

Block A: Unit 3**One port networks and Dependent Sources***G. Rizzoni Ch 3.6, 3.5***Block A: Unit 3**

To see the motivation for this unit, let us first refer to the circuit shown below:

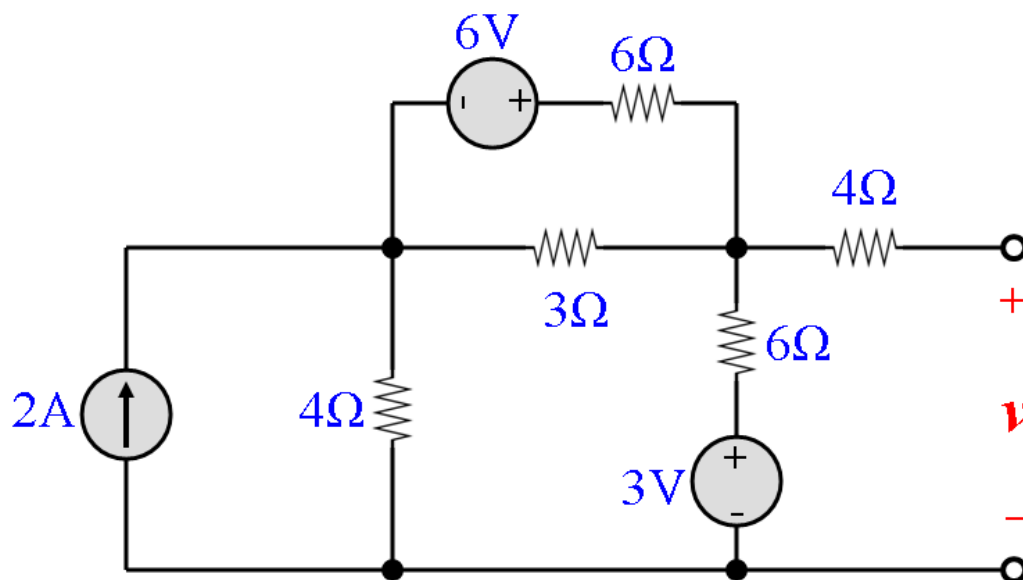


Fig 1: An example of a one port network

If you were given a number of different resistor loads to be connected to the terminal of this device, you would have to recalculate the whole circuit to find the current and voltage at the terminals for each load. A different load will give you difference output current & voltage.

If it was possible to transform any two terminal circuit into a circuit that is as simple as having one resistor and one source, that will make the analysis a lot simpler. If the transformed circuit was like the one below, then to

find the voltage for different loads, we just need to use voltage divider rule each time. Finding the current then requires simple Ohm's law.

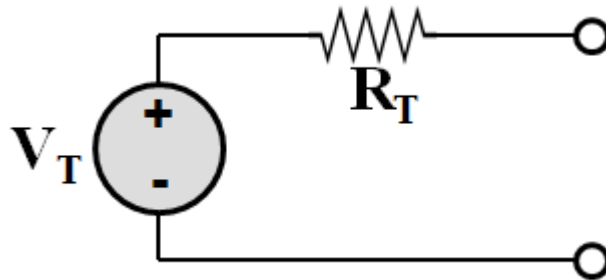


Fig 2: Transformed Thevenin equivalent circuit

It turns out this is indeed possible and you will learn how to do this in this unit. The first part of this unit is organized into the following 3 sections:

- Thevenin equivalent
- Norton equivalent
- Source transformation

In the last part of this unit, we will take a slight detour and revisit all the concepts in Block A and apply them to dependent sources.

I. Thevenin

Definition

Any one port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **voltage**

source V_T in series with an equivalent resistance R_T . An example of the transformed circuit is shown in Fig 2.

Deriving the Thevenin equivalent circuit

Step 1: Remove the load from the rest of the one port network. This is the first and most fundamental step.

Step 2: Find the equivalent resistance R_T

To do this, first disable all ideal sources (refer back to Section III of the previous unit) then find the resistance across the terminals.

For voltage sources – replace with short circuit

For current sources – replace with open

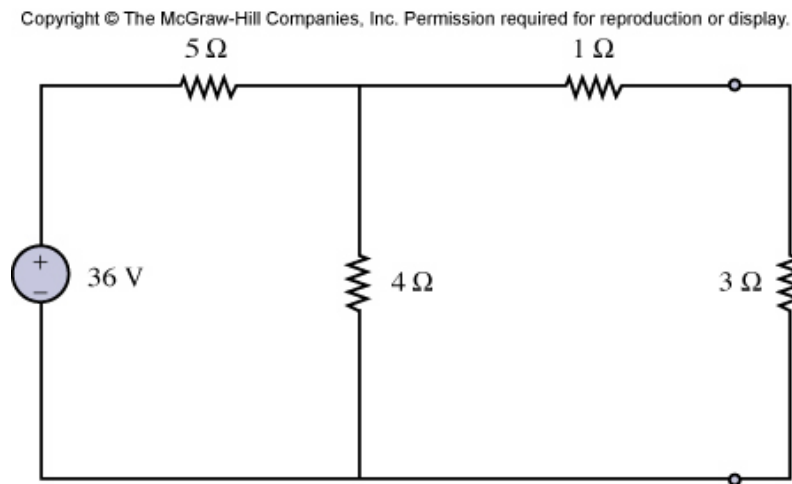
Step 3: Find the Thevenin voltage source

The Thevenin voltage source is equal to the open circuit voltage seen across the terminals (with no load). Solve for this voltage using any preferred method (refer back to previous unit).

The best way to see how to actually do this is to work through some problems.

Worked Example 1 on Thevenin

Derive the Thevenin equivalent circuit seen by the 3Ω load



Answer: $R_{th} = 29/9\Omega$, $V_{th} = 16V$

First re-draw the circuit without the load:

If we are finding R_{th} first, then we short the 36V voltage source and redraw the remaining resistor network:

Then find the equivalent resistance across the relevant terminals:

$$R_{th} = \underline{\hspace{2cm}}$$

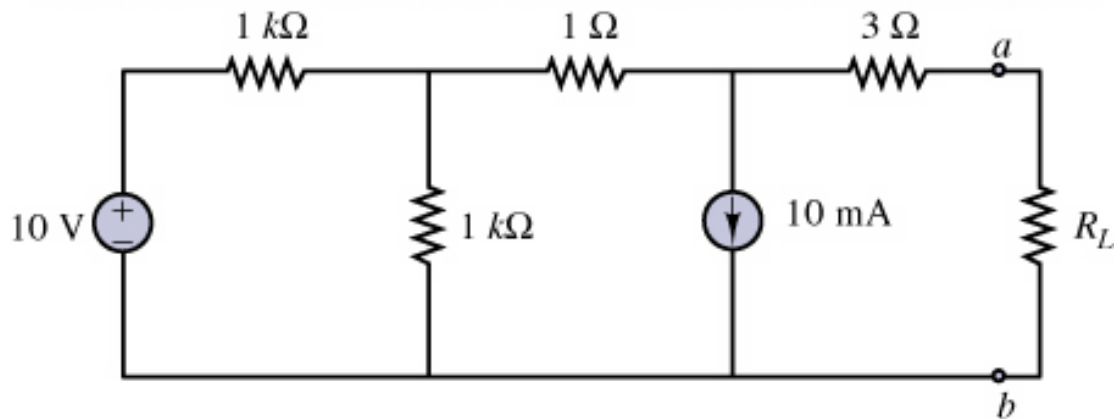
Now to find V_{th} , find the open voltage across terminals where the load was removed (ref pg. 4):

$$V_{th} = \underline{\hspace{2cm}}$$

Worked Example 2 on Thevenin

Derive the Thevenin equivalent circuit seen by the load R_L .

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Answer: $V_{\text{th}} = -10\text{ mV}$, $R_{\text{th}} = 504\text{ }\Omega$

To find R_{th} , we begin again by disabling all sources and re-draw the resulting circuit:

$R_{\text{th}} = \underline{\hspace{2cm}}$

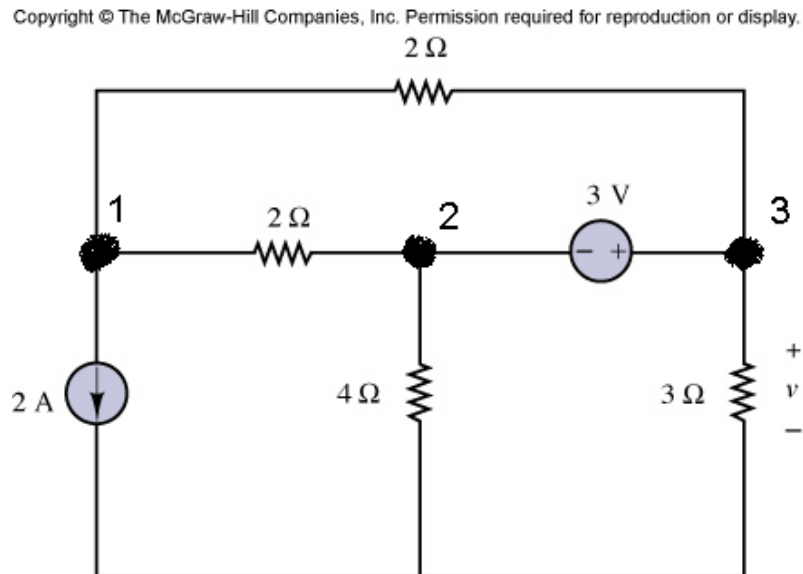
Now to find V_{th} , we put back the sources and find the voltage across the open circuit terminals.

From the above re-drawn circuit with load removed, we can use MCA which requires 1 equation only.

$$V_{th} = \underline{\hspace{2cm}}$$

Worked Example 3 on Thevenin

Derive the Thevenin equivalent circuit seen by the 3Ω load



Answer: $V_{\text{th}} = -5\text{V}$, $R_{\text{th}} = 4\Omega$

Remove 3Ω load and re-draw remaining circuit:

Thevenin voltage = Difference between Node 3 and our chosen reference

$$V_{th} = \underline{\hspace{2cm}}$$

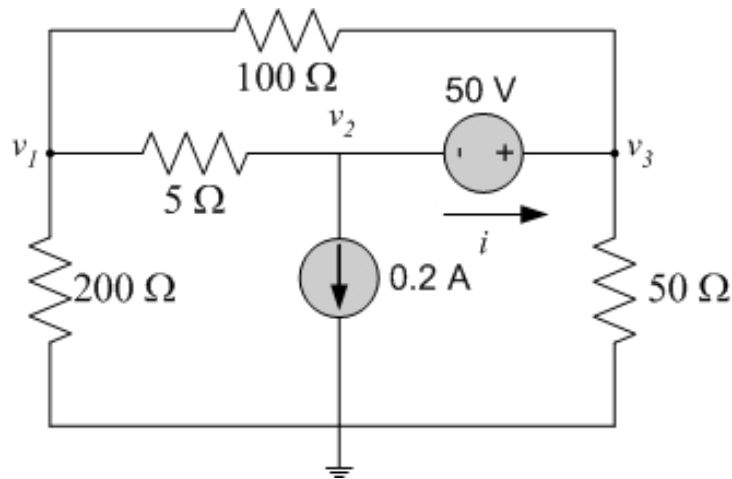
Now short voltage source and open current source

Re-draw the remaining circuit to find the equivalent resistance

$$R_{th} = \underline{\hspace{2cm}}$$

Worked Example 4 on Thevenin

Derive the Thevenin equivalent circuit seen by the 50Ω load



Answer: $V_{\text{th}} = 20/3\text{ V}$, $R_{\text{th}} = 4300/21$

If we find R_{th} first, re-draw circuit with sources disabled and 50Ω removed:

$R_{\text{th}} = \underline{\hspace{2cm}}$

Thevenin voltage = v_3 . Therefore, find v_3 without 50Ω .
Re-draw the circuit to see what this looks like:

MCA requires solving just 1 equation:

$$50 = i \cdot (100 + 5) + 0.2 \cdot 5$$

$$\Rightarrow i = 7/15 \text{ A}$$

Simply add up voltages from one node to the next using Ohm's law:

$$\Rightarrow v_1 = -(200 \cdot 0.2) = -40\text{V}$$

$$\Rightarrow v_3 = -40\text{V} \quad \underline{\hspace{1cm}} \quad (7/15) \cdot 100 = \underline{\hspace{1cm}}$$

$$\text{Therefore: } V_{\text{th}} = \underline{\hspace{1cm}}$$

Norton

Definition

Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **current source** I_N in parallel with an equivalent resistance R_N . An example of the transformed circuit is shown in Fig 3.

The Norton equivalent circuit is simply a current source version of the Thevenin equivalent circuit.

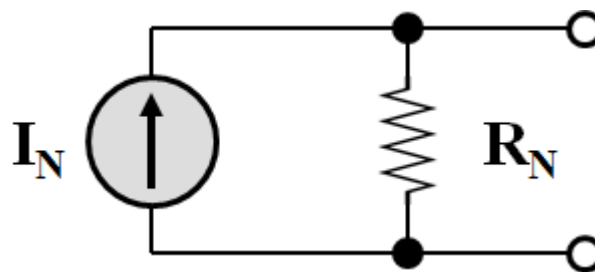


Fig 3: Transformed Thevenin equivalent circuit

Deriving the Norton equivalent circuit

Apart from the last step of deriving the Norton current source, the steps for deriving the Norton equivalent circuit are exactly the same as that for Thevenin.

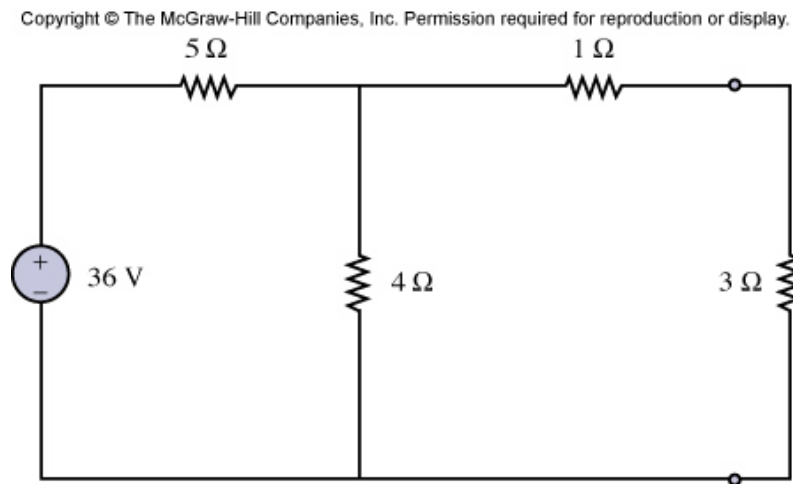
To find the Norton current source:

First, short circuit the terminals to the load.

Next, find the current through the short circuit. This is the Norton current source.

Worked Example 1 on Norton

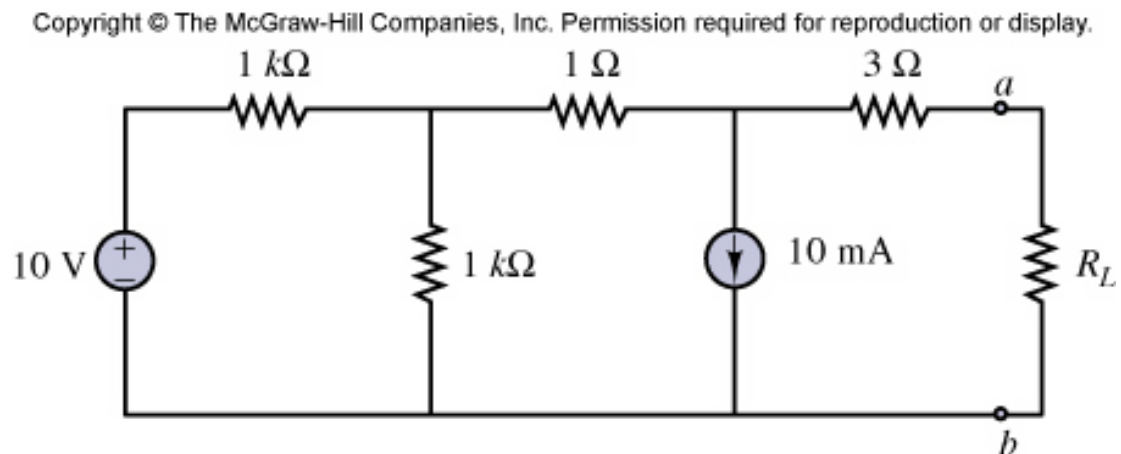
Derive the Norton equivalent circuit seen by the 3Ω load



Norton current source: Current through _____

Worked Example 2 on Norton

Derive the Norton equivalent circuit seen by the load R_L



Redraw the circuit first with a-b shorted!

Use NVA since this requires less solving equations than MCA:

Apply KCL at node v_1 :

$$\frac{10 - v_1}{1000} = \frac{v_1}{1000} + \frac{v_1 - v_2}{1}$$

$$10 - v_1 = v_1 + 1000v_1 - 1000v_2$$

$$1002v_1 - 1000v_2 = 10$$

(1)

Apply KCL at node v_2 :

$$\frac{v_1 - v_2}{1} = 0.01 + \frac{v_2}{3}$$

$$3v_1 - 3v_2 = 0.03 + v_2 \quad (2)$$

$$3v_1 - 4v_2 = 0.03 + v_2$$

STOP & ASK: Do we need to solve for both v_1 and v_2 or just one of them is enough for our objective?

Re-arrange (2) to express v_1 in terms of v_2 :

$$v_1 = \frac{2}{3}v_2 + 0.01$$

Sub into (1):

$$1002 \left(0.01 + \frac{4}{3}v_2 \right) - 1000v_2 = 10$$

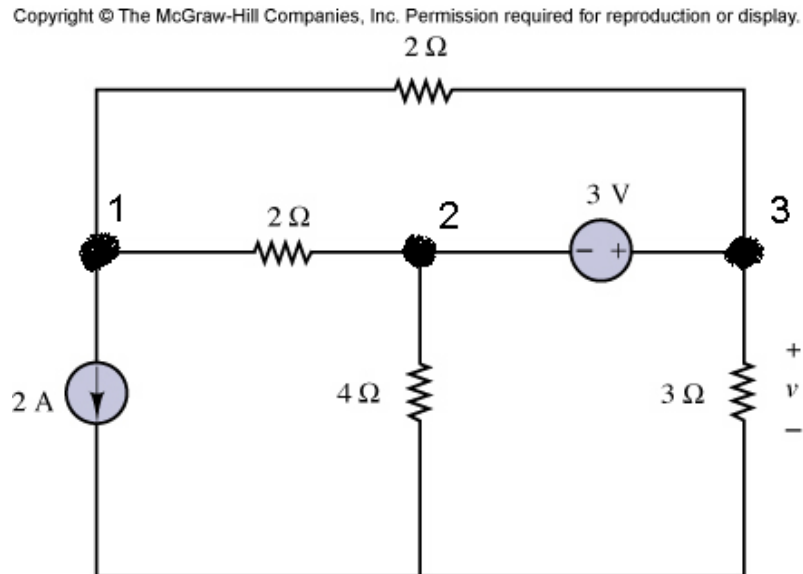
$$0.02 + 336v_2 = 0$$

Norton current:

$$I_N = \frac{v_2}{3\Omega} = \frac{-0.02}{336} / 3 = -\frac{1}{50400} \text{ A}$$

Worked Example 3 on Norton

Derive the Norton equivalent circuit seen by the 3Ω load



Use NVA since this requires less solving equations than MCA:

Apply KCL at node 1:

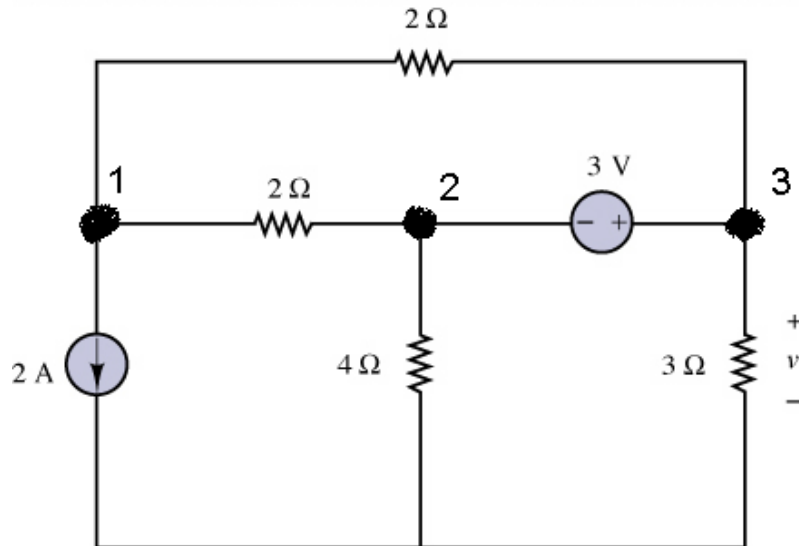
$$\frac{0 - v}{2} + \frac{-3 - v}{2} = 2$$

$$-2v - 3 = 4$$

$$v = -3.5V$$

Redraw the circuit to label the branch currents:

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Evaluate each of the branch currents:

Through 2Ω from 2 to 1: $[-3 - (-3.5)]/2 = 0.25\text{A}$

Through 4Ω from 3 to 2: $3/4 = 0.75\text{A}$

Through 3V source from 2 to 3: $0.75 - 0.25 = 0.5\text{A}$

Through 2Ω from 3 to 1: $3.5/2 = 1.75\text{A}$

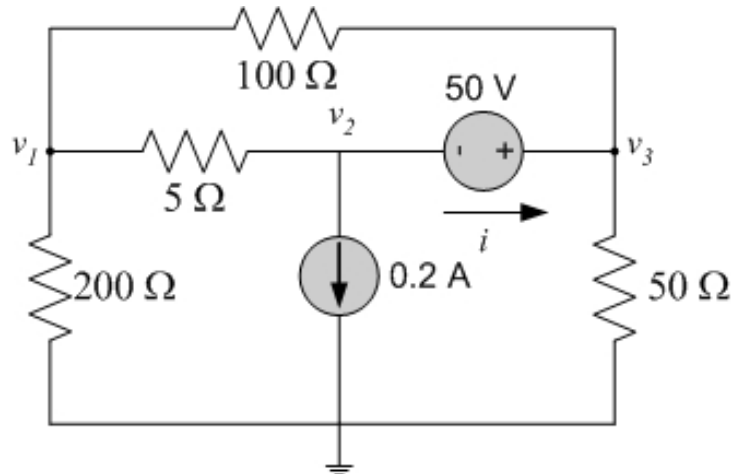
Current from 3 to 1: $1.75 - 0.5 = 1.25\text{A}$

Norton current (1 to 3) = -1.25A

But there is an even shorter way to solve for the current without having to use NVA or MCA: Can you find it?

Worked Example 4 on Norton

Derive the Norton equivalent circuit seen by the 50Ω load



Use NVA since this requires less solving equations than MCA again:

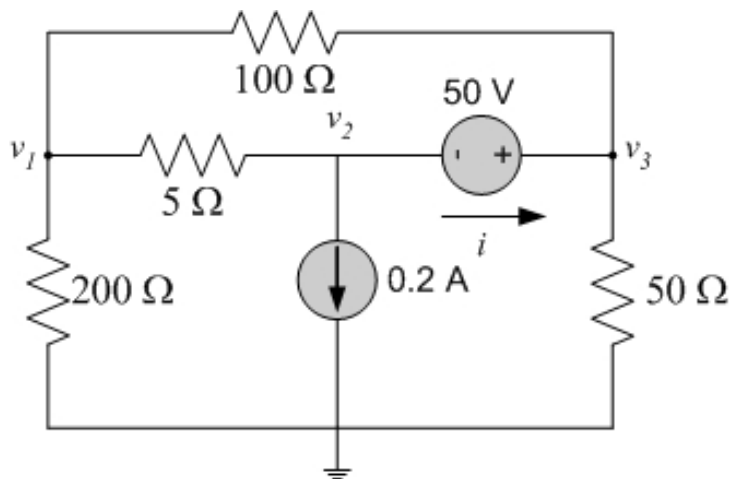
Use node v_3 as reference (0 V) $\Rightarrow v_2 = -50\text{ V}$

Apply KCL to node v_1 :

$$\frac{v_1}{100} + \frac{v_1}{200} + \frac{v_1 - (-50)}{5} = 0$$

$$2v_1 + v_1 + 40v_1 + 2000 = 0$$

$$v_1 = -(2000 / 43)\text{ V}$$



To find the Norton current, one way is to use the current source and current through the 200Ω resistor.

It is advisable to label the currents (including directions) on the diagram as you work out their values.

Current through 200Ω from v_3 to v_1 :
 $(2000/43)/200 = 10/43 \text{ A}$

Current from v_3 to ground:
 $10/43 - 0.2 = 7/215 \text{ A}$

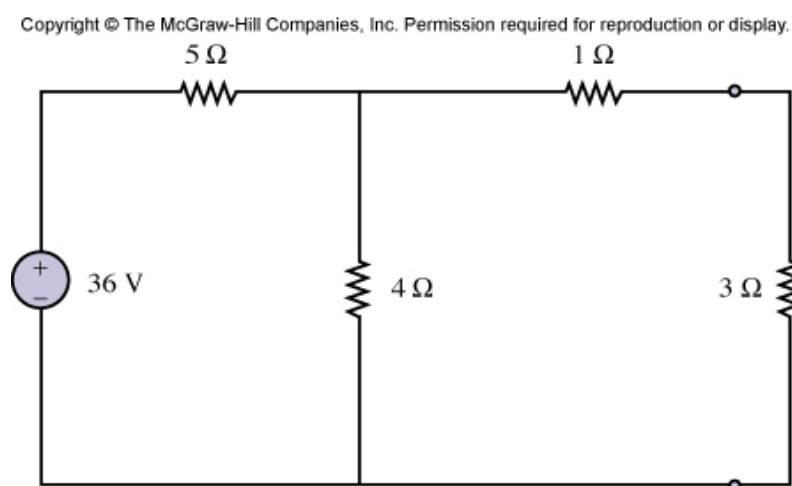
Source transformation

This trick rests on the fact that the Thevenin and Norton forms are equivalent to each other and therefore are also interchangeable. In other words, we switch between the 2 equivalent forms.

Rather than transform the whole circuit in one go to one of the equivalent forms, we can instead transform part of the circuit. From then on, we can attempt to repeat the process of merge transform and merge again. The best way to illustrate this is to work through a problem.

Worked Example 1 on Source transformation

Derive either of the equivalent circuit forms seen by the 3Ω load

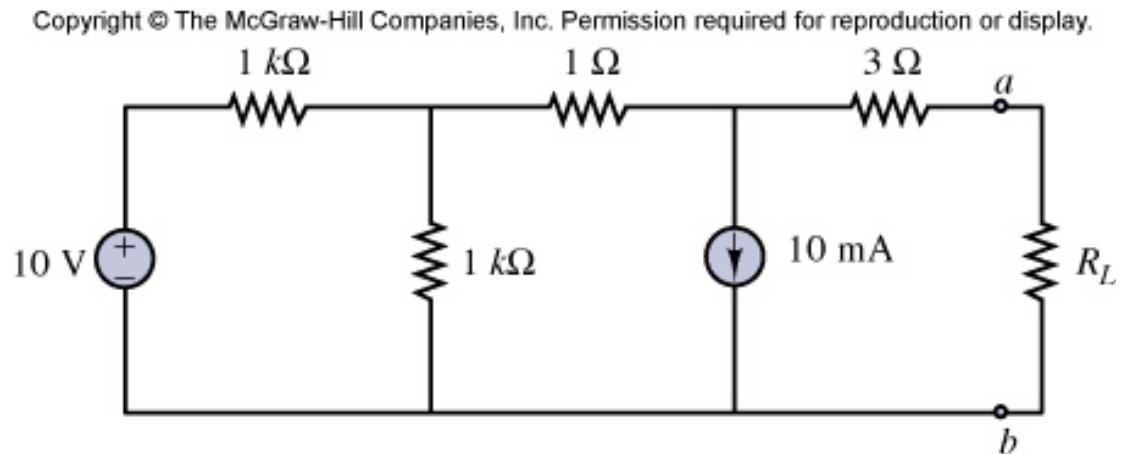


Option 1: Transforming by the smallest steps (e.g. start with 36V and 5Ω)

Option 2: Transform by larger sub-sections (include 4Ω in the transformation).

Worked Example 2 on Source transformation

Derive either of the equivalent circuit forms seen by the load R_L



Transform network with 10V and two $1\text{ k}\Omega$:

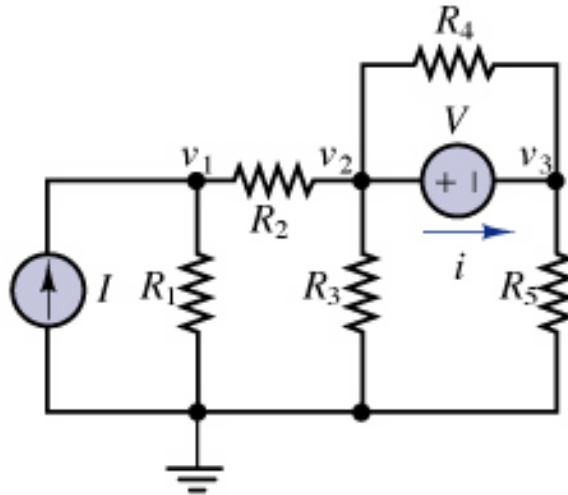
Note that I was careful to remove R_L from the circuit after performing the first transformation.

Option 1: Transform Thevenin to Norton and then back to Thevenin

Option 2: Solve for PD across open circuit terminals

Worked Example 3 on Source transformation

Derive either of the equivalent circuit forms seen by the load R_5 . Let $I = 0.2\text{A}$, $R_1 = 200\Omega$, $R_2 = 75\Omega$, $R_3 = 25\Omega$, $R_4 = 50\Omega$, $R_5 = 100\Omega$, $V = 10\text{V}$.



First transform (Current source I and R_1):

$$V_{th1} = 200 \times 0.2 = 40\text{V}$$

Second transform (absorb R_2 and R_3)

R_1 and R_2 in series: 275Ω

Merge in parallel with R_3 : $275 \parallel 25 = 275/12 \Omega$

Thevenin voltage:

$$V_{th2} = 40 * [25 / (275 + 25)] = 10/3 \text{ V}$$

Now redraw the transformed circuit:

Note that source V and R_4 cannot be transformed. So to proceed further, we fall back on the techniques we have just learned.

What is the current through $275/12\Omega$? _____

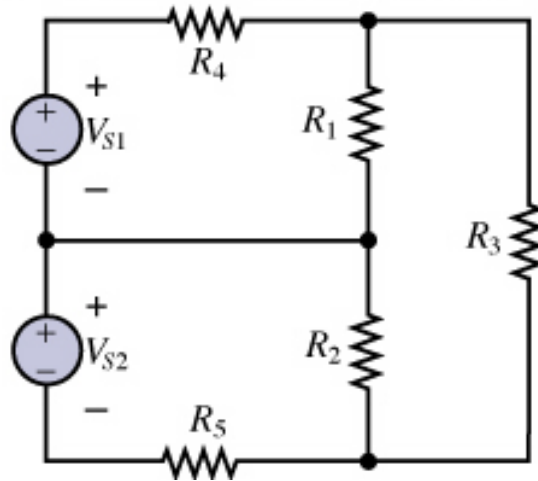
Therefore, terminal voltage = $10/3 - 10 = -20/3 \text{ V}$

$\Rightarrow$ Thevenin voltage = $-20/3 \text{ V}$

Thevenin resistance = $275/12\Omega$

Worked Example 4 on Source transformation

Derive either of the equivalent circuit forms seen by the load R_3 . $V_{S1} = V_{S2} = 50\text{V}$, $R_4 = R_5 = 100\Omega$, $R_1 = 400\Omega$, $R_2 = 150$



First transform V_{S1} with R_4 , R_1 and V_{S2} with R_5 , R_2 .

$$V_{th1} = \frac{R_1}{R_1 + R_4} V_{S1} = \left(\frac{400}{100 + 400} \right) 50 = 40\text{V}$$

$$R_{th1} = R_4 \parallel R_1 = 400 \parallel 100 = 80\Omega$$

$$V_{th2} = \frac{R_2}{R_2 + R_5} V_{S2} = \left(\frac{150}{100 + 150} \right) 50 = 30\text{V}$$

$$R_{th2} = R_5 \parallel R_2 = 100 \parallel 150 = 60\Omega$$

Re-draw the partially transformed circuit:

The next simplification should be obvious once you get to the above stage:

II. Dependent Sources

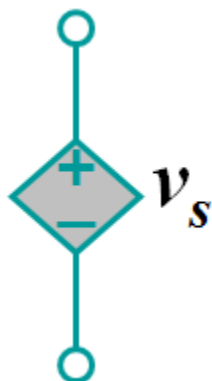
So far in Block A, all the sources we have come across are independent sources. What this means is that for a voltage source, the voltage maintained across the source is fixed. For an independent current source, this means that the current through the source is fixed. However, it is worth noting that there also exist dependent sources in circuit theory. Unlike independent sources, the value of a dependent source is not predetermined, but is set by the current or voltage through a specific branch.

Symbol

The symbol for an independent source is a circle. For a dependent source, the symbol is a diamond. There are a total of 4 basic types of dependent sources.

Dependent Voltage source

The source a voltage across the terminals of v_s . But the value of v_s could be set by either current or voltage from another branch.



Current Controlled Voltage Source

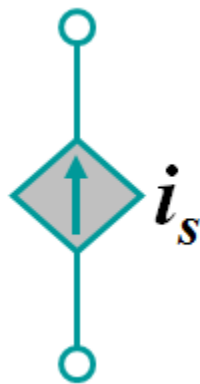
For example, $v_s = 5i_x$. The current i_x is not the current through the source but belongs to another branch. As i_x changes, so also v_s according to the above relation.

Voltage Controlled Voltage Source

For example, $v_s = 7v_x$. The voltage v_x is not the source voltage. As v_x changes, so also v_s according to the above relation.

Dependent Current source

The current through the source is kept at i_s , but the value of i_s is set by either current or voltage from another branch.



Current Controlled Voltage Source

For example, $i_s = 5i_x$. The current i_x is not the current through the source but belongs to another branch. As i_x changes, so also i_s according to the above relation.

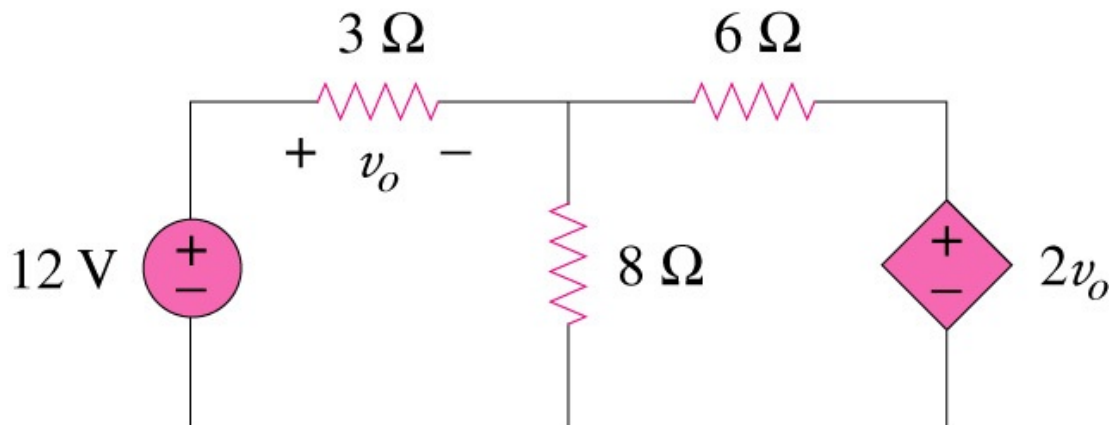
Voltage Controlled Voltage Source

For example, $i_s = 7v_x$. The voltage v_x is not the source voltage but from another branch. As v_x changes, so also will i_s according to the above relation.

Once again, working through a few problems will help to illustrate how these dependent sources work.

Worked Example 1 on Dependent Sources

Calculate v_o in the circuit



The dependent source here in this example is a voltage controlled voltage source

Apply KCL at node X:

$$\frac{v_o}{3} + \frac{2v_o - v_x}{6} = \frac{v_x}{8}$$

But note that $v_x = 12 - v_o$ (Sub into equation)

$$\frac{v_o}{3} + \frac{2v_o - (12 - v_o)}{6} = \frac{12 - v_o}{8}$$

$$\frac{v_o}{3} + \frac{3v_o - 12}{6} = \frac{12 - v_o}{8}$$

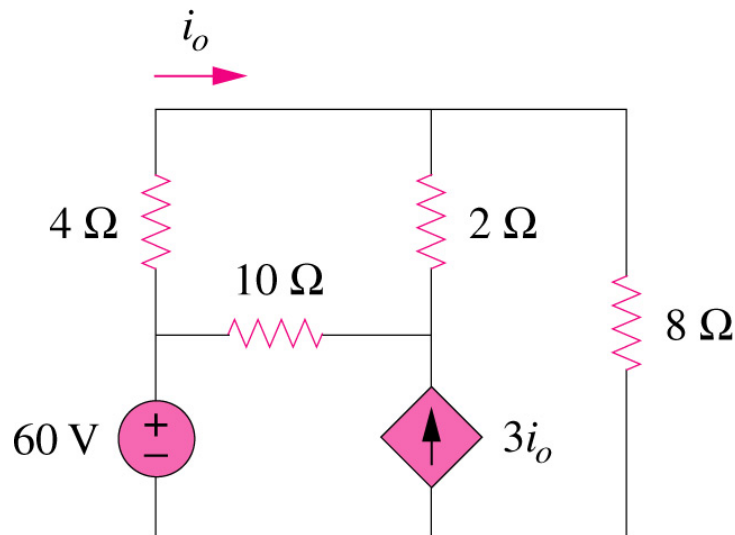
$$8v_o + 4(3v_o - 12) = 3(12 - v_o)$$

$$23v_o = 84$$

$$v_o = 3.65V$$

Worked Example 2 on Dependent Sources

Calculate i_o in the circuit



The dependent source here in this example is a current controlled current source

Identify the unknown node voltages

Apply KCL at node A:

$$\frac{60 - v_A}{4} + \frac{v_B - v_A}{2} = \frac{v_A}{8} \quad (1)$$

Apply KCL at node B:

$$\frac{60 - v_B}{10} + 3i_o = \frac{v_B - v_A}{2} \quad (2)$$

$$\text{But } i_o = \frac{60 - v_A}{4} \quad (3): \text{ sub into (2) then solve for } v_A$$

Simplify (1):

$$2(60 - v_A) + 4(v_B - v_A) = v_A$$

$$7v_A - 4v_B = 120$$

Simplify (2):

$$2(60 - v_B) + 15(60 - v_A) = 10(v_B - v_A)$$

$$5v_A + 12v_B = 1020$$

Solving for v_A :

$$26v_A = 1380$$

$$v_A = 53.077V$$

Finally, use v_A to find i_o :

$$\begin{aligned} i_o &= \frac{60 - 53.077}{4} \\ &= 1.73A \end{aligned}$$