

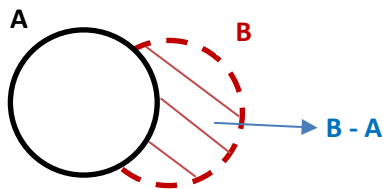
Quiz 1 (Solution)

1. $(a, \alpha), (b, \alpha), (c, \alpha), (a, \beta), (b, \beta), (c, \beta)$

2.

$$\begin{aligned} A \cup (B - A) &= A \cup (B \cap A^c) \\ &= (A \cup B) \cap (A \cup A^c) \\ &= (A \cup B) \cap U \\ &= A \cup B \end{aligned}$$

The following Venn diagram is for reference only:



3. $50x + 30y = 1490.$

50	30		
1	0	50	a
0	1	30	b
1	-1	20	$c = a - b$
-1	2	10	$d = b - c = -a + 2b$

$$50(-1) + 30(2) = 10$$

Multiplying both sides by 149, we obtain $50(-149) + 30(298) = 1490.$

The general solution is given by

$$x = -149 + 3t.$$

$$y = 298 - 5t.$$

Since x, y must be non-negative, we obtain $50 \leq t \leq 59$, so there are 10 combinations.

4.

71	29		
1	0	71	a
0	1	29	b
1	-2	13	$c = a - 2b$
a) -2	5	3	$d = b - 2c = -2a + 5b$
9	-22	1	$e = c - 4d = 9a - 22b$

$$x = (71)(9)(6) + (29)(-22)(5) = 644.$$

5.

a) Yes.

Proof: Suppose $f(n_1) = f(n_2) = c$.

Case 1: c is an odd number.

By the definition of f , n must be an even number.

Therefore, we have $2n_1 + 1 = 2n_2 + 1 = c$, which implies $n_1 = n_2$.

Case 2: c is an even number.

By the definition of f , n must be an odd number.

Therefore, we have $2n_1 = 2n_2 = c$, which implies $n_1 = n_2$.

In both cases, we have $n_1 = n_2$. Hence, f is an injective function.

Q.E.D.

b) No. There does not exist a value for n such that $f(n) = 1$.

6. a) $77 = p \times q = 7 \times 11$. Then $(p - 1)(q - 1) = 6 \times 10$

$$ed \equiv 1 \pmod{60}$$

$$d = 11$$

b) $D(28) = m = c^d \pmod{N} = 28^d \pmod{77}$

$$m = 28^{11} \pmod{77}$$

$$28 \pmod{77} = 28$$

$$28^2 \pmod{77} = 14$$

$$28^4 \pmod{77} = 42$$

$$28^8 \pmod{77} = 70$$

$$28^8 28^2 28 \pmod{77} = 70 \times 14 \times 28 \pmod{77} = 28.$$