# L02: Introduction to Maxwell's Equations

EE3109 Applied Electromagnetics

Lecture Notes

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# Review and preview

- Last class we reviewed the prerequisite mathematics required in this course
- This class we will learn about Maxwell's equations, which govern time-varying electric and magnetic fields
- We will then examine how materials and boundary conditions affect electromagnetic waves

# Outline

- Course Matter
- Intro to Maxwell’s Equations
- Electromagnetic Materials
- Electromagnetic Boundary Relations

# Preview to Maxwell's Equations

## Fundamental EM field quantities:

| | | | |
|---|---|---|---|
| 1. | Electric field intensity | **E** | $(\mathrm{V/m})$ |
| 2. | Electric flux density | **D** | $(\mathrm{C/m^2})$ |
| 3. | Magnetic flux density | **B** | $(\mathrm{T})$ |
| 4. | Magnetic field intensity | **H** | $(\mathrm{A/m})$ |

The principal objective of studying electromagnetism is to understand the interaction between charges and currents at a distance.

We have fundamental postulates relating **E**, **D**, **B**, **H** and the source quantities **J** and $\rho$.

# Electrostatics / Magnetostatics

- Study of electric and magnetic phenomena when quantities do not vary with time
- When is a static solution appropriate?
  - When the **wavelength is long** compared to the device size
  - When the **wave's time period is long** compared to the time scale is interest
  - **If these conditions are not true, we need to consider time-varying electromagnetics**

## Electrostatic postulates

Conservative Electric Field

$$\oint_c \vec{E} \cdot d\vec{l} = 0 \quad \text{or} \quad \nabla \times \vec{E} = 0$$

Gauss' Law

$$\oint_s \vec{D} \cdot d\vec{s} = Q \quad \text{or} \quad \nabla \cdot \vec{D} = \rho$$

## Magnetostatic postulates

Non-existence of magnetic monopole

$$\oint_s \vec{B} \cdot d\vec{s} = 0 \quad \text{or} \quad \nabla \cdot \vec{B} = 0$$

Ampere's Law (ver. 1)

$$\oint_c \vec{H} \cdot d\vec{l} = I \quad \text{or} \quad \nabla \times \vec{H} = \vec{J}$$

## Constitutive relations (linear and isotropic media) :

$D = \varepsilon E$ $\qquad$ $B = \mu H$

$\varepsilon = \varepsilon_r \varepsilon_0$ : permittivity $\qquad$ $\mu = \mu_r \mu_0$ : permeability

$\varepsilon_0$ : $8.854 \times 10^{-12}$ (F/m) $\qquad$ $\mu_0 = 4\pi \times 10^{-7}$ (H/m)

# Quasi-Statics: Faraday's law of Electromagnetic Induction:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$ differential form

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$ integral form

$$V_{emf} = \oint_C \mathbf{E} \cdot d\mathbf{l}$$ = emf induced in circuit with contour C (V)

$$\phi = -\int_S \mathbf{B} \cdot d\mathbf{s}$$ = magnetic flux crossing surface S (Wb)

$$\boxed{V_{emf} = -\frac{\partial \phi}{\partial t}}$$

"Electromagnetic force induced in a stationary closed circuit is equal to the negative rate of increasing of magnetic flux linking the circuit."

# James Clerk Maxwell

- British physicist, 1831-1879
- Maxwell proposed that light is an undulation in the same medium that is the cause of electric and magnetic phenomena.
- The unification of light and electrical phenomena also led to the prediction of the existence of radio waves
- In a millennium poll—a survey of the 100 most prominent physicists—Maxwell was voted the third greatest physicist of all time, behind only Newton and Einstein.

# Electromagnetic Postulates Considered by Maxwell

## Quasi-Electrostatic postulates

Faraday's Law

$$\oint_C \vec{E} \cdot d\vec{l} = -\int_S \frac{d\vec{B}}{dt} \cdot d\vec{s} \quad \text{or}$$

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

Gauss' Law

$$\oint_s \vec{D} \cdot d\vec{s} = Q \quad \text{or} \quad \nabla \cdot \vec{D} = \rho$$

## Magnetostatic postulates

Non-existence of magnetic monopole

$$\oint_s \vec{B} \cdot d\vec{s} = 0 \quad \text{or} \quad \nabla \cdot \vec{B} = 0$$

Ampere's Law (ver. 1)

$$\oint_c \vec{H} \cdot d\vec{l} = I \quad \text{or} \quad \nabla \times \vec{H} = \vec{J}$$

I : Total current through S

Q : Total charge contained in volume V bounded by Surface S

Consider Ampere's Law:

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J}$$

$$0 = \nabla \cdot \vec{J}$$ (from the null identity)

which is inconsistent with the principle of conservation of charge:

$$-\frac{\partial \rho}{\partial t} = \nabla \cdot \vec{J}$$ : equation of continuity

What is missing? Maxwell's Solution....

$$\nabla \cdot (\nabla \times \mathbf{H}) - \frac{\partial \rho}{\partial t} = \nabla \cdot \mathbf{J}$$

$$\nabla \cdot (\nabla \times \mathbf{H}) - \frac{\partial (\nabla \cdot \mathbf{D})}{\partial t} = \nabla \cdot \mathbf{J}$$ (Used Gauss' law)

$$\nabla \cdot (\nabla \times \mathbf{H}) - \nabla \cdot \frac{\partial \mathbf{D}}{\partial t} = \nabla \cdot \mathbf{J}$$

$$\nabla \cdot \left( \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} \right) = \nabla \cdot \mathbf{J}$$

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$$\nabla \cdot \left( \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} \right) = \nabla \cdot \mathbf{J}$$

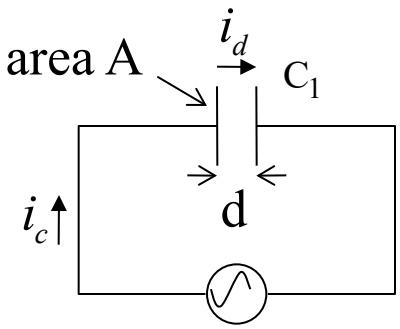
$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$$

Hence Maxwell rewrote Ampere's Law to $\boxed{\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}}$ to establish consistency with current continuity.

He called $\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$ the displacement current density.

The modified Ampere's Law shows that a time-varying E-field can give rise to an H-field, even in the absence of **J**.

# Example 0: Visualizing the Displacement Current




$v_c = V_o \sin \omega t$

## Conduction current

$$i_c = C_1 \frac{d\, v_c}{dt} = C_1 V_o \omega \cos \omega t$$

$$C_1 = \varepsilon \frac{A}{d}$$

$$\boxed{\therefore i_c = \frac{\varepsilon A}{d} V_o \omega \cos \omega t}$$

## Displacement current

$$E = \frac{v_c}{d}$$

$$D = \varepsilon E = \varepsilon \frac{V_o}{d} \sin \omega t$$

$$\therefore i_d = \int_A \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$$

$$= \varepsilon \frac{V_o}{d} \omega \cos \omega t \times A$$

$$= \frac{\varepsilon A}{d} V_o \omega \cos \omega t = i_c$$

# Maxwell's Equations

| Differential Form | | Integral Form |
|---|---|---|
| $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ | Faraday's Law | $\oint_C \mathbf{E} \cdot d\boldsymbol{l} = -\int_S \frac{d\mathbf{B}}{dt} \cdot d\mathbf{s}$ |
| $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ | Ampere's Law | $\oint_C \mathbf{H} \cdot d\boldsymbol{l} = \int_S \left(\mathbf{J} + \frac{d\mathbf{D}}{dt}\right) \cdot d\mathbf{s} = I_c + I_d$ |
| $\nabla \cdot \mathbf{D} = \rho$ | Gauss' Law | $\oint_S \mathbf{D} \cdot d\mathbf{s} = \mathrm{Q}$ |
| $\nabla \cdot \mathbf{B} = 0$ | Non-existence of Magnetic monopole | $\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$ |

**Current Continuity**

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

**Constitutive Relationship**

$$\mathbf{D} = \varepsilon \mathbf{E} \qquad \mathbf{B} = \mu \mathbf{H}$$

**Question:**

*How many equations and unknowns do we have?*

# Maxwell's Equations: Equations and Unknowns

- The Maxwell's equations are not all independent: the divergence equations can be derived by the curl equations and the continuity relationship.
- Further, the continuity relationship relates current to charge in an environment.
- Hence when the material and source are known, the curl equations and constitutive relationships can be solved to find the fields and flux densities (i.e. 12 equations and 12 unknowns!)
- **Conclusion**: When the material and source are known, one can uniquely determine $\{\mathbf{E}, \mathbf{H}, \mathbf{B}, \mathbf{D}\}$ for all space and all time

# Example 1

If $\mathbf{E}(t) = A_0 \sin(\omega t - \beta z + \phi)\ \hat{\mathbf{x}}$, find $\mathbf{H}(t)$

Solution

Use Faraday's Law: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t}$

Let $E_0 = A_0 \sin(\omega t - \beta z + \phi)$, s.t. $\mathbf{E} = E_0\ \hat{\mathbf{x}}$

$$\text{LS} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times (E_0, 0,0)$$

$$= \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_0 & 0 & 0 \end{vmatrix} = \frac{\partial E_0}{\partial z} \hat{\mathbf{y}} = \frac{\partial}{\partial z}(A_0 \sin(\omega t - \beta z + \phi))\ \hat{\mathbf{y}} = -\beta A_0 \cos(\omega t - \beta z + \phi)\ \hat{\mathbf{y}}\ .$$

RS = LS: $-\mu \frac{\partial \mathbf{H}}{\partial t} = -\beta A_0 \cos(\omega t - \beta z + \phi)\ \hat{\mathbf{y}}$

$$\Rightarrow \mathbf{H}(t) = \frac{\beta}{\mu} A_0 \int \cos(\omega t - \beta z + \phi)\ dt\, \hat{\mathbf{y}} = \frac{\beta}{\omega \mu} A_0 \sin(\omega t - \beta z + \phi)\, \hat{\mathbf{y}} + \mathbf{C}$$

Cont'd:

$$\mathbf{H}(t) = \frac{\beta}{\omega\mu} A_0 \sin(\omega t - \beta z + \phi)\,\hat{\mathbf{y}} + \mathbf{C} = \frac{\beta}{\omega\mu} E_0 \hat{\mathbf{y}} + \mathbf{C}$$

Remarks:

1. It is usually easier to let the known quantity be the argument inside the curl operation.
2. We will often see the term $\frac{\omega\mu}{\beta}$ (written in various ways). It is in some ways related to the "impedance" of the material.
3. What is the integration constant $\mathbf{C}$?

   *It is a constant w.r.t. time (but not w.r.t. space), i.e. a static H field that can exist independent of the E field.*

   *For the purpose of this course, we will ignore its presence to concentrate on the time varying field.*

# Simplifying Maxwell's Equations with Phasors

| Time Domain Differential Form | | Phasor Differential Form |
|---|---|---|
| $\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$ | Faraday's Law | $\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$ |
| $\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t}$ | Ampere's Law | $\nabla \times \mathbf{H} = \mathbf{J} + j\omega\varepsilon\mathbf{E}$ |
| $\nabla \cdot \mathbf{E} = \rho/\varepsilon$ | Gauss' Law | $\nabla \cdot \mathbf{E} = \rho/\varepsilon$ |
| $\nabla \cdot \mathbf{H} = 0$ | Non-existence of Magnetic monopole | $\nabla \cdot \mathbf{H} = 0$ |

When waves of only one frequency are considered, Maxwell's Equations are greatly simplified using phasors!

# Example 2

If $\mathbf{H}(t) = A_0 \sin(\omega t - \beta z + \phi)\ \hat{\mathbf{x}}$, find $\mathbf{E}(t)$ (Assume there is no source in the environment.

Solution

First convert into a phasor: $\mathbf{H} = A_0 e^{j(\phi - \frac{\pi}{2})} e^{-j\beta z}\ \hat{\mathbf{x}} = H_0\ \hat{\mathbf{x}}$

Use Ampere's Law (phasor form): $\nabla \times \mathbf{H} = J + j\omega\varepsilon\mathbf{E}$

$$\Rightarrow \mathbf{E} = \frac{1}{j\omega\varepsilon}\nabla \times \mathbf{H} = \frac{1}{j\omega\varepsilon}\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \times\ (H_0, 0, 0)$$

$$= \frac{1}{j\omega\varepsilon}\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_0 & 0 & 0 \end{vmatrix} = \frac{1}{j\omega\varepsilon}\frac{\partial H_0}{\partial z}\ \hat{\mathbf{y}} = \frac{1}{j\omega\varepsilon}\frac{\partial}{\partial z}\left(A_0 e^{j\left(\phi - \frac{\pi}{2}\right)} e^{-j\beta z}\right)\ \hat{\mathbf{y}}$$

$$= \frac{-j\beta}{j\omega\varepsilon}\left(A_0 e^{j\left(\phi - \frac{\pi}{2}\right)} e^{-j\beta z}\right)\hat{\mathbf{y}} = -\frac{\beta}{\omega\varepsilon}\mathbf{H}_0\ \hat{\mathbf{y}}$$

Convert back into time domain: $\mathbf{E}(t) = -\frac{\beta}{\omega\varepsilon} A_0 \sin(\omega t - \beta z + \phi)\ \hat{\mathbf{y}}$

## Remarks:

1. Computation is made much easier when phasor notation is used!

2. There is no ambiguity with the integration constant, because by using the phasor formulation we are just examining one frequency.

3. In both Examples 1 and 2, **E** and **H** have the same magnitude ratio (meaning the quantity $|\mathbf{E}|/|\mathbf{H}|$ does not depend on space or time), and are orthogonal to one-another. We shall see next time why this is the case.

# Summary

- In this class section we
  - Reviewed electro/magneto/quasi-statistics
  - Introduced Maxwell and Maxwell's equations
  - Used Maxwell's equations to solve for electromagnetic fields
  - Introduced and demonstrated the usage of Maxwell's equations in phasor formulation

# Outline

- Course Matter
- Intro to Maxwell’s Equations
- Electromagnetic Materials
- Electromagnetic Boundary Relations

# Simplifying Maxwell's Equations with Phasors

| Time Domain Differential Form | | Phasor Differential Form |
|---|---|---|
| $\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$ | Faraday's Law | $\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$ |
| $\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t}$ | Ampere's Law | $\boxed{\nabla \times \mathbf{H} = \mathbf{J} + j\omega\varepsilon\mathbf{E}}$ |
| $\nabla \cdot \mathbf{E} = \rho/\varepsilon$ | Gauss' Law | $\nabla \cdot \mathbf{E} = \rho/\varepsilon$ |
| $\nabla \cdot \mathbf{H} = 0$ | Non-existence of Magnetic monopole | $\nabla \cdot \mathbf{H} = 0$ |

Take a look at Ampere's Law… (what is the unit of $j\omega\varepsilon\mathbf{E}$ ?)

# A complex permittivity

$$\nabla \times \mathbf{H} = \mathbf{J}_\mathrm{c} + \mathbf{J}_\mathrm{d} = \sigma \mathbf{E} + j\omega\varepsilon\mathbf{E} = \mathrm{j}\omega\left(\varepsilon - j\frac{\sigma}{\omega}\right)\mathbf{E} \equiv j\omega\varepsilon_c\mathbf{E}$$

Define:

$$\boxed{\varepsilon_c = \varepsilon' - j\varepsilon'' \equiv \varepsilon - j\frac{\sigma}{\omega}}$$

Good insulator: $\varepsilon' \gg \varepsilon''$ $(\omega\varepsilon \gg \sigma)$

Good conductor: $\varepsilon'' \gg \varepsilon'$ $(\sigma \gg \omega\varepsilon)$

# Example 1: Ulaby 6.18

**6.18** In wet soil, characterized by $\sigma = 10^{-2}$ (S/m), $\mu_r = 1$, and $\varepsilon_r = 36$, at what frequency is the conduction current density equal in magnitude to the displacement current density?

**Solution:** For sinusoidal wave variation, the phasor electric field is

$$E = E_0 e^{j\omega t}.$$

$$J_c = \sigma E = \sigma E_0 e^{j\omega t}$$

$$J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = j\omega\varepsilon E_0 e^{j\omega t}$$

$$\left|\frac{J_c}{J_d}\right| = 1 = \frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2\pi\varepsilon f}$$

or

$$f = \frac{\sigma}{2\pi\varepsilon} = \frac{10^{-2}}{2\pi \times 36 \times 8.85 \times 10^{-12}} = 5 \times 10^6 = 5 \text{ MHz}.$$

# Charge Dissipation from a Conductor

Question 1: What happens if you place a certain amount of free charge inside of a material?
Answer: The charge will move to the surface of the material, thereby returning its interior to a neutral state.

Question 2: How fast will this happen?
Answer: It depends on the material; in a good conductor, the charge dissipates in less than a femtosecond, whereas in a good dielectric, the process may take several hours.

Derivation of charge density equation:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} . \tag{6.58}$$

In a conductor, the point form of Ohm's law, given by Eq. (4.63), states that $\mathbf{J} = \sigma \mathbf{E}$. Hence,

$$\sigma \nabla \cdot \mathbf{E} = -\frac{\partial \rho_v}{\partial t} . \tag{6.59}$$

Next, we use Eq. (6.1), $\nabla \cdot \mathbf{E} = \rho_v / \varepsilon$, to obtain the partial differential equation

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon} \rho_v = 0. \tag{6.60}$$

# Solution of Charge Dissipation Equation

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon}\rho_v = 0$$

Given that $\rho_v = \rho_{vo}$ at $t = 0$, the solution of Eq. (6.60) is

$$\rho_v(t) = \rho_{vo} e^{-(\sigma/\varepsilon)t} = \rho_{vo} e^{-t/\tau_r} \qquad (\mathrm{C/m^3}),$$

where $\tau_r = \varepsilon/\sigma$ is called the ***relaxation time constant***.

For copper: $\tau_r = 1.53 \times 10^{-19}$ s

For mica: $\tau_r = 5.31 \times 10^{4}$ s = 15 hours

# "Complicated" Permittivity and Permeability

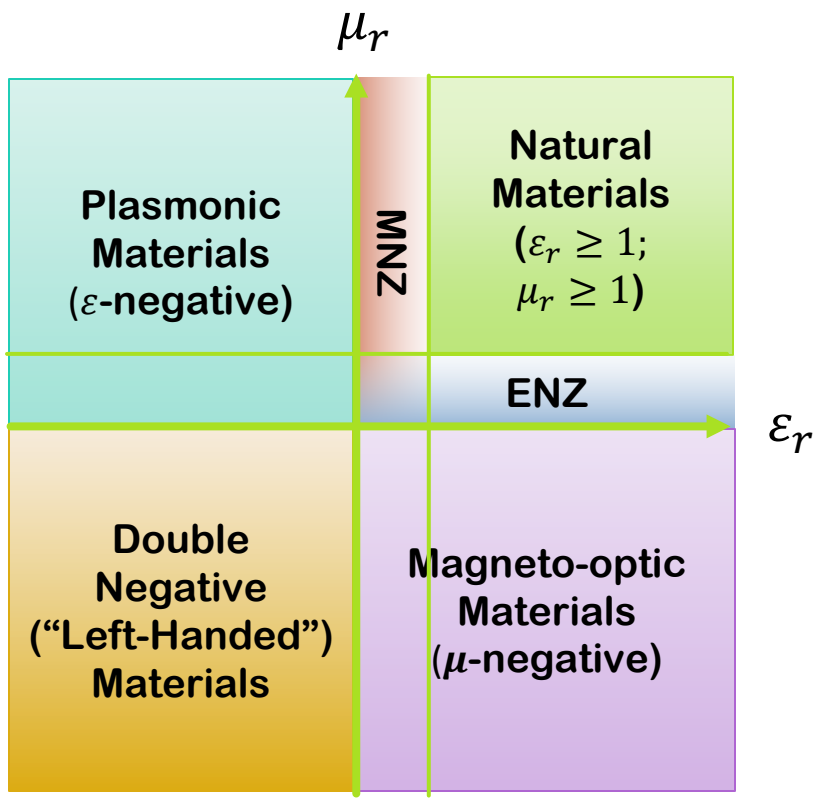
- For simple medium, this applies:

Constitutive Relationship

$$\mathbf{D} = \varepsilon \mathbf{E} \qquad \mathbf{B} = \mu \mathbf{H}$$

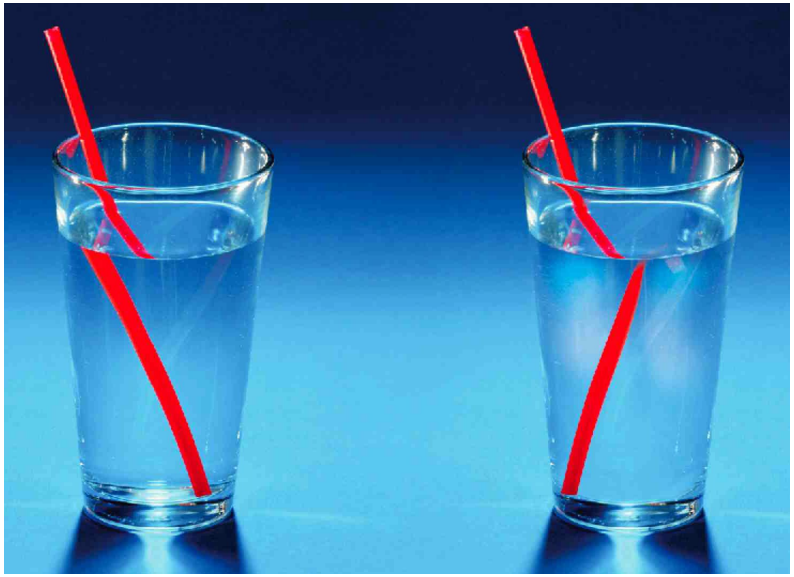
- More complicated constitutive relationships exist:
  - Anisotropic material: $\mathbf{D} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \mathbf{E}$
  - Tensorial permittivity material: $\mathbf{D} = \bar{\bar{\varepsilon}} \mathbf{E}$
  - Bi-anisotropic material: $\mathbf{D} = \bar{\bar{\varepsilon}} \mathbf{E} + \bar{\bar{\kappa}} \mathbf{H}$
  - Non-linear material: $\mathbf{D} = \varepsilon_0 \left(1 + \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 \ldots\right)$
    - Notice: this means **new frequencies** are generated!

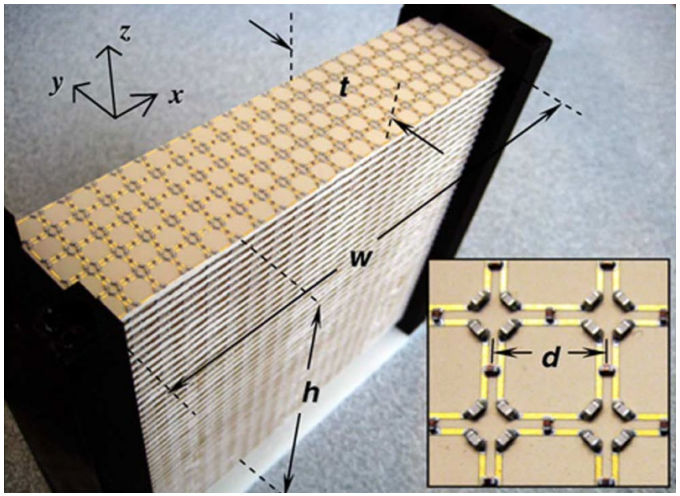
# Metamaterials

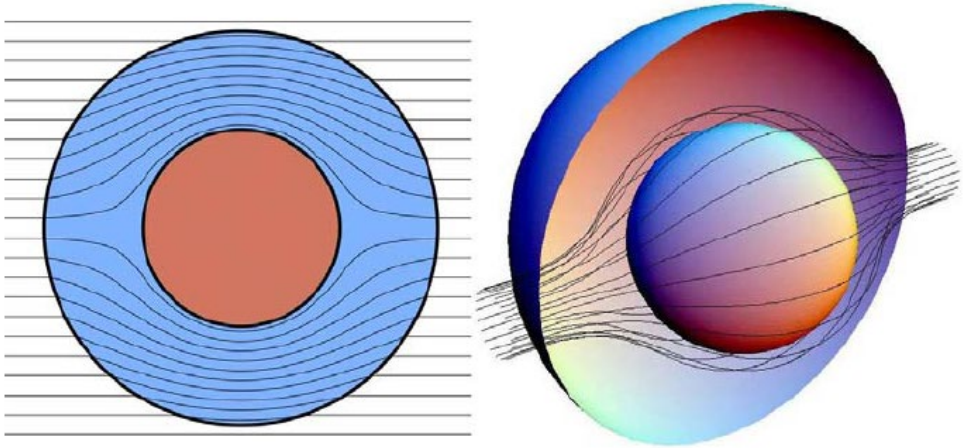


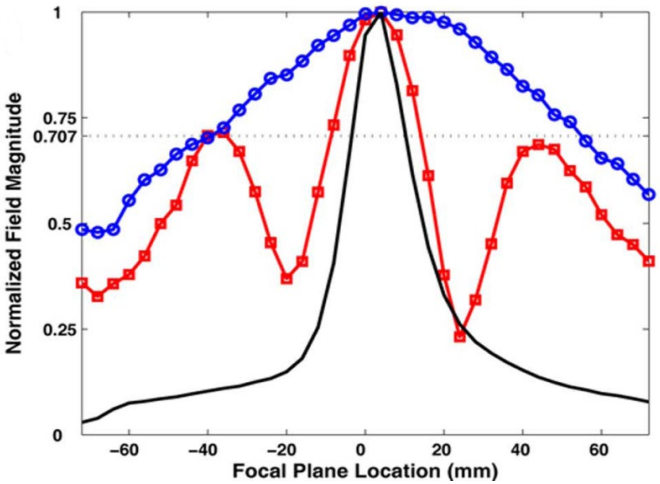

- **Permittivity ($\varepsilon$, "epsilon")** and **permeability ($\mu$, "mu")** denote a material's response to **electric** and **magnetic** fields
- By engineering the material structure, one can achieve material properties unfound in nature

# Some Example Metamaterials







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# Electric Boundary Conditions

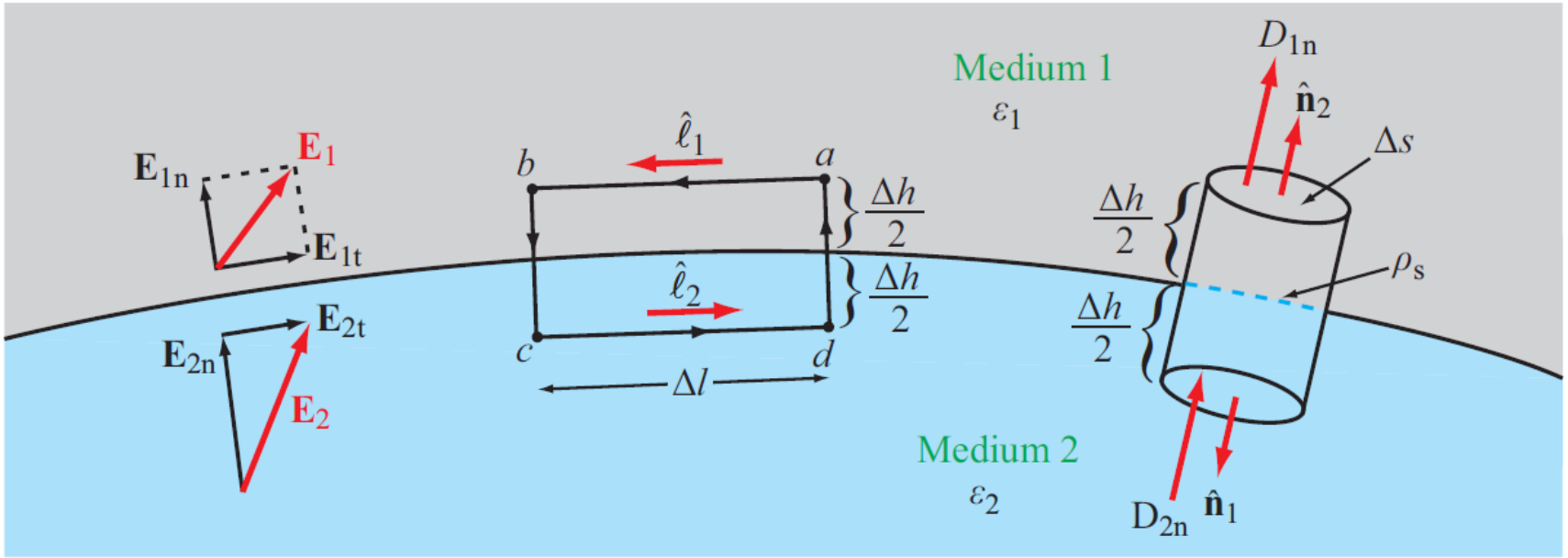



**Figure 4-18:** Interface between two dielectric media.

$$\mathbf{E}_{1t} = \mathbf{E}_{2t} \qquad \text{(V/m)}. \qquad (4.90)$$

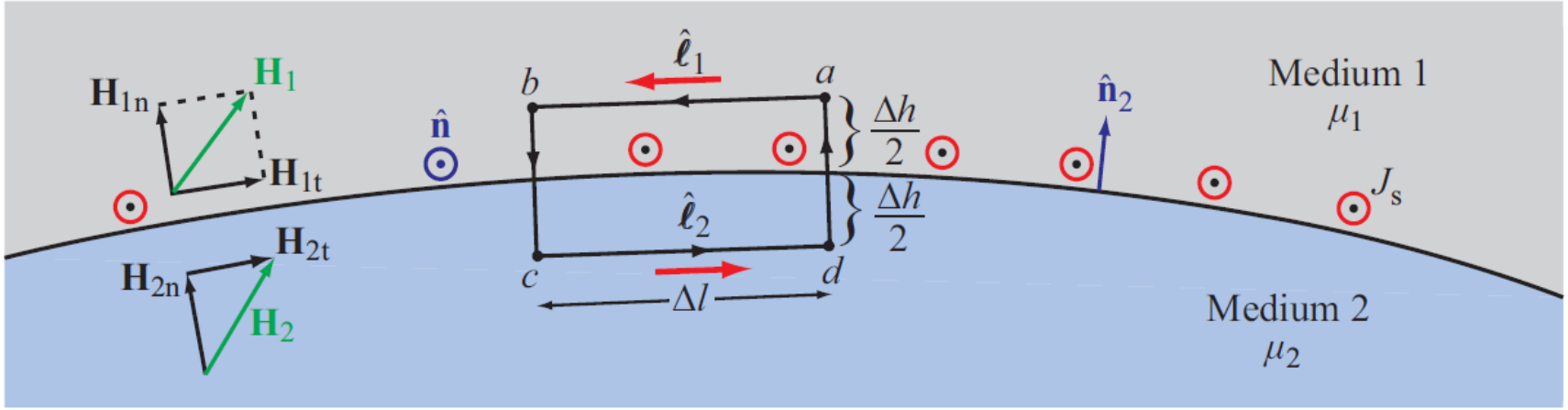
$$\frac{\mathbf{D}_{1t}}{\varepsilon_1} = \frac{\mathbf{D}_{2t}}{\varepsilon_2}. \qquad (4.91)$$

$$\hat{\mathbf{n}}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s \qquad (\text{C/m}^2).$$

$$D_{1n} - D_{2n} = \rho_s \qquad (\text{C/m}^2). \qquad (4.94)$$

*The normal component of* **D** *changes abruptly at a charged boundary between two different media in an amount equal to the surface charge density.*

# Magnetic Boundary Conditions




$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q \quad \rightarrow \quad \boxed{D_{1n} - D_{2n} = \rho_s.} \qquad (5.78)$$

$$\hat{\mathbf{n}}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s.$$

By analogy, application of Gauss's law for magnetism, as expressed by Eq. (5.44), leads to the conclusion that

$$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0 \quad \rightarrow \quad \boxed{B_{1n} = B_{2n}.} \qquad (5.79)$$

*Thus the normal component of* **B** *is continuous across the boundary between two adjacent media.*

Surface currents can exist only on the surfaces of perfect conductors and superconductors. Hence, ***at the interface between media with finite conductivities,*** $\mathbf{J}_s = 0$ and

$$\boxed{H_{1t} = H_{2t}. \qquad (5.85)}$$

# Boundary Conditions

**Table 6-2:** Boundary conditions for the electric and magnetic fields.

| Field Components | General Form | Medium 1 Dielectric | Medium 2 Dielectric | Medium 1 Dielectric | Medium 2 Conductor |
|---|---|---|---|---|---|
| Tangential E | $\hat{\mathbf{n}}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$ | $E_{1t} = E_{2t}$ | | $E_{1t} = E_{2t} = 0$ | |
| Normal D | $\hat{\mathbf{n}}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$ | $D_{1n} - D_{2n} = \rho_s$ | | $D_{1n} = \rho_s$ | $D_{2n} = 0$ |
| Tangential H | $\hat{\mathbf{n}}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$ | $H_{1t} = H_{2t}$ | | $H_{1t} = J_s$ | $H_{2t} = 0$ |
| Normal B | $\hat{\mathbf{n}}_2 \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$ | $B_{1n} = B_{2n}$ | | $B_{1n} = B_{2n} = 0$ | |

Notes: (1) $\rho_s$ is the surface charge density at the boundary; (2) $\mathbf{J}_s$ is the surface current density at the boundary; (3) normal components of all fields are along $\hat{\mathbf{n}}_2$, the outward unit vector of medium 2; (4) $E_{1t} = E_{2t}$ implies that the tangential components are equal in magnitude and parallel in direction; (5) direction of $\mathbf{J}_s$ is orthogonal to $(\mathbf{H}_1 - \mathbf{H}_2)$.

- Summary: Boundary Conditions remain the same for electro/magneto-statics and electromagnetics

# Summary

- In this class we looked at
  - Maxwell's equations and how they relate electric and magnetic fields
  - Electromagnetic material properties ($\varepsilon_c, \mu, \sigma$)
  - Relationship between electromagnetic fields across a boundary of two different materials