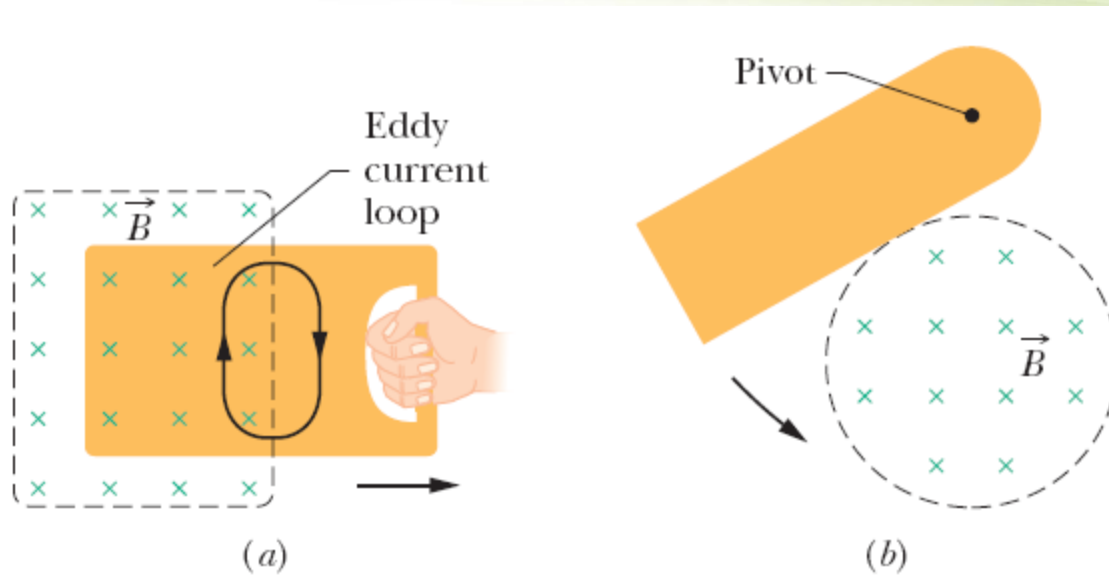


Fig. 30-10 (a) As you pull a solid conducting plate out of a magnetic field, *eddy currents* are induced in the plate. A typical loop of eddy current is shown. (b) A conducting plate is allowed to swing like a pendulum about a pivot and into a region of magnetic field. As it enters and leaves the field, eddy currents are induced in the plate.

Many applications:
braking,
heating,
magnetic levitation...

30.6: Induced Electric Fields:



A changing magnetic field produces an electric field.

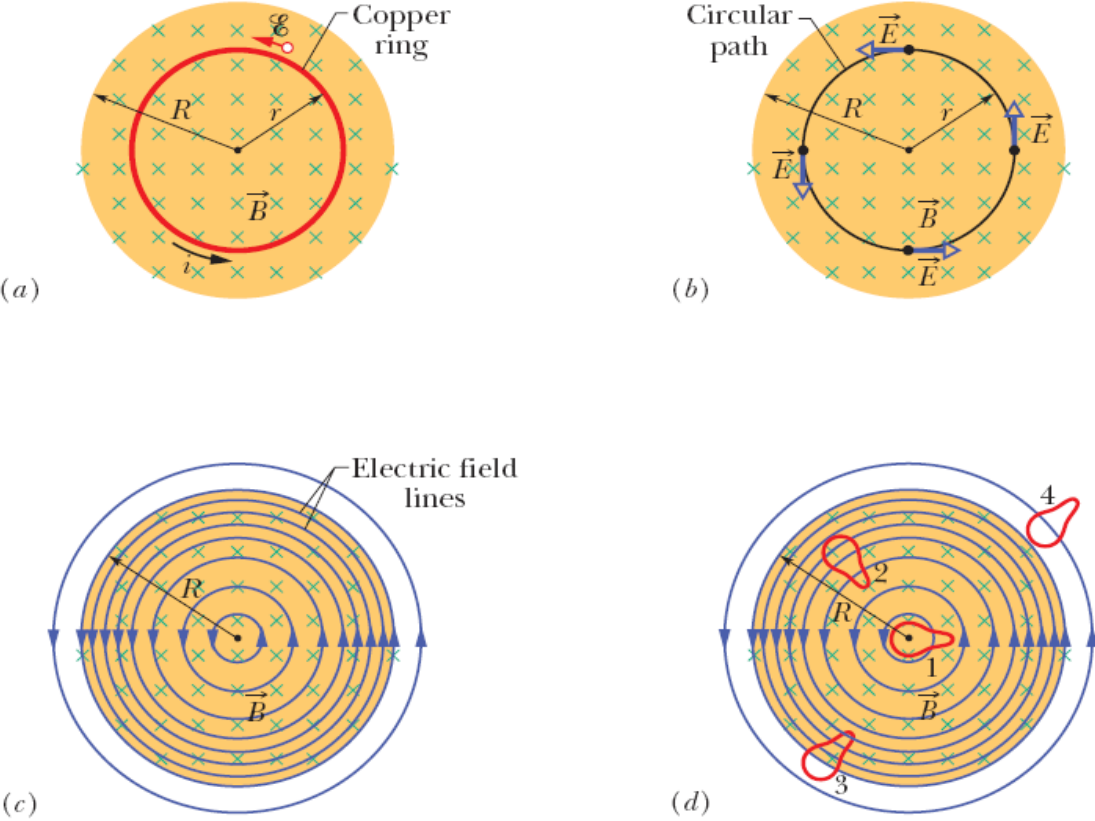


Fig. 30-11 (a) If the magnetic field increases at a steady rate, a constant induced current appears, as shown, in the copper ring of radius r . (b) An induced electric field exists even when the ring is removed; the electric field is shown at four points. (c) The complete picture of the induced electric field, displayed as field lines. (d) Four similar closed paths that enclose identical areas. Equal emfs are induced around paths 1 and 2, which lie entirely within the region of changing magnetic field. A smaller emf is induced around path 3, which only partially lies in that region. No net emf is induced around path 4, which lies entirely outside the magnetic field.

30.6: Induced Electric Fields, Reformulation of Faraday's Law:

$$W = \oint \vec{F} \cdot d\vec{s} = q_0 \oint \vec{E} \cdot d\vec{s}.$$

$$\Rightarrow \mathcal{E} = \oint \vec{E} \cdot d\vec{s}.$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt} \quad (\text{Faraday's law}).$$

$$\oint (\nabla \times \vec{E}) \cdot d\vec{a} = - \frac{\partial \oint \vec{B} \cdot d\vec{a}}{\partial t}$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (\text{Faraday's law in differential form})$$

30.6: Induced Electric Fields, A New Look at Electric Potential:



Electric potential has meaning only for electric fields that are produced by static charges; it has no meaning for electric fields that are produced by induction.

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt} \quad (\text{Faraday's law}).$$

The integration over a closed path is NOT zero!

The induced electric force is not a conservative force!

Recall: Net work done by a conservative force on a particle moving around every closed path is ZERO.

L05 slide 8



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Example, Induced electric field from changing magnetic field:

In Fig. 30-11*b*, take $R = 8.5$ cm and $dB/dt = 0.13$ T/s.

(a) Find an expression for the magnitude E of the induced electric field at points within the magnetic field, at radius r from the center of the magnetic field. Evaluate the expression for $r = 5.2$ cm.

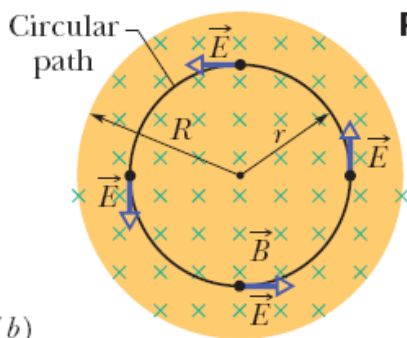


Fig. 30-11

Calculations: To calculate the field magnitude E , we apply Faraday's law in the form of Eq. 30-20. We use a circular path of integration with radius $r \leq R$ because we want E for points within the magnetic field. We assume from the symmetry that $\vec{E}$ in Fig. 30-11*b* is tangent to the circular path at all points. The path vector $d\vec{s}$ is also always tangent to the circular path; so the dot product $\vec{E} \cdot d\vec{s}$ in Eq. 30-20 must have the magnitude $E ds$ at all points on the path. We can also assume from the symmetry that E has the same value at all points along the circular path. Then the left side of Eq. 30-20 becomes

$$\oint \vec{E} \cdot d\vec{s} = \oint E ds = E \oint ds = E(2\pi r). \quad (30-23)$$

(The integral $\oint ds$ is the circumference $2\pi r$ of the circular path.)

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$$\Phi_B = BA = B(\pi r^2). \quad (30-24)$$

Substituting this and Eq. 30-23 into Eq. 30-20 and dropping the minus sign, we find that

$$E(2\pi r) = (\pi r^2) \frac{dB}{dt}$$

or
$$E = \frac{r}{2} \frac{dB}{dt}. \quad (\text{Answer}) \quad (30-25)$$

Equation 30-25 gives the magnitude of the electric field at any point for which $r \leq R$ (that is, within the magnetic field). Substituting given values yields, for the magnitude of $\vec{E}$ at $r = 5.2$ cm,

$$\begin{aligned} E &= \frac{(5.2 \times 10^{-2} \text{ m})}{2} (0.13 \text{ T/s}) \\ &= 0.0034 \text{ V/m} = 3.4 \text{ mV/m}. \quad (\text{Answer}) \end{aligned}$$

Example, Induced electric field from changing magnetic field, cont.:

(b) Find an expression for the magnitude E of the induced electric field at points that are outside the magnetic field, at radius r from the center of the magnetic field. Evaluate the expression for $r = 12.5$ cm.

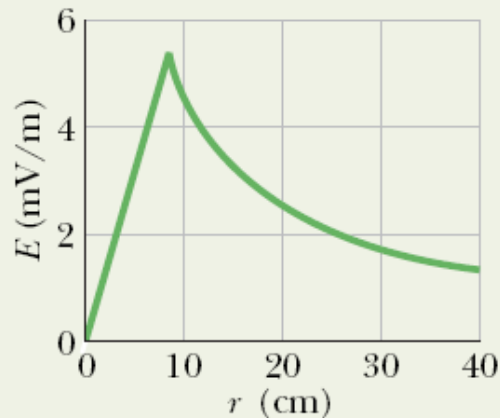
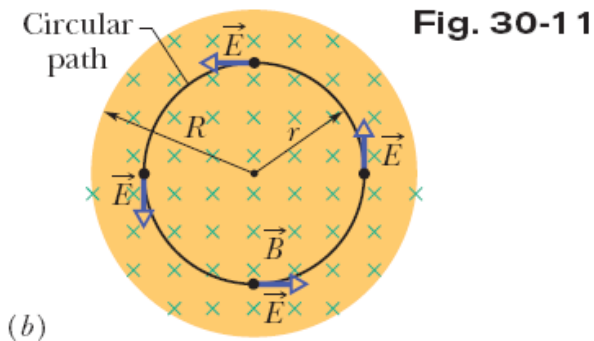


Fig. 30-12 A plot of the induced electric field $E(r)$.
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Calculations: We can now write

$$\Phi_B = BA = B(\pi R^2). \quad (30-26)$$

Substituting this and Eq. 30-23 into Eq. 30-20 (without the minus sign) and solving for E yield

$$E = \frac{R^2}{2r} \frac{dB}{dt}. \quad (\text{Answer}) \quad (30-27)$$

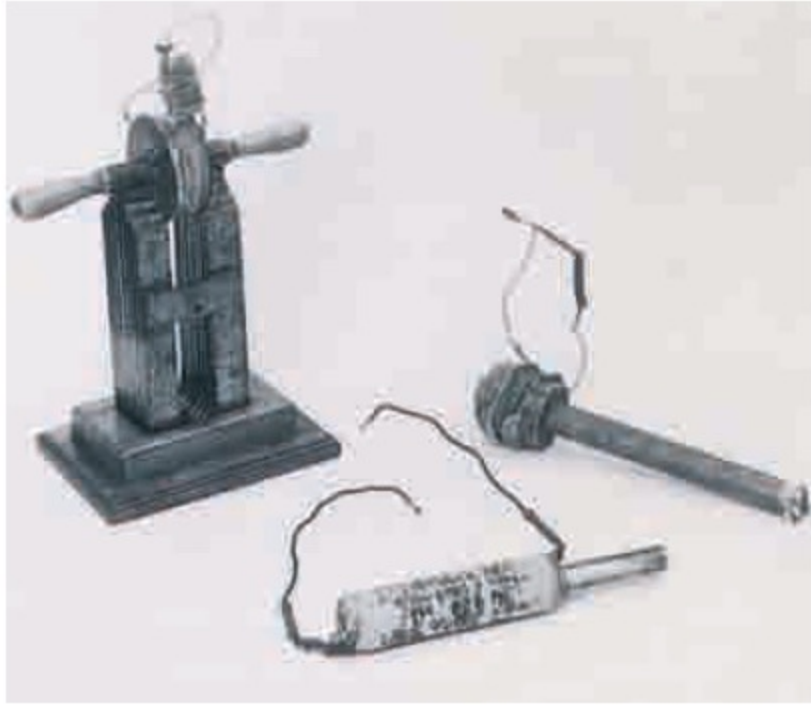
Because E is not zero here, we know that an electric field is induced even at points that are outside the changing magnetic field, an important result that (as you will see in Section 31-11) makes transformers possible.

With the given data, Eq. 30-27 yields the magnitude of $\vec{E}$ at $r = 12.5$ cm:

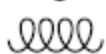
$$\begin{aligned} E &= \frac{(8.5 \times 10^{-2} \text{ m})^2}{(2)(12.5 \times 10^{-2} \text{ m})} (0.13 \text{ T/s}) \\ &= 3.8 \times 10^{-3} \text{ V/m} = 3.8 \text{ mV/m}. \quad (\text{Answer}) \end{aligned}$$

Equations 30-25 and 30-27 give the same result for $r = R$. Figure 30-12 shows a plot of $E(r)$. Note that the inside and outside plots meet at $r = R$.

Inductors and Inductance:



The crude inductors with which Michael Faraday discovered the law of induction. In those days amenities such as insulated wire were not commercially available. It is said that Faraday insulated his wires by wrapping them with strips cut from one of his wife's petticoats. (*The Royal Institution/Bridgeman Art Library/NY*)

An **inductor** (symbol ) can be used to produce a desired magnetic field.

If we establish a current i in the windings (turns) of the solenoid which can be treated as our inductor, the current produces a magnetic flux Φ_B through the central region of the inductor.

The inductance of the inductor is then

$$L = \frac{N\Phi_B}{i} \quad (\text{inductance defined})$$

The SI unit of inductance is the tesla-square meter per ampere ($\text{T m}^2/\text{A}$). We call this the **henry (H)**, after American physicist Joseph Henry.

Inductance of a Solenoid:

Consider a long solenoid of cross-sectional area A , with number of turns N , and of length l . The flux is $N\Phi_B = (nl)(BA)$,

Here n is the number of turns per unit length.

The magnitude of B is given by: $B = \mu_0 in$,

Therefore,
$$L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 in)(A)}{i}$$
$$= \mu_0 n^2 l A.$$

The *inductance per unit length near the center* is therefore:

$$\frac{L}{l} = \mu_0 n^2 A \quad (\text{solenoid}).$$

$$\begin{aligned}\mu_0 &= 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A} \\ &= 4\pi \times 10^{-7} \text{ H/m}.\end{aligned}$$

Self-Induction:



An induced emf $\mathcal{E}_L$ appears in any coil in which the current is changing.

This process (see Fig. 30-13) is called **self-induction**, and the emf that appears is called a **self-induced emf**. It obeys Faraday's law of induction just as other induced emfs do.

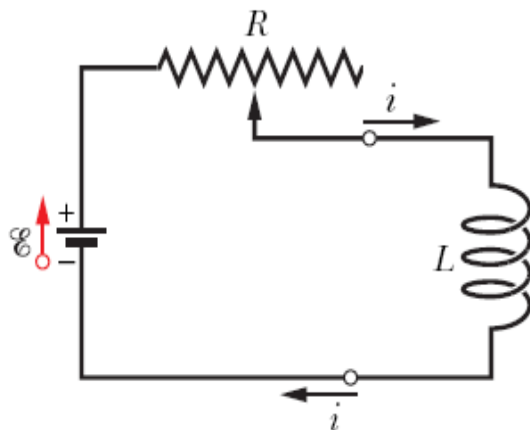


Fig. 30-13 If the current in a coil is changed by varying the contact position on a variable resistor, a self-induced emf $\mathcal{E}_L$ will appear in the coil *while the current is changing*.

$$N\Phi_B = Li.$$

$$\mathcal{E}_L = - \frac{d(N\Phi_B)}{dt}.$$

$$\mathcal{E}_L = -L \frac{di}{dt} \quad (\text{self-induced emf}).$$

This is defined in the direction of the current.

Inductors and lightning strikes

- If lightning strikes part of an electrical power transmission system, it causes a sudden spike in voltage that can damage the components of the system.
- To minimize these effects, large **inductors** are incorporated into the transmission system.
- These use the principle that an inductor opposes and suppresses any rapid changes in the current.



- Magnetic Flux:

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \quad (\text{magnetic flux through area } A).$$

$$\Phi_B = BA \quad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform}).$$

- Faraday's Law of Induction

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday's law}),$$

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2.$$

- Lenz's Law
 - An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.
 - The induced emf has the same direction as the induced current.

- Emf and the induced Electric Field

- The changing magnetic field induces an electric field $\vec{E}$ at every point of the closed path; the induced emf is related to $\vec{E}$ by

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}.$$

- The general expression of Faraday's law:

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt} \quad (\text{Faraday's law}).$$

- Inductors

$$L = \frac{N\Phi_B}{i} \quad (\text{inductance defined})$$

$$\frac{L}{l} = \mu_0 n^2 A \quad (\text{solenoid}).$$

- Self-induction

$$\mathcal{E}_L = -L \frac{di}{dt} \quad (\text{self-induced emf}).$$

Practice Questions:

Chapter 30 - Induction and Inductance

Questions: *3, *7, **10, *29, *37, *45