

Name: _____

CityU ID# _____

EE1002 Test 1

Instructions: 1. Attend all questions.

2. Show all necessary steps and provide full justification for your answers.

Question 1 (20%)

Calculate the limit of the following functions. If limit does not exist, show it/provide justification.

$$a) \lim_{t \rightarrow 1} \frac{t^3 - 1}{t^2 - 1} = \lim_{t \rightarrow 1} \frac{(t-1)(t^2+t+1)}{(t-1)(t+1)} = \lim_{t \rightarrow 1} \frac{t^2+t+1}{t+1} = \frac{3}{2} = 1\frac{1}{2}$$

$$b) \lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right) = \lim_{x \rightarrow 1} \frac{x+1-2}{(x^2-1)} = \lim_{x \rightarrow 1} \frac{1}{x+1} = \frac{1}{2}$$

$$c) \lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} = \lim_{h \rightarrow 0} \frac{(\sqrt{2+h} - \sqrt{2})(\sqrt{2+h} + \sqrt{2})}{h(\sqrt{2+h} + \sqrt{2})} = \lim_{h \rightarrow 0} \frac{2+h-2}{h(\sqrt{2+h} + \sqrt{2})}$$

$$d) \lim_{x \rightarrow \infty} \sqrt{4x^2 - x} + 2x = \lim_{x \rightarrow \infty} (\sqrt{4x^2 - x} + 2x) \times \frac{\sqrt{4x^2 - x} - 2x}{\sqrt{4x^2 - x} - 2x} = \lim_{x \rightarrow \infty} \frac{-x}{\sqrt{4x^2 - x} - 2x} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{4 - \frac{1}{x}} - 2} = \frac{1}{4} //$$

Question 2 (20%)

Calculate each of the following derivatives:

$$a) f(x) = (x^2 + 3x + 1)(x - 1)$$

$$f'(x) = (2x+3)(x-1) + (x^2+3x+1) \cdot 1 = 2x^2 + x - 3 + x^2 + 3x + 1 = 3x^2 + 4x - 2$$

$$b) y = e^{-x^2}$$

$$y' = e^{-x^2} \cdot (-2x) = -2xe^{-x^2}$$

$$c) f(t) = \frac{1+t^2}{\sqrt{t}}$$

$$f'(t) = \frac{1}{\sqrt{t}} \left(\sqrt{t} \cdot 2t - (1+t^2) \cdot \frac{1}{2} t^{-\frac{1}{2}} \right) = 2\sqrt{t} - \frac{1+t^2}{2} t^{-\frac{3}{2}} = \frac{\sqrt{t}}{2} \left(3 - \frac{1}{t^2} \right)$$

$$d) f(x) = \cos(\sqrt{x+2} - 1)$$

$$f'(x) = -\sin(\sqrt{x+2} - 1) \times \frac{1}{2}(x+2)^{-\frac{1}{2}}$$

$$= -\frac{\sin(\sqrt{x+2} - 1)}{2\sqrt{x+2}}$$

Q.3 $f(x) = x^5 + x + 1$

Let the inverse function be $x = g(y) = g(f(x))$

$$\frac{dg}{dx} = \frac{dg}{dy} \cdot \frac{dy}{dx}$$

$$1 = \frac{dg}{dy} \cdot \frac{dy}{dx}$$

$$\therefore \frac{dg}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{5x^4 + 1}$$

$$\therefore g'(x) = \frac{1}{5x^4 + 1}$$

$$\text{For } x=1, \quad g'(1) = \frac{1}{5+1} = \frac{1}{6}$$

$$x=5 \quad g'(x=5) = \frac{1}{5 \cdot 5^4 + 1} = \frac{1}{3126}$$

Question 4

$$f(x) = x^3 + 3x^2 + x + 1$$

$$(a) \quad f'(x) = 3x^2 + 6x + 1$$

$$(b) \quad f''(x) = 6x + 6$$

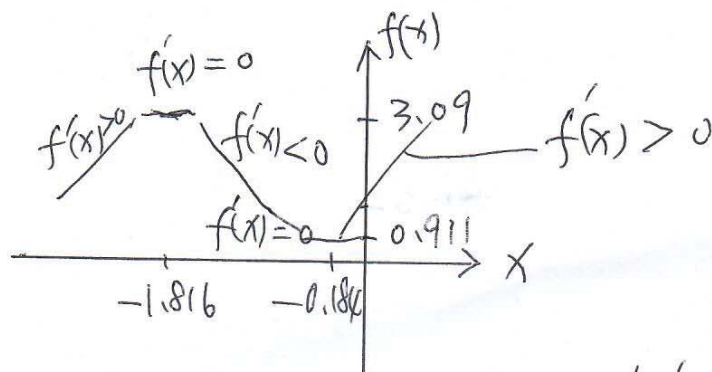
$$(c) \quad f'(x) = 0, \text{ i.e. } 3x^2 + 6x + 1 = 0$$

$$x_1 = -1.816, \quad x_2 = -0.184$$

(d)

x	$(-\infty, -1.816)$	-1.816	$(-1.816, -0.184)$	-0.184	$(-0.184, +\infty)$
$f'(x)$	+	0	-	0	+
$f''(x)$	$(-\infty, -4.896)$	-4.896	$(-4.896, 4.896)$	4.896	$(4.896, \infty)$
$f(x)$	—	3.09	—	0.911	—

(e)



It can be noted from the above sketch that f is concave upward in $(-1.816, +\infty)$ and concave downward in $(-\infty, -0.184)$.

(f) During $(-\infty, -0.184)$, $f'(x)$ changes from positive to zero, and negative, it is monotonically decrease, and $f''(x)$ is always negative, i.e. $f''(x) < 0$ indicates local maxima. Similarly, $f''(x) > 0$ implies a local minima in the interval $(-1.816, +\infty)$.