

CITY UNIVERSITY OF HONG KONG

Course code & title : **EE 3114 Systems and Control**
Session : **Semester A 2020/21**
Time allowed : **Two hours**

This paper has **FIVE** pages (including this cover page and an appendix).

Answer **ALL** questions.

This is an **open-book** examination. Materials allowed during the examination include only:

- Calculator
- Textbook
- Lecture notes, homework assignments, midterm test scripts

I pledge that the answers in this exam are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

- *I will not plagiarize from any source;*
- *I will not communicate or attempt to communicate with any other person during the exam, neither will I give or attempt to give assistance to another student taking the exam; and*
- *I will use only approved devices (e.g., calculators) and/or approved materials models.*

I understand that any act of academic dishonesty can lead to disciplinary action.

Signature:

Student ID:

Q1: [10 Marks] For the circuit shown in Figure Q1, e_i is the input voltage and e_o is the output voltage.

(i) [5 Marks] Find the system's transfer function $\frac{e_o(s)}{e_i(s)}$.

(ii) [5 Marks] Find a differential equation relating the input $e_i(t)$ to the output $e_o(t)$.

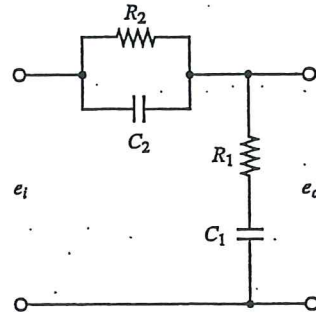


Figure Q1

Q2: [10 Marks] Consider the block diagram shown in Figure Q2. Find the transfer function $\frac{C(s)}{R(s)}$.

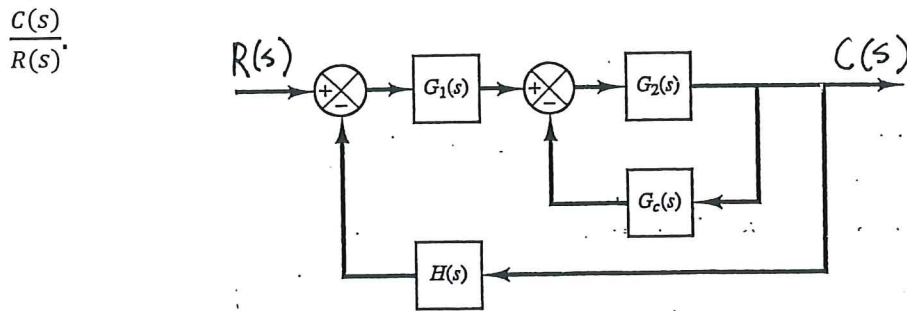


Figure Q2

Q3: [15 Marks] Consider the feedback system shown in Figure Q3, in which $J = 1$.

(i) [10 Marks] Determine the value of k and K such that the system's maximum overshoot $M_p = 0.25$ and the peak time $t_p = 2$.

(ii) [5 Marks] Find, accordingly,

- rise time t_r
- settling time t_s

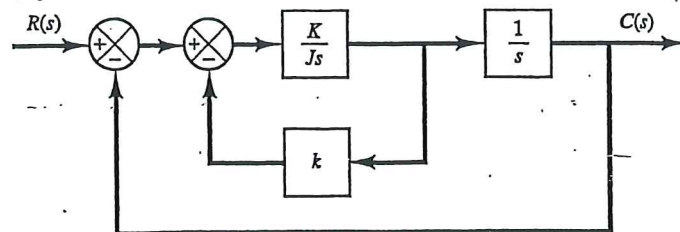
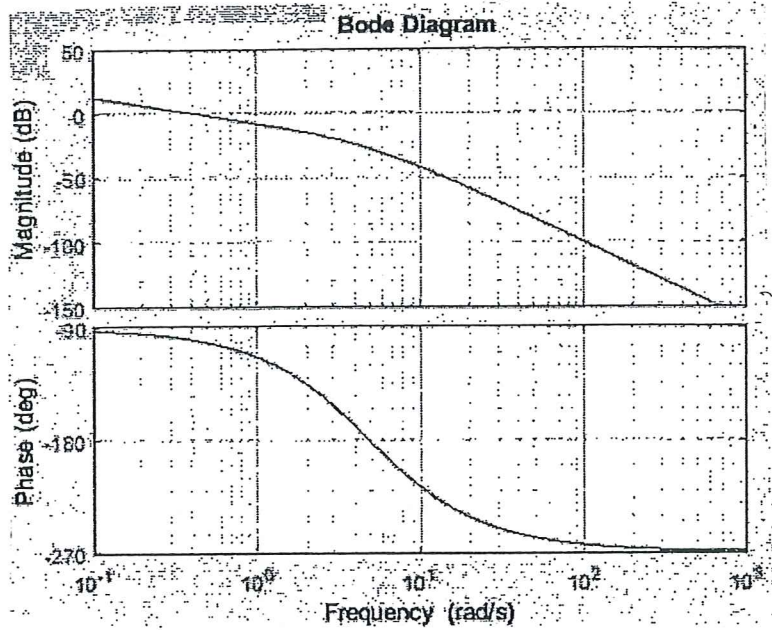


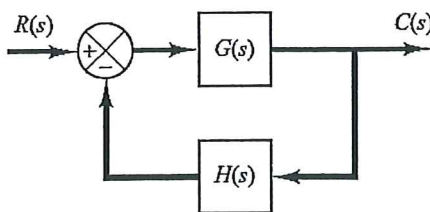
Figure Q3

Q4: [10 Marks]

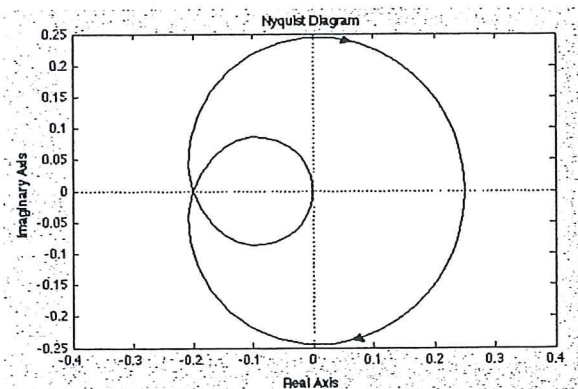
- (i) [5 Marks] Given a Bode plot shown in Figure Q4(a), find the gain margin and the phase margin.
- (ii) [5 Marks] Consider the feedback system shown in Figure Q4(b), where $G(s) = \frac{1-s}{1+s}$ and $H(s) = \frac{1}{s+4}$. Given its Nyquist plot shown in Figure Q4(c), determine if the system is stable or not. Show your reasoning.



(a)



(b)



(c)

Figure Q4

Q5: [20 Marks] Consider the system shown in Figure Q5.

- (i) [5 Marks] Determine the closed-loop characteristic polynomial.
- (ii) [10 Marks] For what ranges of a , the system is stable?
- (iii) [5 Marks] Sketch the root locus of the system as the parameter a varies from 0 to infinity.

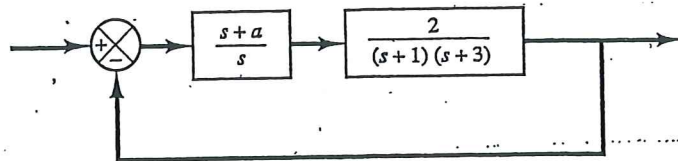


Figure Q5

Q6: [15 Marks] Given an LTI system with transfer function

$$G(s) = \frac{s+3}{s^2+7s+12}$$

- (i) [5 Marks] Find its state-space matrices (A, B, C, D) in controllable canonical form
- (ii) [5 Marks] Verify, using the controllability matrix, if this state-space realization is controllable?
- (iii) [5 Marks] Is this state-space realization a minimal realization? Explain why or why not.

Q7: [10 Marks] Consider the system

$$\dot{x}(t) = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t),$$

$$y(t) = [1 \ 1]x(t)$$

- (i) [5 marks] Design a state feedback which has eigenvalues at -2, -4.
- (ii) [5 marks] Determine the transfer function of the closed loop system.

Q8: [10 Marks] Consider a system with the transfer function

$$G(s) = \frac{(s-1)(s+2)}{(s+1)(s-2)(s+3)}$$

- (i) [5 Marks] Is it possible to change the transfer function to

$$G_f(s) = \frac{(s-1)}{(s+2)(s+3)}$$

by state feedback? Explain why or why not.

- (ii) [5 Marks] Is the resulting system BIBO stable? Is it asymptotically stable? Explain why or why not.

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Appendix

Table of Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$	2. e^{at}	$\frac{1}{s-a}$
3. $t^n, n=1,2,3,\dots$	$\frac{n!}{s^{n+1}}$	4. $t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. $\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	6. $t^{n-1/2}, n=1,2,3,\dots$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+1/2}}$
7. $\sin(at)$	$\frac{a}{s^2+a^2}$	8. $\cos(at)$	$\frac{s}{s^2+a^2}$
9. $t \sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10. $t \cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11. $\sin(at) - at \cos(at)$	$\frac{2a^3}{(s^2+a^2)^3}$	12. $\sin(at) + at \cos(at)$	$\frac{2as^2}{(s^2+a^2)^3}$
13. $\cos(at) - at \sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^3}$	14. $\cos(at) + at \sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^3}$
15. $\sin(at+b)$	$\frac{s \sin(b) + a \cos(b)}{s^2+a^2}$	16. $\cos(at+b)$	$\frac{s \cos(b) - a \sin(b)}{s^2+a^2}$
17. $\sinh(at)$	$\frac{a}{s^2-a^2}$	18. $\cosh(at)$	$\frac{s}{s^2-a^2}$
19. $e^{at} \sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20. $e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21. $e^{at} \sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22. $e^{at} \cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23. $t^n e^{at}, n=1,2,3,\dots$	$\frac{n!}{(s-a)^{n+1}}$	24. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
25. $u_c(t) = u(t-c)$ Heaviside Function	$\frac{e^{-cs}}{s}$	26. $\delta(t-c)$ Dirac Delta Function	e^{-cs}
27. $u_c(t) f(t-c)$	$e^{-cs} F(s)$	28. $u_c(t) g(t)$	$e^{-cs} \mathcal{L}\{g(t+c)\}$
29. $e^{ct} f(t)$	$F(s-c)$	30. $t^n f(t), n=1,2,3,\dots$	$(-1)^n F^{(n)}(s)$
31. $\frac{1}{t} f(t)$	$\int_s^\infty F(u) du$	32. $\int_a^t f(v) dv$	$\frac{F(s)}{s}$
33. $\int_0^t f(t-\tau) g(\tau) d\tau$	$F(s) G(s)$	34. $f(t+T) = f(t)$	$\frac{\int_0^T e^{-st} f(t) dt}{1-e^{-sT}}$
35. $f'(t)$	$sF(s) - f(0)$	36. $f''(t)$	$s^2 F(s) - sf'(0) - f''(0)$
37. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$		