

EE1001 2022-23 Sem B, Exercise (Q+A)

Q1

Determine the following statements whether they are true or false. (10 marks)

- (i) “Oxygen is one of the basic elements of life” is a valid and sound argument.
- (ii) A valid argument can go from all false premises to a true conclusion.
- (iii) Given the statement of “If you study hard, you get an A in this course”, “Studying hard” is a necessary condition for “getting an A in this course”.
- (iv) “If you study hard, you get an A in this course” is logically equivalent to “If you do not get an A in this course, you did not study hard”.
- (v) Given $A = \{a_1, a_2, a_3\}$. $\exists x \in A, T(x)$ is logically equivalent to $T(a_1) \wedge T(a_2) \wedge T(a_3)$.

Solution of Q1

- (i) **False**
(An argument should be composed of at least one premise and a conclusion.)
- (ii) **True**
(Unless the situation of all true premises with a false conclusion, a valid argument can go from all false premises to a true conclusion.)
- (iii) **False**
(“Study hard” is a sufficient condition for “getting an A in this course”)
- (iv) **True**
(The converse is equivalent to inverse/ verify using truth table)
- (v) **False**
(It should be $T(a_1) \vee T(a_2) \vee T(a_3)$)

Q2

Determine whether the following statement is a tautology, a contradiction, or neither.

$$\sim[(a \rightarrow b) \wedge \sim b \wedge (a \vee c) \rightarrow c]$$

Solution of Q2

$$\begin{aligned}
 & \sim[(a \rightarrow b) \wedge \sim b \wedge (a \vee c) \rightarrow c] \\
 \equiv & \sim[(\sim a \vee b) \wedge \sim b \wedge (a \vee c) \rightarrow c] && \text{(Definition of } \rightarrow \text{)} \\
 \equiv & \sim[\sim b \wedge (\sim a \vee b) \wedge (a \vee c) \rightarrow c] && \text{(Commutative laws)} \\
 \equiv & \sim\{[(\sim b \wedge \sim a) \vee (\sim b \wedge b)] \wedge (a \vee c) \rightarrow c\} && \text{(Distributive laws)} \\
 \equiv & \sim\{[(\sim b \wedge \sim a) \vee \mathbf{c}] \wedge (a \vee c) \rightarrow c\} && \text{(Negation laws)} \\
 \equiv & \sim[(\sim b \wedge \sim a) \wedge (a \vee c) \rightarrow c] && \text{(Identity laws)} \\
 \equiv & \sim\{\sim b \wedge [(\sim a \wedge a) \vee (a \wedge c)] \rightarrow c\} && \text{(Distributive laws)} \\
 \equiv & \sim\{\sim b \wedge [\mathbf{c} \vee (a \wedge c)] \rightarrow c\} && \text{(Negation laws)} \\
 \equiv & \sim[\sim b \wedge (a \wedge c) \rightarrow c] && \text{(Identity laws)} \\
 \equiv & \sim\{\sim[\sim b \wedge (a \wedge c)] \vee c\} && \text{(Definition of } \rightarrow \text{)} \\
 \equiv & \sim[b \vee \sim a \vee \sim c \vee c] && \text{(De Morgan's laws)} \\
 \equiv & \sim[b \vee \sim a \vee \mathbf{t}] && \text{(Negation laws)} \\
 \equiv & \sim \mathbf{t} && \text{(Universal bound laws)} \\
 \equiv & \mathbf{c} && \text{(Negation of t)}
 \end{aligned}$$

It's a contradiction.

Q3

Given two types of devices, namely normal and hacked devices, the normal devices always give **true** (“**T**”) messages, whereas the hacked devices always give **false** (“**F**”) messages.

There are THREE (3) devices, and their messages are as follows:

Device A: “All of us are normal.”

Device B: “If A is hacked, then B is normal and C is hacked.”

Device C: “I’m normal, and at least one of the others is hacked”

Let a , b , and c be

a = “Device A is normal”

b = “Device B is normal”

c = “Device C is normal”

Q3(i) Formulate the statements of Devices A, B, and C, respectively.

Q3(ii) Using truth table to determine whether Devices A, B, and C are normal or hacked, respectively.

(Hint: There are THREE (3) possible situations)

Solution of Q3

Q3(i)

The statement of the devices:

“ $a \wedge b \wedge c$ ” = A’s statement = “All of us are normal”

“ $\sim a \rightarrow (b \wedge \sim c)$ ” = B’s statement = “If A is hacked, then B is normal and C is hacked”

“ $c \wedge (\sim a \vee \sim b)$ ” = C’s statement = “I’m normal, and at least one of the others is hacked”

Q3(ii)

The situation is satisfied only when the truth values of the devices’ statuses and the truth values of the devices’ statements are the same, i.e.,

$[a \leftrightarrow (a \wedge b \wedge c)] \wedge [b \leftrightarrow \sim a \rightarrow (b \wedge \sim c)] \wedge [c \leftrightarrow c \wedge (\sim a \vee \sim b)]$ must be true

a	b	c	$a \wedge b \wedge c$	$\sim a \rightarrow (b \wedge \sim c)$	$c \wedge (\sim a \vee \sim b)$	$[a \leftrightarrow (a \wedge b \wedge c)] \wedge [b \leftrightarrow \sim a \rightarrow (b \wedge \sim c)] \wedge [c \leftrightarrow c \wedge (\sim a \vee \sim b)]$
T	T	T	T	T	F	F
T	T	F	F	T	F	F
T	F	T	F	T	T	F
T	F	F	F	T	F	F
F	T	T	F	F	T	F
F	T	F	F	T	F	T
F	F	T	F	F	T	T
F	F	F	F	F	F	T

Therefore, there are three possible situations:

1. Devices A and C are hacked, and Device B is normal.

2. Devices A and B are hacked and Device C is normal.
3. All Devices A, B, and C are hacked.

Q4

A series of experiments is carried out that involves the use of increasing number of non-reusable capacitors (i.e., the capacitors used in each round of experiment cannot be re-used in the next round).

In the first experiment, the testbed involves 50 capacitors. In the second experiment, 65 capacitors are involved.

- (i) It is given that the numbers of the capacitors used form an arithmetic progression. Determine the total number of the capacitors in the first 25 experiments.
- (ii) Instead, it is given that the number of the capacitors used form a geometric progression. There are a total of 23,000 capacitors available for the testbed. Determine the greatest number of possible experiment (N).

Solution of Q4

Q4(i)

$$d = 65 - 50 = 15$$

Total number of the devices for the first 25 experiments
 $= 50 + 65 + 80 + \dots + L$

$$S_n = \frac{n}{2} [2a_1 + (n-1)d]$$

$$S_{25} = \frac{25}{2} [2(50) + (25-1)15]$$

$$S_{25} = 5750$$

Q4(ii)

For N experiments with $S_N \leq 23,000$

Given the geometric series of $50 + 65 + \dots$

$$r = 65/50 = 1.3$$

$$\frac{a(r^N - 1)}{r - 1} \leq 23000$$

$$\frac{50(1.3^N - 1)}{1.3 - 1} \leq 23000$$

$$1.3^N - 1 \leq 138$$

$$N \log 1.3 \leq \log 139$$

$$N \leq 18.80$$

Thus, the maximum N is 18.

Q5.

Analysis the following situations and find the answer for each question with numbers or permutation/combination expression (e.g. ${}^4C_1$, ${}^7P_3$...).

(i) Lucy plans to wear a shirt, pants, socks, and shoes to join a party. She owns 8 pairs of socks, 3 pairs of shoes, 5 pairs of pants, and 4 shirts.
How many different outfits can John make?

(ii) Martin claims that he can read minds. To test Martin's abilities, Kelvin draws 5 cards without replacement from a standard deck of 52 playing cards. Kelvin then asks Martin to identify in any order which 5 cards he drew without looking.

Assume that Luka has no special abilities and is randomly guessing the cards.

(a) What is the probability that Martin correctly identifies all 5 cards in any order?

(b) If the order the matters, what is the probability that Martin correctly identifies all 5 cards in the exactly right order?

(iii) Leo is 1 of 30 students in a class. Every month, Leo's teacher randomly selects 4 students from their class to act as class president, vice president, secretary, and treasurer. No one student can hold two positions.

In a given month, what is the probability that Leo is chosen as president?

(iv) AC3 restaurant is planning to offer a special price on pizzas with 2 toppings. They offer the toppings below:

Pepperoni; Chicken; Mushroom; Sausage; Green pepper; Pineapple; Ham; Onion

Suppose that your favorite is *Chicken* and *Mushroom*. Your classmate Steven don't know this, so he is going to randomly choose 2 different toppings.

What is the probability that Stefen chooses your favorite topping? With this question, discuss why it is better to make a clear order when ask others to help you buy food.

Solution of Q5

(i) 480

This is a simple permutation problem, the answer is $8 \times 3 \times 5 \times 4 = 480$

(ii) (a) $1/{}_{52}C_5$; (b) $1/{}_{52}P_5$;

If order doesn't matter, there are ${}_{52}C_5$ total ways to select 5 cards from a deck of 52cards.

Martin is trying to identify all 5 of the 52 cards (in any order) that Kelvin drew.

We could use combinations to count here, which is $1/{}_{52}C_5$

If the order matters, we can use the permutation and there are total ${}_{52}P_5$ total ways to select 5 cards from a deck of 52cards.

We could use permutation to count here, which is $1/{}_{52}P_5$

(iii) ${}_{29}P_3/{}_{30}P_4$

For selecting 4 students from 30 to act as 4 roles, the order matters. Therefore, this is a permutation problem, and there are ${}_{30}P_4$ total ways.

If Leo is selected as the president, the problems become “selecting 3 students from 29 to act as the rest 3 roles”. There are ${}_{29}P_3$ total ways.

Hence, the probability is ${}_{29}P_3/{}_{30}P_4$;

(iv) $1/8C_2 = 3.6\%$ Since the probability is so low, it is difficult to get our favorite food buy a friend’s “randomly selecting”. Therefore, it is better to tell our friend what we exactly like. (2mark)

Since the order doesn't matter, there are $8C_2$ total ways to select 2 toppings from a list of 8 toppings. We want to count how many of those groups contain *Chicken* and *Mushroom*.. It is exactly 1 of the total possible groups of 2 toppings is *Chicken* and *Mushroom*. Therefore, the probability is $1/8C_2 = 3.6\%$.

Q6.

If Set A contains all even numbers larger than 0 and less than 45, and Set B contains all numbers larger than 0 and less than 45 that are multiples of 5.

- Find the set C which is the intersection of Set A and Set B?
- What is $A-B$?
- What is $B-A$?
- What is $B-A \cup B$?

Solution of Q6

With the given information, we can find that

$$A = \{2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38,40,42,44\}$$

$$B = \{5,10,15,20,25,30,35,40\}$$

Thus,

$$(i) A \cap B = \{10,20,30,40\}$$

$$(ii) A-B = \{2,4,6,8,12,14,16,18,22,24,26,28,32,34,36,38,42,44\}$$

$$(iii) B-A = \{5,15,25,35\}$$

$$(iv) B-A \cup B = \emptyset$$

Q7.

- The coefficient of x^2 in the expansion of $(1+2x)^n$ is 40. Given that n is a positive integer, find the value of n .
- In the binomial expansion of $(1 + 3x)^n$ ($n>0$), If the coefficient of x^2 is six times the coefficient of x , find the value of n .

Solution of Q7

$$Q7(i): {}_nC_2 (2x)^2 = 40 x^2 \rightarrow n=5 \text{ or } -4; \text{ positive solution is required, } n=5$$

$$Q7(ii): (1 + 3x)^n = 1 + {}_nC_1(3x) + {}_nC_2(3x)^2 + \dots$$

$$6 \cdot {}_nC_1 = 9 {}_nC_2 \rightarrow n=0 \text{ or } 5. \text{ Since } n>0, n=5.$$

Q8.

There are 125 lights with each light labelled by a serial number in the following sequence:

1, 2, 3, ..., 125.

Each light has a button, which can switch/change (toggle) the state of the light between “on” and “off”. All lights are turned off at the beginning. To examine the function of these lights. An engineer clicks the buttons of the lights with serial numbers which are multiple of 4. Secondly, he clicks the buttons of the lights with serial numbers which are multiple of 6.

(i) How many lights are on?

(ii) In the third time, the engineer clicks the buttons of lights with the serial numbers which are multiple of 4, but not multiple of 6. How many lights are on?

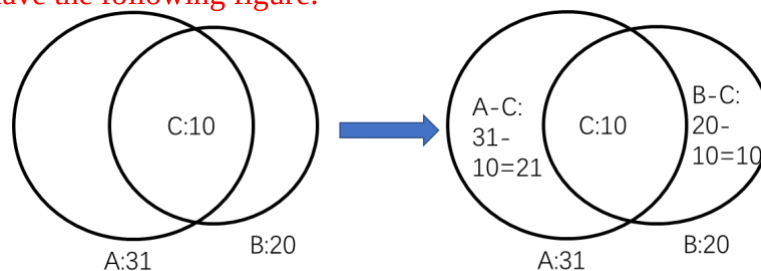
Solution of Q8

Define the lights with serial multiple of 4 as set A, the number of element is $\lfloor 125/4 \rfloor = 31$

Define the lights with serial multiple of 6 as set B, the number of element is $\lfloor 125/6 \rfloor = 20$

Define the lights with serial multiple of 12 as set C, the number of element is $\lfloor 125/12 \rfloor = 10$

Then, we can have the following figure:



Noted, if the light has been selected ONLY once, it will be on;

if the light has been selected twice, it will be off.

Hence, the lights still on in the final are the lights only selected once, which is:

$$(A-C) + (B-C) = 31 - 10 + 20 - 10 = 31$$

(ii)

In this case, we can separate the light in set A into two groups, which are:

(A-C): the lights are on, and their serial number are multiple of 4 but not 6

C: the lights are off, and their serial number are multiple of 4 and 6

According to the question, “the engineer clicks the buttons of lights with the serial numbers which are multiple of 4, but not multiple of 6”

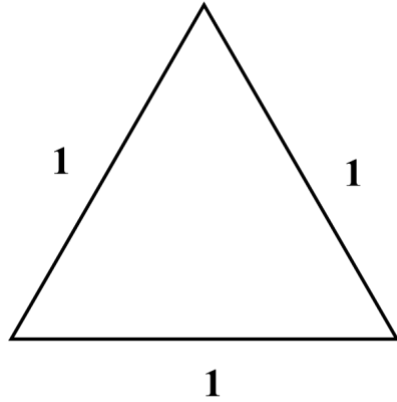
Hence, only the light in (A-C) are selected, and they are changed to off.

The rest lights are still on are the set (B-C), which is $20 - 10 = 10$

Q9.

It is required to put several points (more than 3) at random in a triangular with side length 1. In these points, any two of them can form a short-line. The more points you place, the more lines can be formed. The formed lines may have different lengths, depending on the point positions.

Now, you need to guarantee that there is at least one formed line with a length equal to or less than $1/2$. What is the minimal number of points needed to be placed?

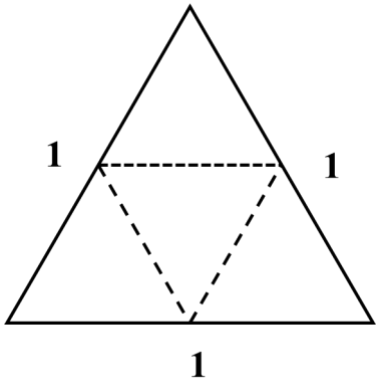


Solution of Q9

We can divide the square into 4 sub-triangular with length equal to 0.5. If we can ensure the line is formed in one of the sub- triangular, the length will be less than $1/2$.

To ensure there at least have 2 points in one sub- triangular,

By pigeonhole principle, 5 points are required.



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