

EE3210 Signals and Systems

Semester B 2023-2024

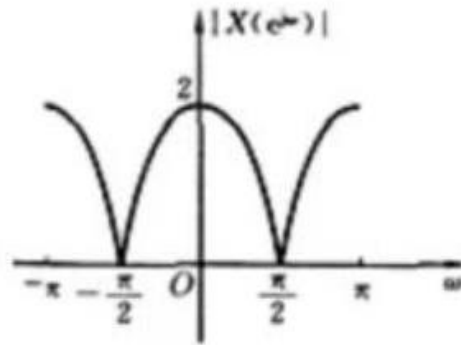
Assignment 2 Solution

1. (9 marks)

Based on the definition of the DTFT, we have

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \delta[n-1]e^{-j\omega n} + \sum_{n=-\infty}^{\infty} \delta[n+1]e^{-j\omega n} = e^{-j\omega} + e^{j\omega} = 2\cos \omega$$

The magnitude of the frequency spectrum should be $|X(e^{j\omega})| = 2|\cos \omega|$



2. (8 marks)

Using the Euler formula $\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$, we have

$$X(j\Omega) = \frac{e^{-j(3\Omega+1)} + e^{j(3\Omega+1)}}{2} = \frac{1}{2e^j} e^{-3j\Omega} + \frac{e^j}{2} e^{3j\Omega}.$$

By taking the inverse Fourier transfer, we have $x(t) = \frac{1}{2e^j} \delta(t-3) + \frac{e^j}{2} \delta(t+3)$.

3. (10 marks)

The maximum frequency of $x(t)$ is 100Hz. The signal $x(t/2)$ is obtained by stretching the signal $x(t)$. The maximum frequency of $x(t)$ is 50Hz. Therefore, the minimum sampling frequency is 100Hz, and the maximum sampling period is 0.01s.

4. (16 marks)

(a) Since signals $x_1(1-t)$ and $x_1(t-1)$ are obtained by the time shift and time reversal of the original signal $x_1(t)$, they have the same fundamental period with $x_1(t)$. $x_2(t)$ is the linear superposition of $x_1(1-t)$ and $x_1(t-1)$. Therefore, $x_2(t)$ has the same fundamental frequency with $x_1(t)$, i.e., $\Omega_2 = \Omega_1$.

(b)

$$x_1(t) \longleftrightarrow a_k$$

$$x_1(t-1) \longleftrightarrow a_k e^{-jk\Omega_1}$$

$$x_1(t+1) \longleftrightarrow a_k e^{jk\Omega_1}$$

$$x_1(-t+1) \longleftrightarrow a_{-k} e^{-jk\Omega_1}$$

$$\text{So } x_2(t) = x_1(1-t) + x_1(t-1) \longleftrightarrow (a_k + a_{-k}) e^{-jk\Omega_1}$$

$$b_k = (a_k + a_{-k}) e^{-jk\Omega_1}$$

5. (21 marks)

(a) Based on the conclusion in Lecture notes Example 5.2, we have

$$h_1(t) = \frac{\sin 4t}{\pi t} \longleftrightarrow H_1(j\Omega) = \begin{cases} 1, & -4 < \Omega < 4 \\ 0, & \text{else} \end{cases}$$

$$\text{So } h(t) = h_1(t-1) \longleftrightarrow H_1(j\Omega) e^{-j\Omega} = \begin{cases} e^{-j\Omega}, & -4 < \Omega < 4 \\ 0, & \text{else} \end{cases}$$

$$(b) \text{ Using the Euler formular, we have } x(t) = \cos(6t + \frac{\pi}{2}) = \frac{1}{2} e^{j6t} e^{j\frac{\pi}{2}} + \frac{1}{2} e^{-j6t} e^{-j\frac{\pi}{2}}$$

$$X(j\Omega) = \frac{1}{2} e^{j\frac{\pi}{2}} 2\pi\delta(\Omega-6) + \frac{1}{2} e^{-j\frac{\pi}{2}} 2\pi\delta(\Omega+6)$$

$$= \pi e^{j\frac{\pi}{2}} \delta(\Omega-6) + \pi e^{-j\frac{\pi}{2}} \delta(\Omega+6)$$

$$= j\pi\delta(\Omega-6) - j\pi\delta(\Omega+6)$$

$$(c) Y(j\Omega) = H(j\Omega)X(j\Omega) = 0$$

$$y(t) = 0$$

6. (15 marks)

The fundamental frequency of the input signal is $\Omega_0 = \frac{2\pi}{T} = 12$.

The output signal is $y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\Omega_0) e^{jk\Omega_0 t}$.

The system frequency response is $H(j\Omega) = 0$ when $|\Omega| > 100$.

To ensure $y(t) = x(t)$, we should have

$$|k| \Omega_0 > 100$$

Therefore, $|k| > 8$ for $a_k = 0$.

7. (21 marks)

(a) In a denoising system, the input contains a signal-of-interest and additive noise, and the system attempts to extract the signal-of-interest while removing/suppressing the noise, thus it is expected that the output is a good approximation of the signal-of-interest. The principle is that, under a certain transform, the signal-of-interest is sparse, meaning that there are only a few non-zero entries and we only need to keep them and ignoring the rest.

(b) In theory, we can transform the continuous-time signal to frequency domain using Fourier transform. The $\cos(100\pi t)$ corresponds to two impulses at -100π and 100π in the frequency domain. We keep only the two components at -100π and 100π , and assign the rest to zero, and convert this resultant frequency-domain signal to time domain using the inverse Fourier transform.

Compression is easily achieved. Instead of storing $\cos(100\pi t)$, we only need to store the amplitudes and locations of the two impulses in the frequency-domain signal.