

CITY UNIVERSITY OF HONG KONG

Course code & title : EE3121 Differential Equations for Electrical Engineering

Session : Semester B 2023/24

Time allowed : Two hours

This paper has THREE pages (including this cover page).

1. This paper consists of Five questions.
 2. Answer ALL questions. The questions are ALL compulsory. Make sure that you attempt all of them
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This is an open-book examination.

Students are allowed to use the following materials/aids:

Only printed course materials and lecture note will be allowed. Using electronic device will be strictly prohibited.

Materials/aids other than those stated above are not permitted. Students will be subject to disciplinary action if any unauthorized materials or aids are found on them.

Question 1

Find the general solutions of the following differential equations. Please provide derivations to receive full marks.

(a) (10 pts)

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} - 8y = e^{4x}$$

(b) (10 pts)

$$\frac{d^2 y}{dx^2} + \omega^2 y = \exp(i\omega_0 x)$$

(1 - \omega^2)(1 - \omega_0^2)x

\omega^2 \neq \omega_0^2

(Remark: consider two cases ($\omega = \omega_0$ and $\omega \neq \omega_0$) separately.)

$$(1 - \omega^2) = \omega^2 x$$

Question 2

Find the power series solutions of the following differential equation centered at $t = 0$. Please provide derivations to receive full marks.

(a) (10 pts)

$$\frac{dy^2}{dt^2} - 6t \frac{dy}{dt} + 10y = 0$$

(b) (10 pts)

$$\frac{dy}{dt} = y + \frac{8}{1-t}$$

(Hint: Use the following series formulation $\frac{1}{1-t} = \sum_{n=0}^{\infty} t^n$ for $|t| < 1$.)

Question 3

(a) (10 pts) Compute the matrix exponential e^{At} for the following two matrices.

$$A_1 = \begin{bmatrix} 5-\lambda & -3 \\ 1 & 1-\lambda \end{bmatrix}, \quad A_2 = \begin{bmatrix} 5-\lambda & -1 \\ 5 & 1-\lambda \end{bmatrix}$$

(5-\lambda)(1-\lambda) - (-3) = \lambda^2 - 6\lambda + 11

(5-\lambda)(1-\lambda) - (-5) = \lambda^2 - 6\lambda + 10

(b) (10 pts) Use the matrix exponential to solve the following initial value problem.

$$\frac{d\bar{y}}{dt} = A \bar{y}(t), \quad A = \begin{bmatrix} 6 & -3 \\ 1 & 2 \end{bmatrix}, \quad \bar{y}(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

\lambda^2 - 8\lambda + 12 = 0

\lambda = 2, 6

\bar{y}(t) = e^{2t} \begin{bmatrix} 1 \\ 3 \end{bmatrix} + e^{6t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}

Question 4

Find the solutions of the following differential equations using Laplace transform. Please provide derivations to receive full marks.

(a) (10 pts)

$$\frac{d^2x}{dt^2} = -3x + y, \quad \frac{d^2y}{dt^2} = 2x - 2y + 40 \sin(3t),$$

$$x(0) = x'(0) = 0, \quad y(0) = y'(0) = 0$$

(b) (10 pts)

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = \delta(t) - \delta(t-2), \quad x(0) = x'(0) = 0$$

Question 5

Find the solutions of the following systems of differential equations. Please provide derivations to receive full marks.

(a) (10 pts)

$$\frac{dx}{dt} = -y, \quad \frac{dy}{dt} = 10x - 7y, \quad x(0) = 2, \quad y(0) = -7$$

$$\frac{dx}{dt} = -1.2x$$

$$x = -1.2$$

$$x =$$

(b) (10 pts)

$$\frac{dx}{dt} = 3x + 4y, \quad \frac{dy}{dt} = 3x + 2y, \quad x(0) = 1, \quad y(0) = 1$$

$$f(x) = 1.6x$$

$$f(x) = 1.4x$$

$$f(x) = \frac{1}{2}x$$

$$\frac{1}{4}(x^2 - 3x)$$