

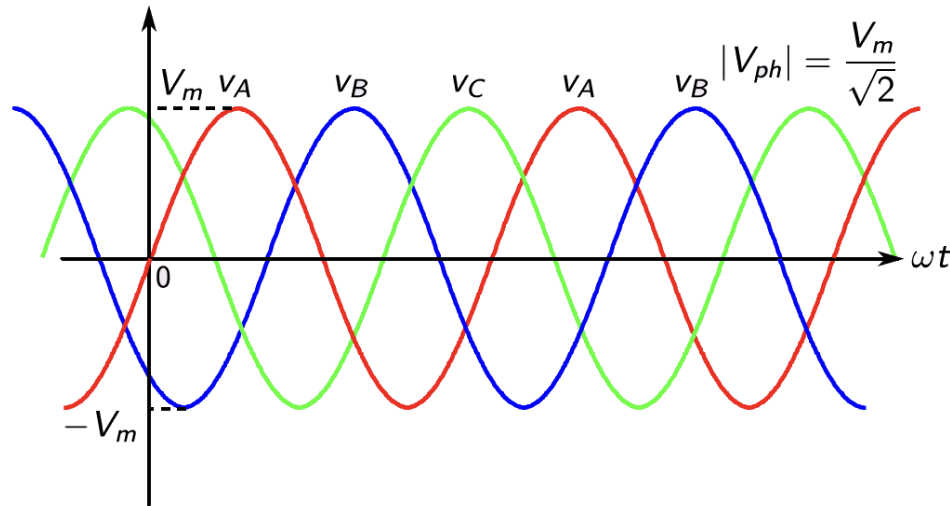
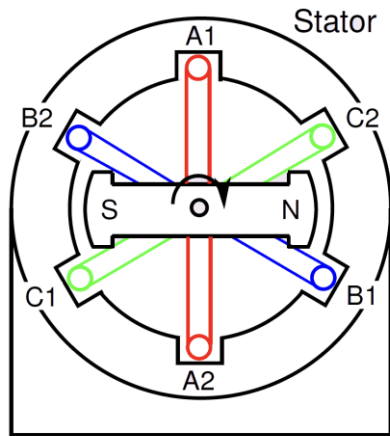
EE3123 Introduction to Electric Power Systems

Three-phase power circuit analysis

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Many thanks to Prof. Michael Tse

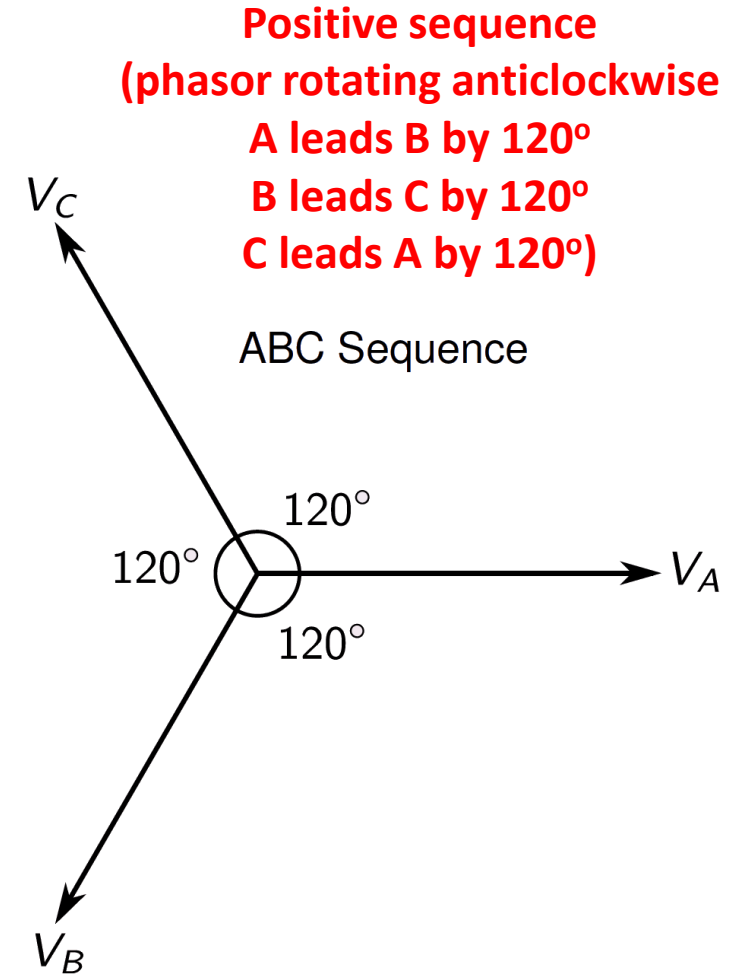
Three-phase voltages



$$v_A = V_m \sin \omega t \Rightarrow V_A = |V_{ph}| \angle 0^\circ$$

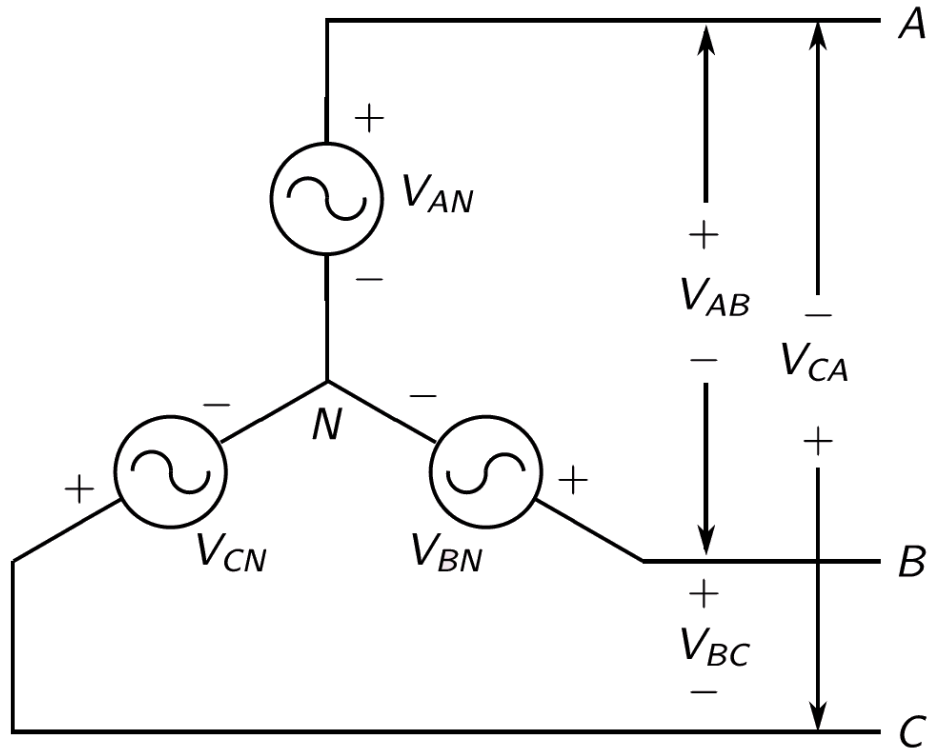
$$v_B = V_m \sin(\omega t - 120^\circ) \Rightarrow V_B = |V_{ph}| \angle -120^\circ$$

$$v_C = V_m \sin(\omega t - 240^\circ) \Rightarrow V_C = |V_{ph}| \angle -240^\circ$$

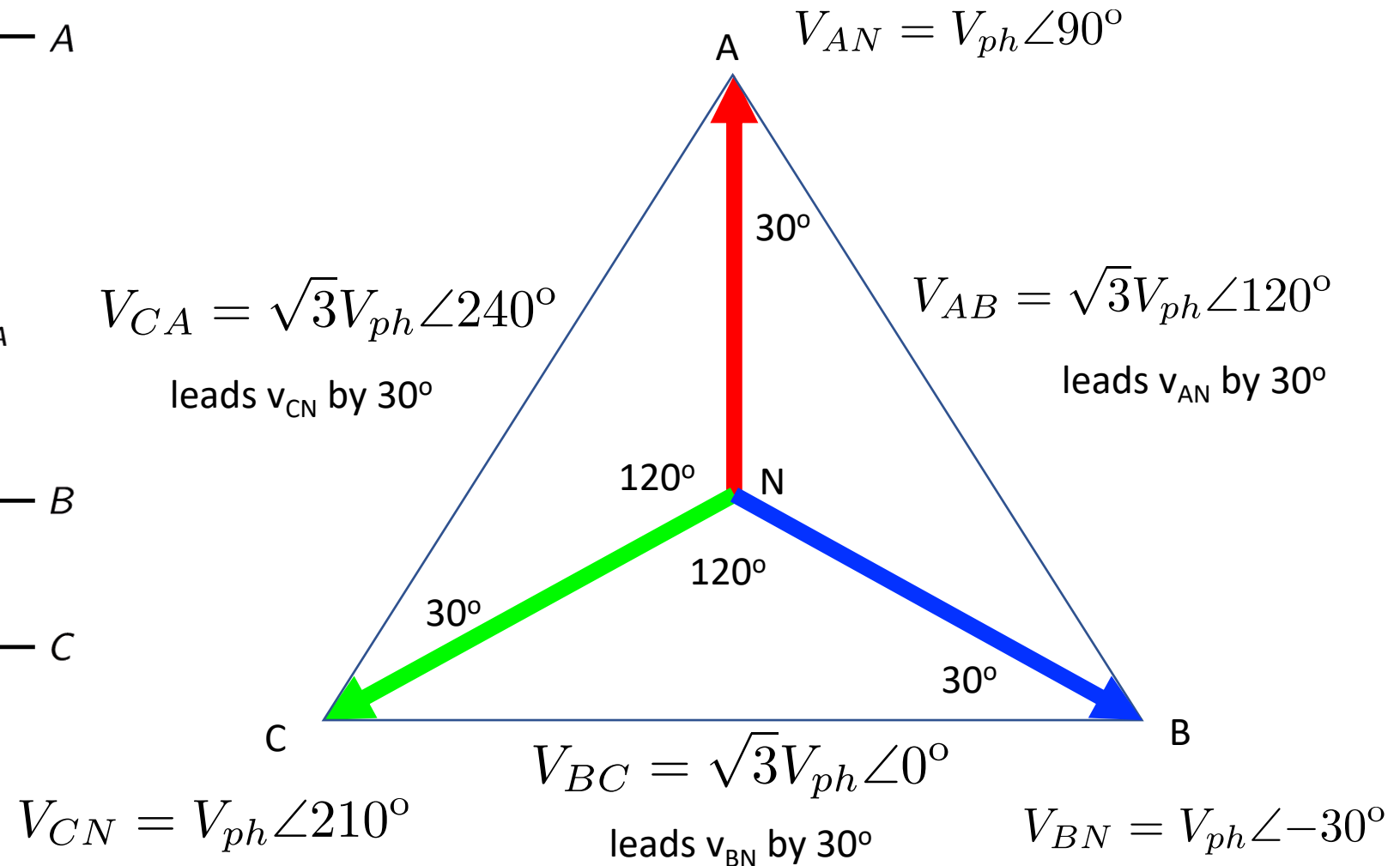


If $|V_A| = |V_B| = |V_C| = |V_{ph}|$ and all phase differences are equal, the three-phase voltages are BALANCED.

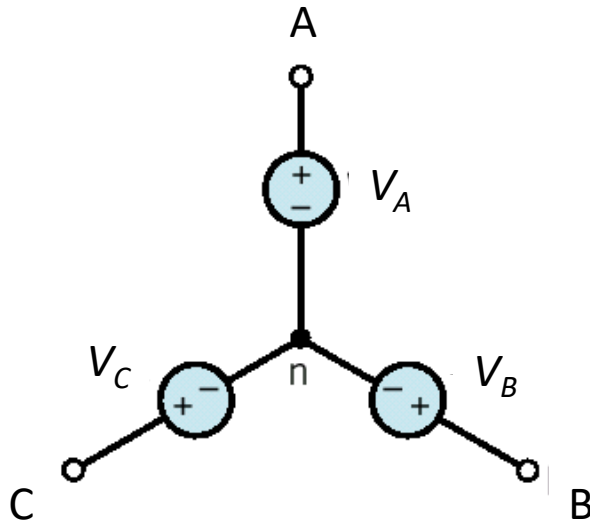
POSITIVE SEQUENCE or ABC SEQUENCE



$$\begin{aligned} E_{ab} &= \sqrt{3}E_{an} \angle +30^\circ \\ E_{bc} &= \sqrt{3}E_{bn} \angle +30^\circ \\ E_{ca} &= \sqrt{3}E_{cn} \angle +30^\circ \end{aligned}$$



Three-phase connections

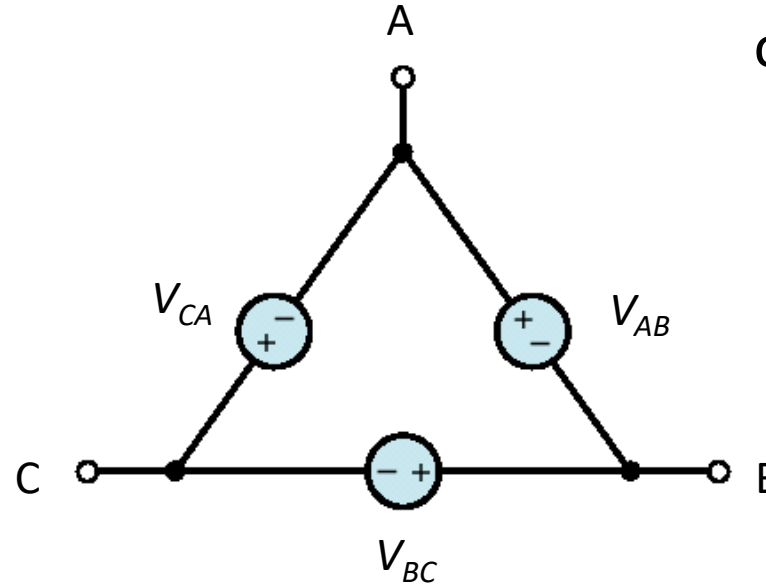


$$V_A = 110 \text{ V}$$

$$V_A = 220 \text{ V}$$

$$V_A = 240 \text{ V}$$

Phase voltage



$$V_{AC} = 190 \text{ V}$$

$$V_{AC} = 380 \text{ V}$$

$$V_{AC} = 416 \text{ V}$$

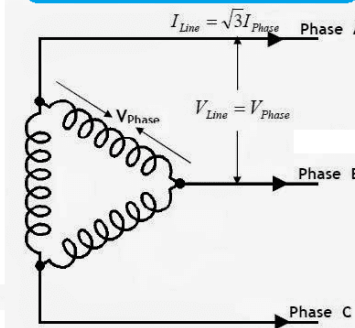
Line voltage

$$V_{\text{line}} = \sqrt{3}V_{\text{phase}}$$

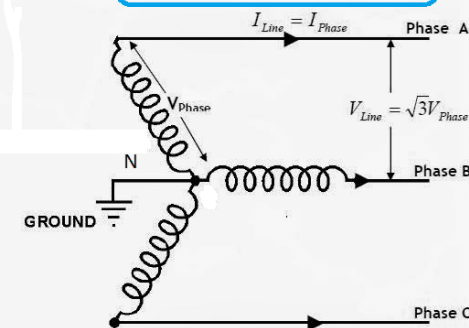
or

$$V_{LL} = \sqrt{3}V_{LN}$$

Delta Connction



Star Connection

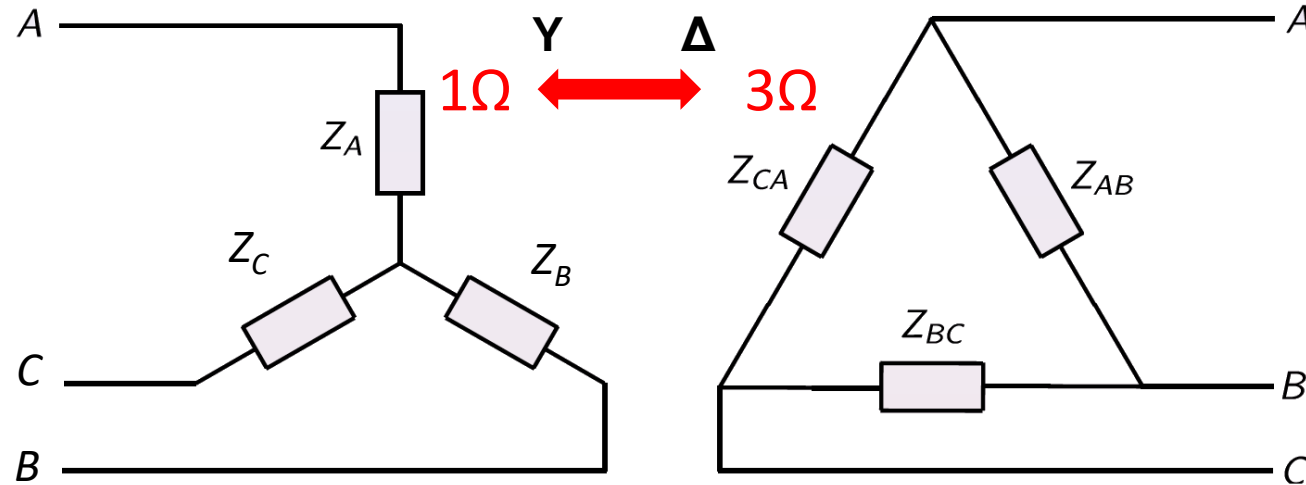


Three-phase balanced loads

If all loads are equal in a Y or Δ circuit, then the three-phase load is BALANCED.

For balanced loads,

$$Z_{\Delta} = 3Z_Y$$



$$Z_A = \frac{Z_{CA}Z_{AB}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_B = \frac{Z_{AB}Z_{BC}}{Z_{BC} + Z_{CA} + Z_{AB}}$$

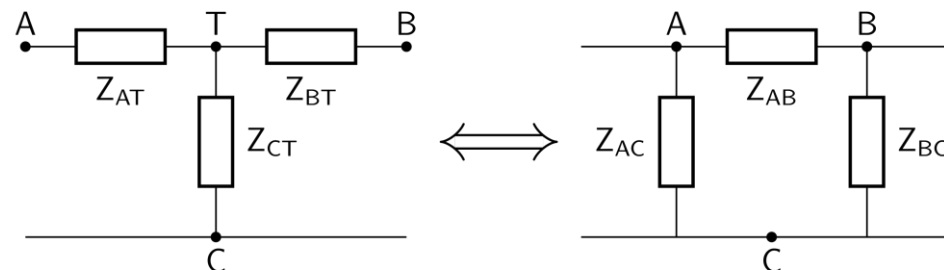
$$Z_C = \frac{Z_{BC}Z_{CA}}{Z_{CA} + Z_{AB} + Z_{BC}}$$

$$Z_{AB} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_C}$$

$$Z_{BC} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A}$$

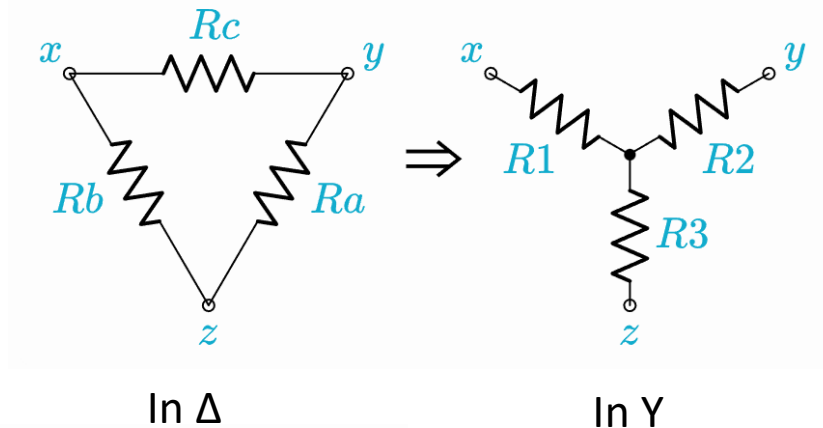
$$Z_{CA} = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_B}$$

Pi - T transformation



Proof of Wye (Y) – Delta (Δ) Transformation

Δ – Y derivation



$$R_{xy} = Rc \parallel (Ra - Rb) = \frac{Rc(Ra + Rb)}{Rc + (Ra + Rb)}$$

$$R_{xy} = R1 + R2$$

$$R_{xy} : R1 + R2 = Rc(Ra + Rb)/(Ra + Rb + Rc)$$

The same for other two sub circuits

$$R_{xy} : R1 + R2 = Rc(Ra + Rb)/(Ra + Rb + Rc)$$

$$R_{yz} : R2 + R3 = Ra(Rb + Rc)/(Ra + Rb + Rc)$$

$$R_{zx} : R3 + R1 = Rb(Rc + Ra)/(Ra + Rb + Rc)$$

$$R1 = (R_{xy} + R_{zx} - R_{yz}) / 2$$

$$= \frac{Rc(Ra + Rb) + Rb(Rc + Ra) - Ra(Rb + Rc)}{2(Ra + Rb + Rc)}$$

$$R1 = \frac{Rb Rc}{Ra + Rb + Rc}$$

The same to get R2 and R3.

The Δ to Y transformation is,

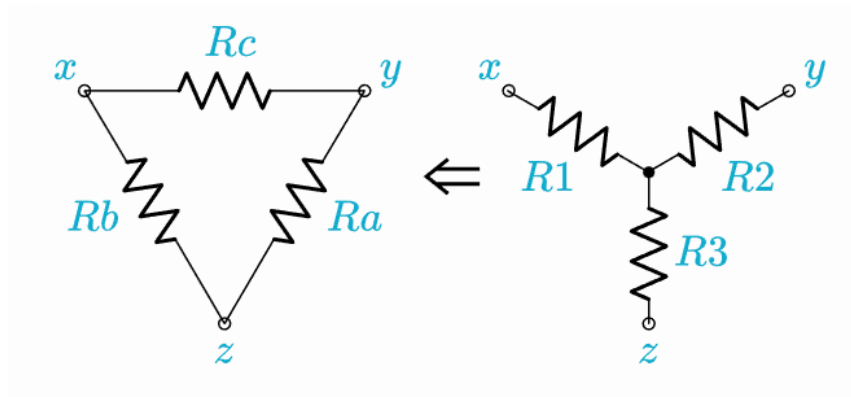
$$R1 = \frac{Rb Rc}{Ra + Rb + Rc}$$

$$R2 = \frac{Ra Rc}{Ra + Rb + Rc}$$

$$R3 = \frac{Ra Rb}{Ra + Rb + Rc}$$

Proof of Wye (Y) – Delta (Δ) Transformation

Y – Δ derivation



The Y to Δ transformation is,

$$R_a = \frac{R_3 R_2 + R_3 R_1 + R_1 R_2}{R_1}$$

$$R_b = \frac{R_3 R_2 + R_3 R_1 + R_1 R_2}{R_2}$$

$$R_c = \frac{R_3 R_2 + R_3 R_1 + R_1 R_2}{R_3}$$

The Δ to Y transformation is,

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$\frac{R_3}{R_1} = \frac{\frac{R_a R_b}{R_b R_c}}{\frac{R_a + R_b + R_c}{R_b R_c}}$$

$$\frac{R_3}{R_1} = \frac{R_a R_b}{R_b R_c} = \frac{R_a}{R_c}$$

$$R_a = \frac{R_3 R_c}{R_1}$$

Repeat the previous

$$\frac{R_3}{R_2} = \frac{R_a R_b}{R_a R_c} = \frac{R_b}{R_c}$$

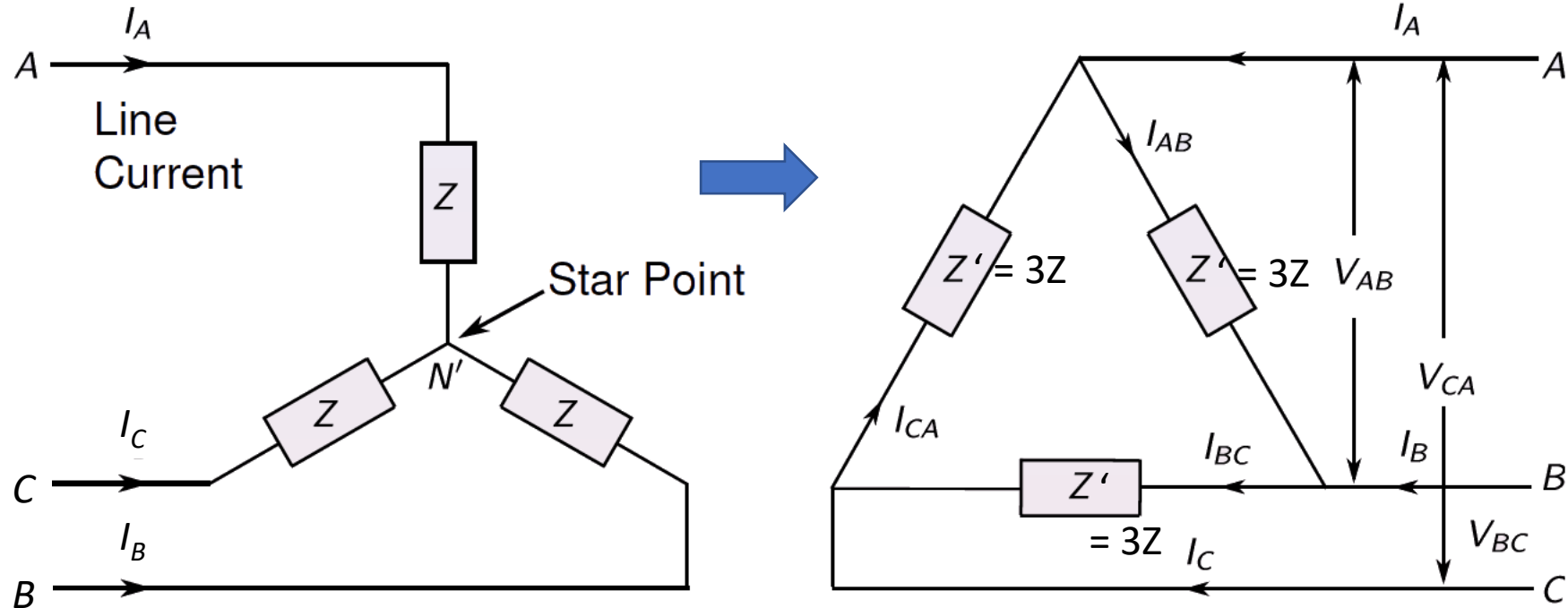
$$R_b = \frac{R_3 R_c}{R_2}$$

$$R_2 = \frac{\frac{R_3 R_c}{R_1} R_c}{\frac{R_3 R_c}{R_1} + \frac{R_3 R_c}{R_2} + R_c}$$

$$R_2 = \frac{\frac{R_3 R_c}{R_1}}{\frac{R_3}{R_1} + \frac{R_3}{R_2} + 1}$$

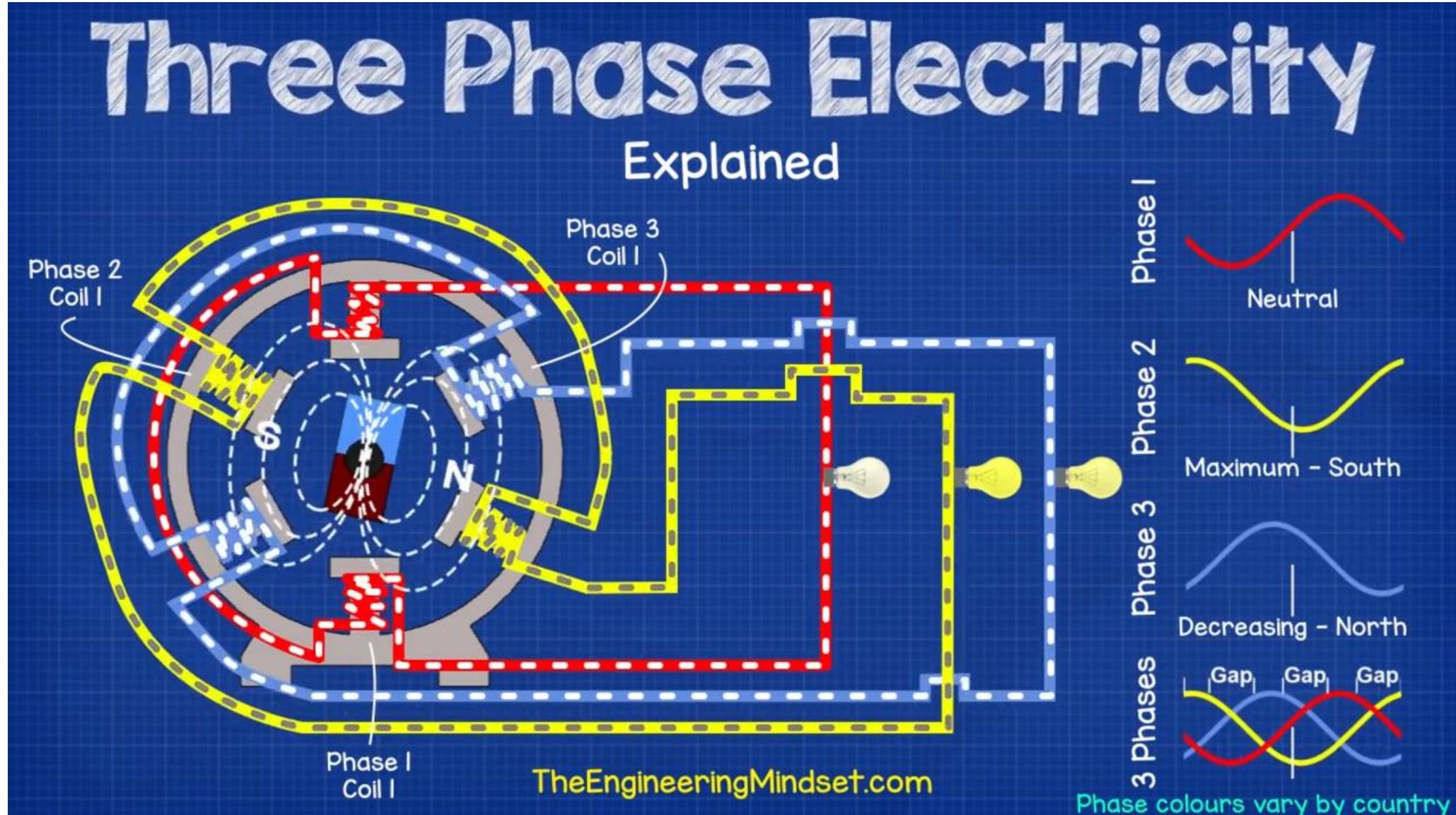
$$R_c = \frac{R_3 R_2 + R_3 R_1 + R_1 R_2}{R_3}$$

Line currents



$$I_A = \sqrt{3}I_{AB}$$

Three-phase electricity

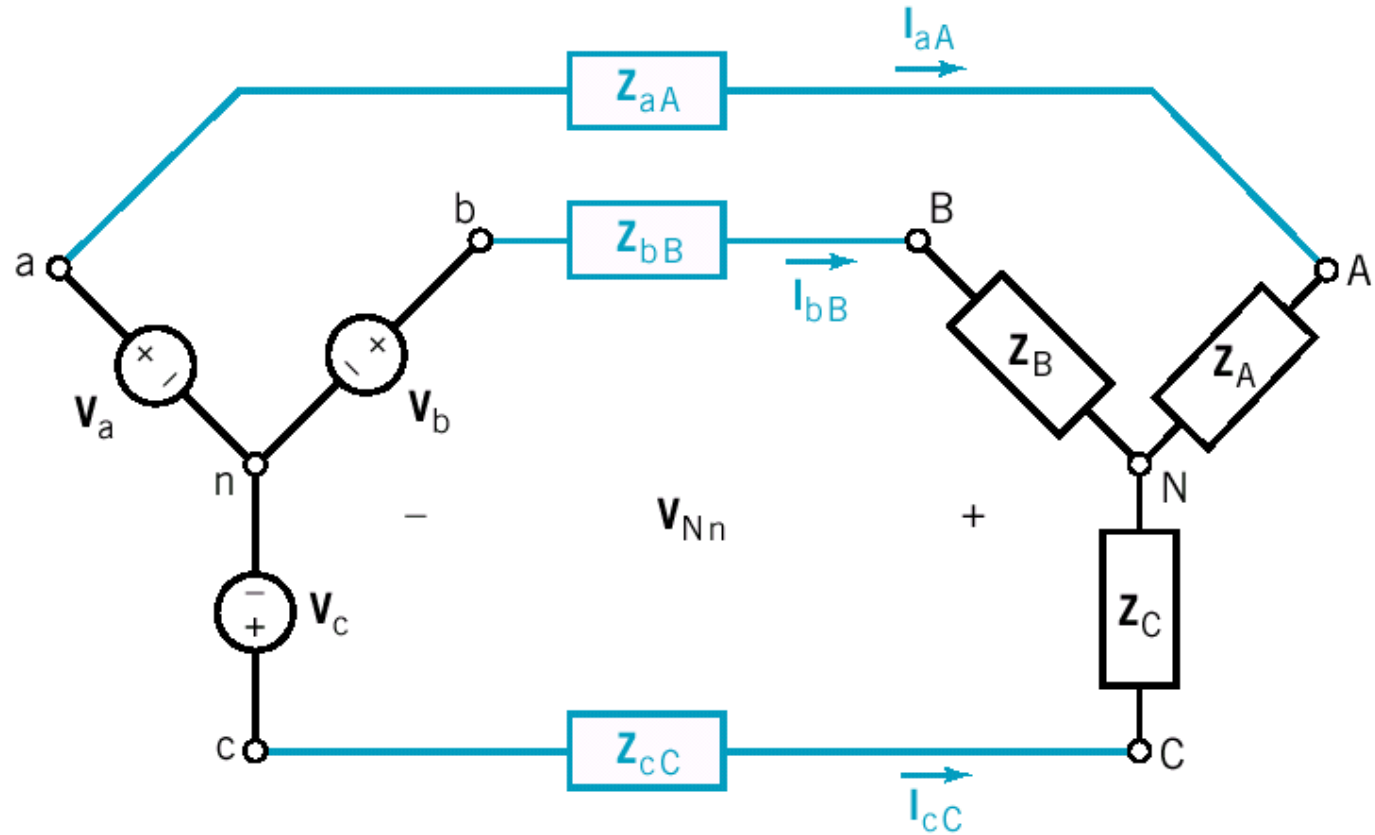


Y-to-Y circuit

If the circuit is balanced, V_{Nn} will be 0.

In other words, there will be no returning current from N to n .

The neutral cable is thus NOT needed!



Suppose $Z_A = Z_B = Z_C = Z \angle \theta$ and $V_{AN} = V_{ph}$ (voltage in rms)

$$\text{Power in each impedance} = \frac{V_{ph}^2}{Z} \cos \theta$$

$$\text{Total power delivered to load} = \frac{3V_{ph}^2}{Z} \cos \theta$$

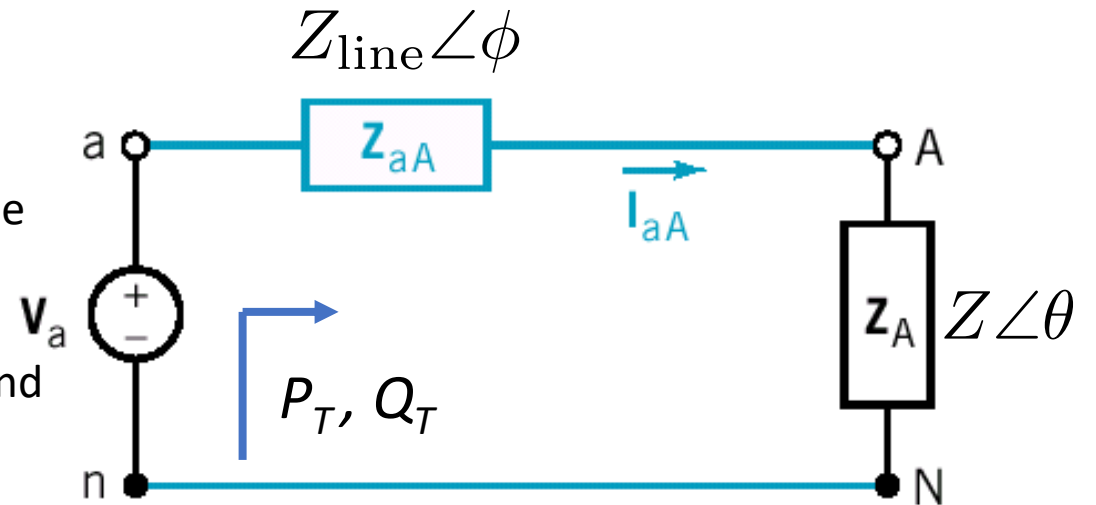
Single-phase equivalent

For a balanced circuit, we know that

- $V_{Nn} = 0$.
- All line currents are equal and differ by 120° .
- Equal power is dissipated in each load.

So, we can simplify the representation to a single-phase circuit, like considering only one of the phases.

Solve the circuit like a single-phase circuit, and then extend all the phase voltages and currents accordingly.



$$Z_T = (Z_{\text{line}} \cos \phi + Z \cos \theta) + j(Z_{\text{line}} \sin \phi + Z \sin \theta)$$

$$|Z_T| = \sqrt{(Z_{\text{line}} \cos \phi + Z \cos \theta)^2 + (Z_{\text{line}} \sin \phi + Z \sin \theta)^2}$$

$$I_{aA} = \frac{V_a}{|Z_T|} \quad (\text{current and voltage in rms})$$

$$P_T = I_{aA}^2 (Z_{\text{line}} \cos \phi + Z \cos \theta)$$

$$Q_T = I_{aA}^2 (Z_{\text{line}} \sin \phi + Z \sin \theta)$$

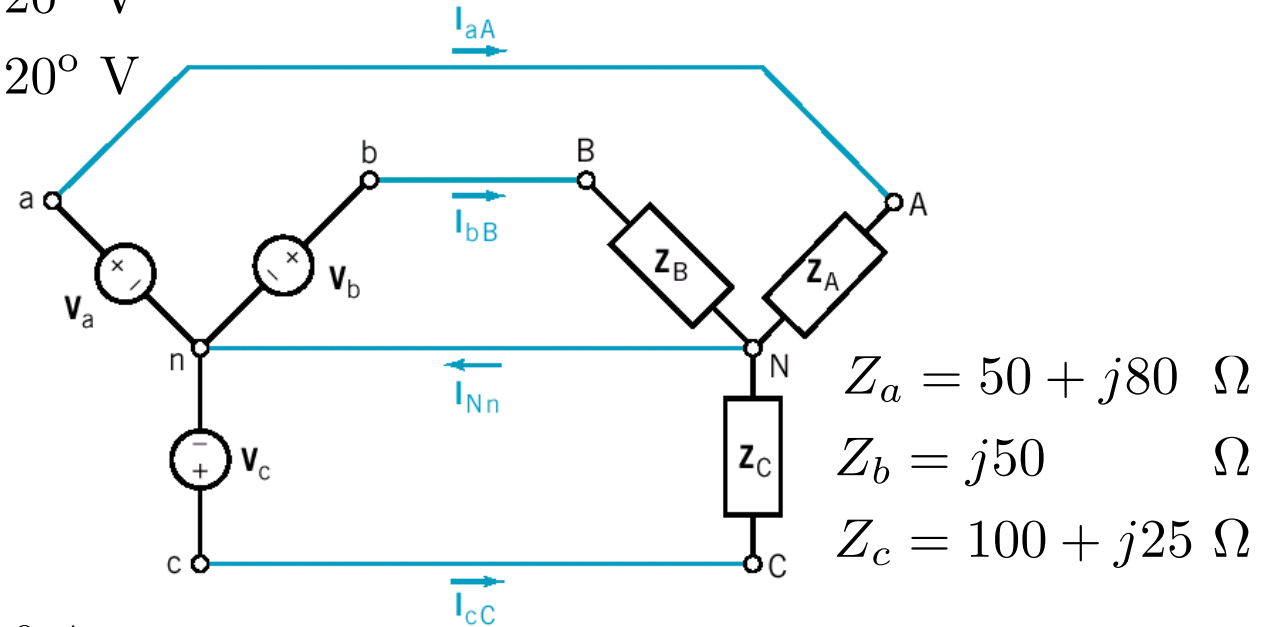
$$S_T = \sqrt{P_T^2 + Q_T^2}$$

$$\theta_T = \tan^{-1}(Q_T/P_T)$$

Unbalanced loads

What is the apparent power delivered to the load by the source?

$$\begin{aligned} V_a &= 110\angle 0^\circ \text{ V} \\ V_b &= 110\angle -120^\circ \text{ V} \\ V_c &= 110\angle +120^\circ \text{ V} \end{aligned} \quad \text{all in rms}$$



$$I_{aA} = \frac{110\angle 0^\circ}{50 + j80} = 1.16\angle -58^\circ \text{ A}$$

$$I_{bB} = \frac{110\angle -120^\circ}{j50} = 2.2\angle 150^\circ \text{ A}$$

$$I_{cC} = \frac{110\angle 120^\circ}{100 + j25} = 1.07\angle -106^\circ \text{ A}$$

$$I_{Nn} = I_{aA} + I_{bB} + I_{cC} = 1.8292\angle 150^\circ \text{ A}$$

$$S_A = V_a I_{aA}^* = 68 + j109 \text{ VA}$$

$$S_B = V_b I_{bB}^* = j242 \text{ VA}$$

$$S_C = V_c I_{cC}^* = 114 + j28 \text{ VA}$$

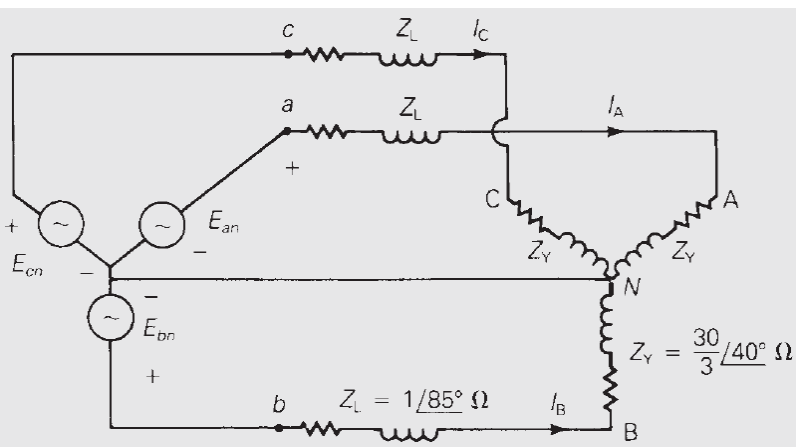
$$S = S_A + S_B + S_C = 182 + j379 \text{ VA}$$

Example: Balanced- Δ and-Y loads

A balanced, positive-sequence, Y-connected voltage source with $E_{ab} = 480 \angle 0^\circ$ volts is applied to a balanced- Δ load with $Z_\Delta = 30 \angle 40^\circ \Omega$. The line impedance between the source and load is $Z_L = 1 \angle 85^\circ \Omega$ for each phase. Calculate the line currents, the Δ -load currents, and the voltages at the load terminals.

SOLUTION

First, convert the Δ load to an equivalent Y. Then connect the source and Y-load neutrals with a zero-ohm neutral wire. The connection of the neutral wire has no effect on the circuit, since the neutral current $I_n = 0$ in a balanced system.



$$\begin{aligned}
 I_A &= \frac{E_{an}}{Z_L + Z_Y} = \frac{\frac{480}{\sqrt{3}} \angle -30^\circ}{1 \angle 85^\circ + \frac{30}{3} \angle 40^\circ} \\
 &= \frac{277.1 \angle -30^\circ}{(0.0872 + j0.9962) + (7.660 + j6.428)} \\
 &= \frac{277.1 \angle -30^\circ}{(7.748 + j7.424)} = \frac{277.1 \angle -30^\circ}{10.73 \angle 43.78^\circ} = 25.83 \angle -73.78^\circ \text{ A} \\
 I_B &= 25.83 \angle 166.22^\circ \text{ A} \\
 I_C &= 25.83 \angle 46.22^\circ \text{ A}
 \end{aligned}$$

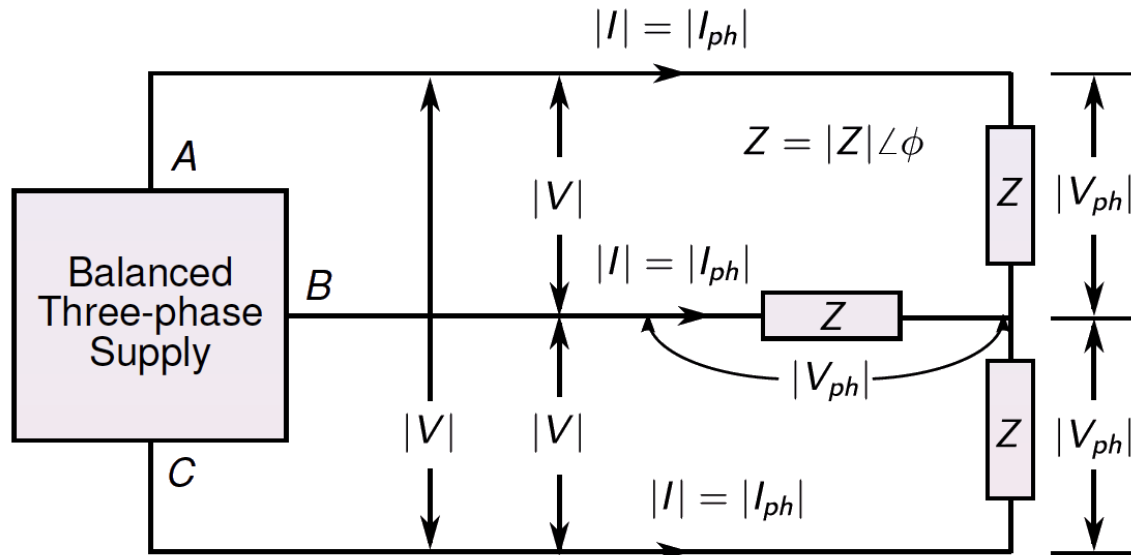
$$\begin{aligned}
 I_{AB} &= \frac{I_a}{\sqrt{3}} \angle +30^\circ = \frac{25.83}{\sqrt{3}} \angle -73.78^\circ + 30^\circ = 14.91 \angle -43.78^\circ \text{ A} \\
 I_{BC} &= 14.91 \angle -163.78^\circ \text{ A} \\
 I_{CA} &= 14.91 \angle +76.22^\circ \text{ A}
 \end{aligned}$$

The voltages at the load terminals are

$$\begin{aligned}
 E_{AB} &= Z_\Delta I_{AB} = (30 \angle 40^\circ)(14.91 \angle -43.78^\circ) = 447.3 \angle -3.78^\circ \text{ V} \\
 E_{BC} &= 447.3 \angle -123.78^\circ \text{ V} \\
 E_{CA} &= 447.3 \angle 116.22^\circ \text{ V}
 \end{aligned}$$

Instantaneous power

Balanced three-phase circuits



$$v_{an} = V_{an} \sin \omega t$$

$$v_{bn} = V_{bn} \sin(\omega t - 120^\circ)$$

$$v_{cn} = V_{cn} \sin(\omega t + 120^\circ)$$

$$p(t) = \frac{v_{ac}^2 + v_{bn}^2 + v_{cn}^2}{|Z|}$$

* Here, the V_{ph} is the maximum value before $\sin()$.

Suppose $V_{an} = V_{bn} = V_{cn} = V_{ph}$

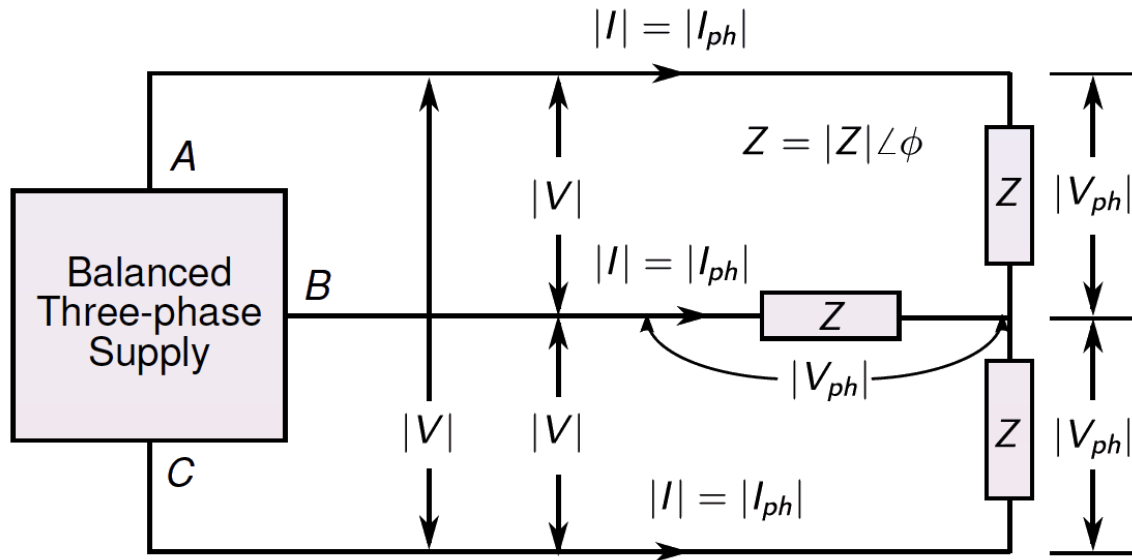
$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

SMOOTH FLOW

$$p(t) = \frac{V_{ph}^2}{2|Z|} (1 - \cos 2\omega t + 1 - \cos(2(\omega t - 120^\circ)) + 1 - \cos(2(\omega t + 120^\circ))) = \frac{3V_{ph}^2}{2|Z|} - \frac{V_{ph}^2}{2|Z|}(0)$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

(Average) Power



* Here, the V_{ph} is the maximum value before $\sin()$.

$$P = \frac{3V_{ph}^2}{2|Z|} = \frac{3V_{ph,rms}^2}{|Z|}$$

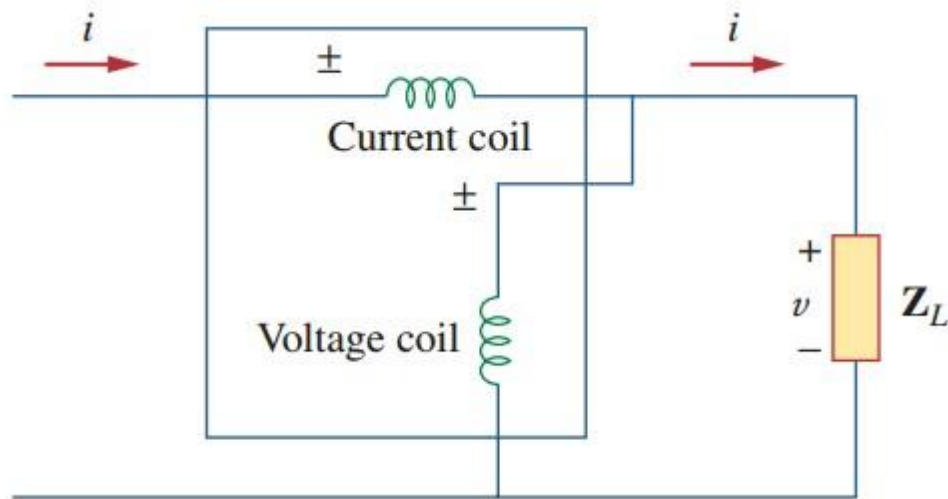
$$\text{or } P = 3V_{ph,rms}I_{ph,rms}\cos\phi$$

$$\text{Since } V_{line} = \sqrt{3}V_{ph,rms}$$

$$P = \sqrt{3}V_{line}I_{ph}\cos\phi \text{ with } V \text{ and } I \text{ in rms values}$$

$$P = \sqrt{3}VI\cos\phi \text{ with } V \text{ and } I \text{ in rms values}$$

Wattmeter



Current coil picks up the current through
Voltage coil picks up the voltage across
Meter reading gives the average product.

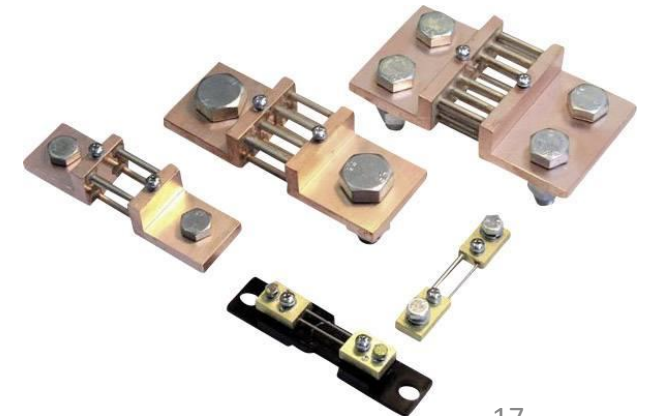
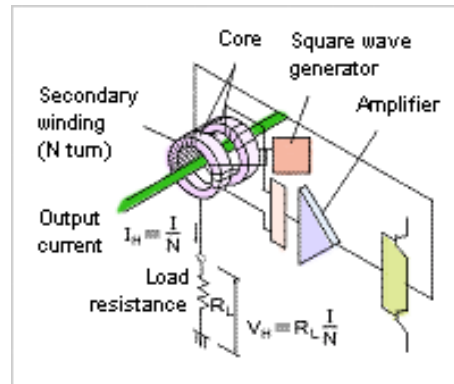
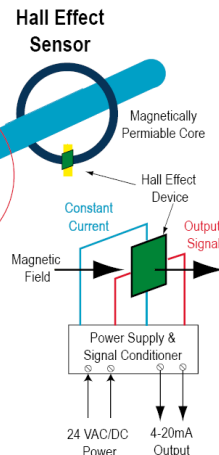
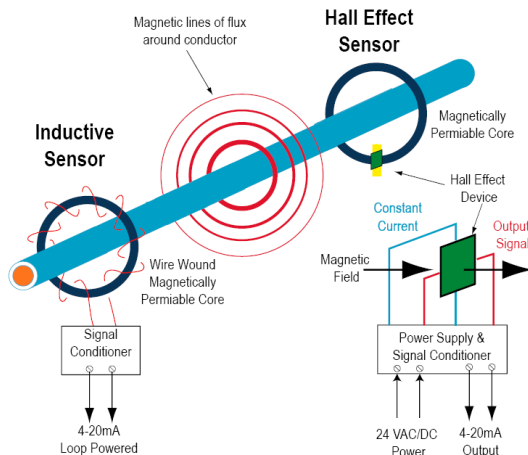
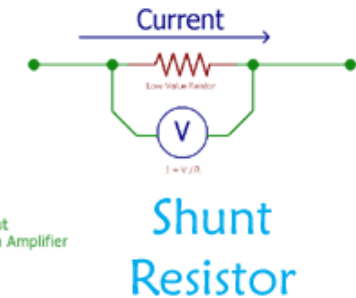
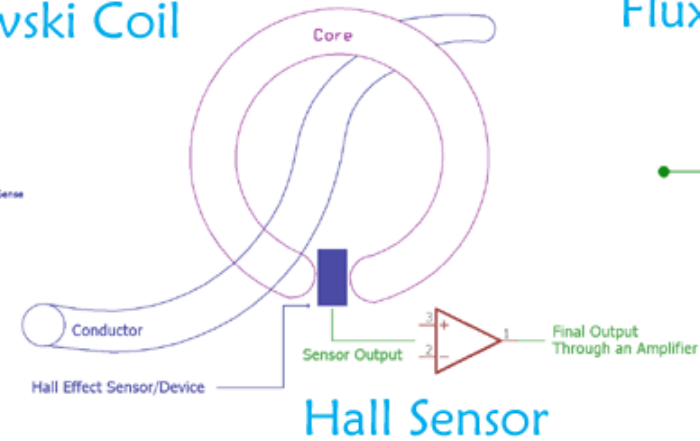
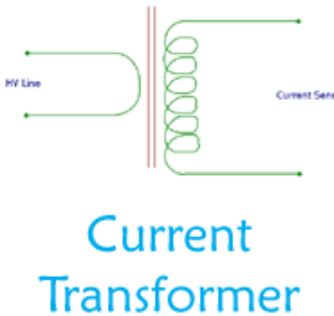
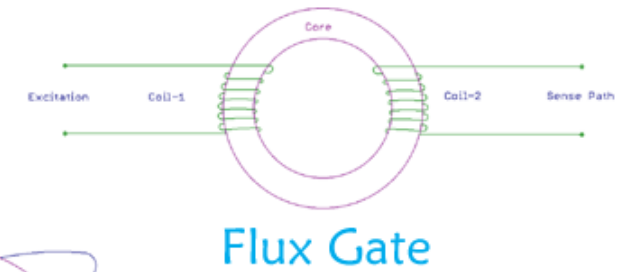
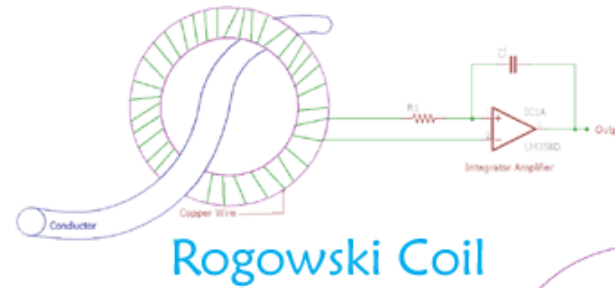
$$P = P_{\text{average}} = VI \cos \phi$$

where ϕ is the angle between V and I

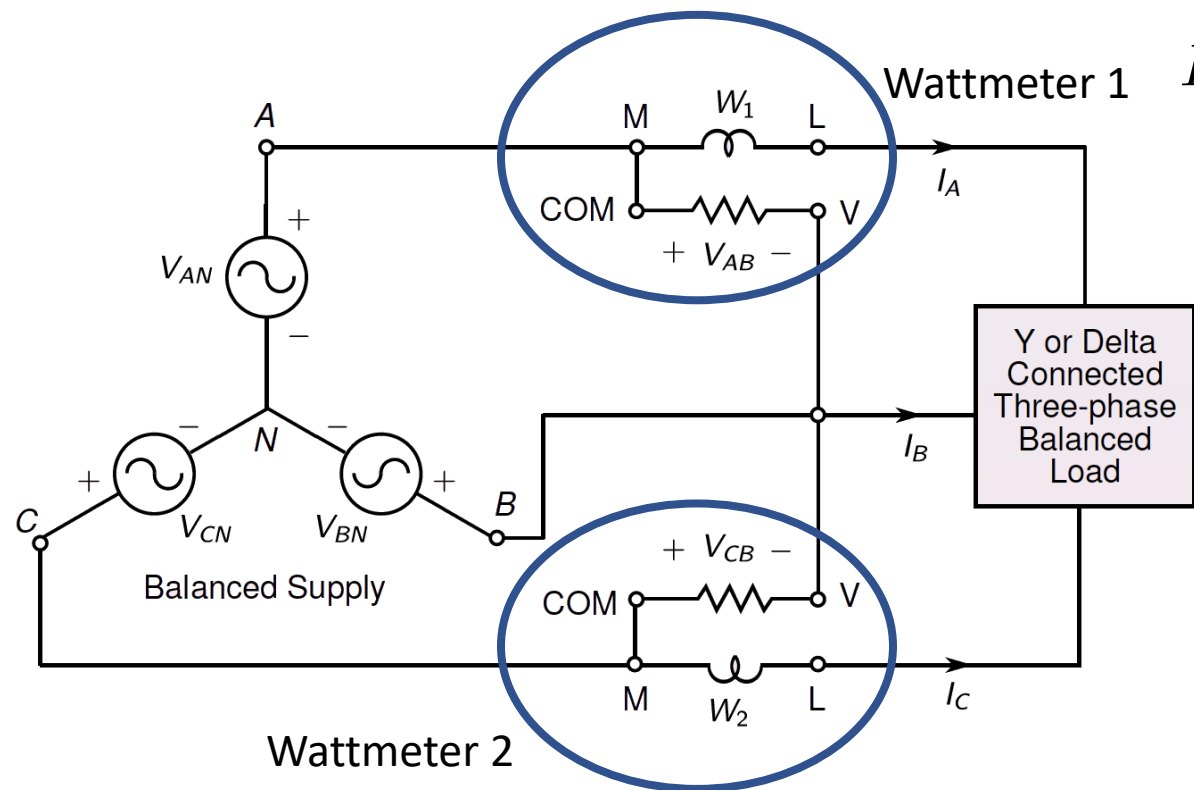
Wattmeter: Current sensing techniques

Measurement methods:

1. Resistive (direct)
 - a. Shunt resistors
2. Magnetic (indirect)
 - a. Current transformer
 - b. Rogowski coil
 - c. Hall effect device- hall sensor
3. Transistor (direct)
 - a. RDS(ON)
 - b. Ratio-metric



Measuring power

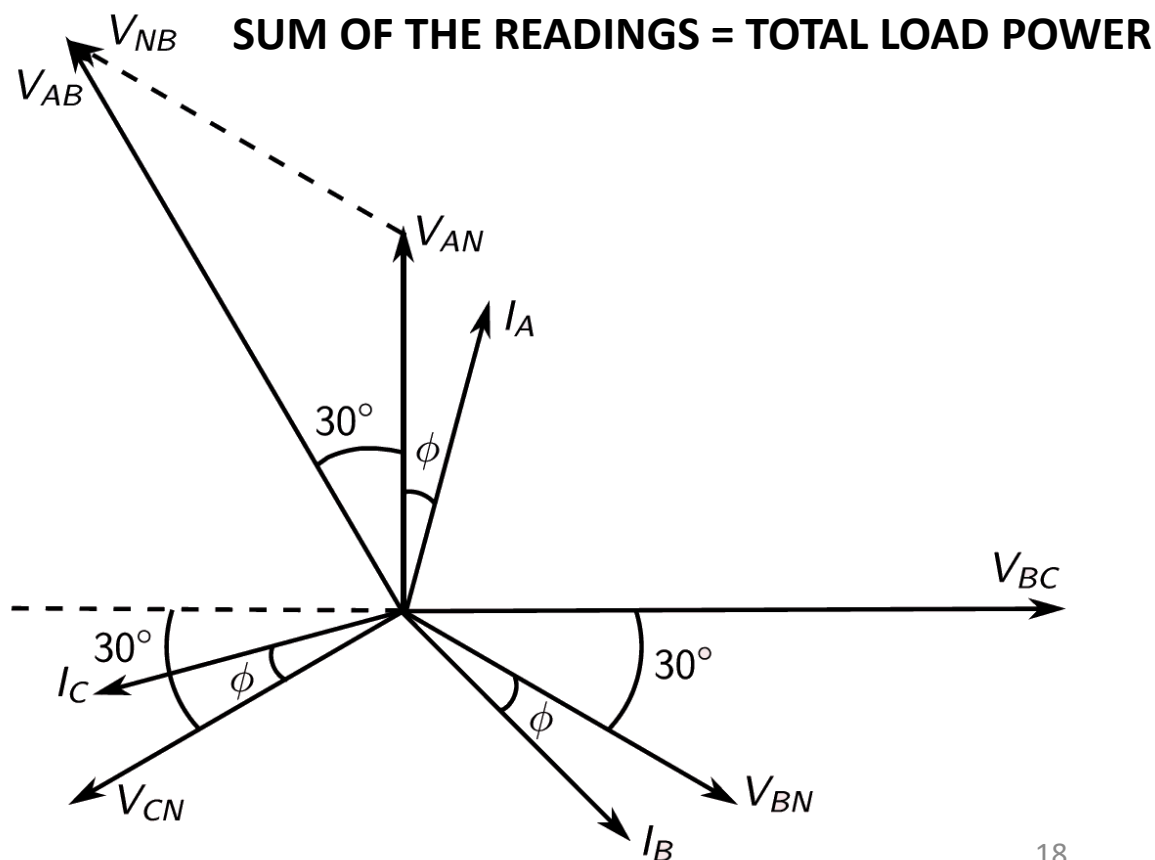


SUM OF THE READINGS W1 and W2 = TOTAL LOAD POWER

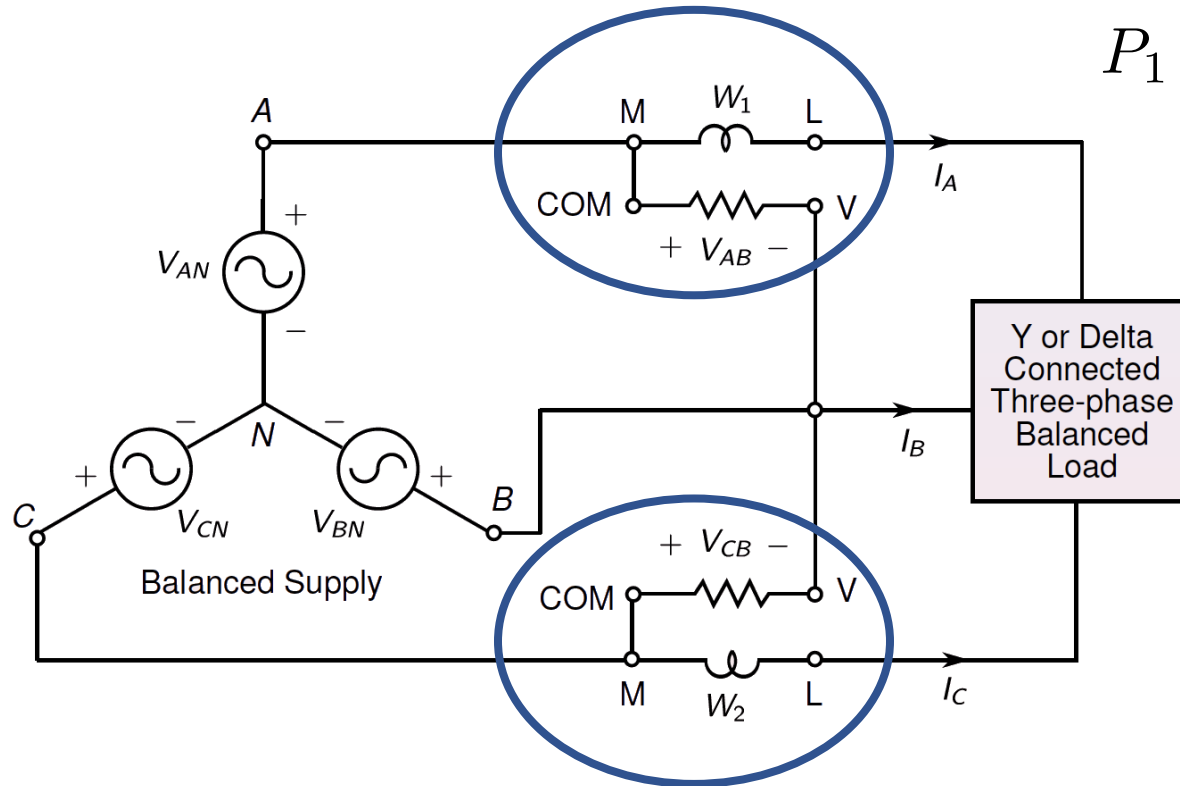
$$P_1 = |V_{AB}| |I_A| \cos(\phi + 30^\circ) = |V| |I| \cos(\phi + 30^\circ)$$

$$P_2 = |V_{BC}| |I_C| \cos(30^\circ - \phi) = |V| |I| \cos(30^\circ - \phi)$$

$$\begin{aligned} P_1 + P_2 &= |V| |I| (\cos(\phi + 30^\circ) + \cos(30^\circ - \phi)) \\ &= 2|V| |I| \cos 30^\circ \cos \phi = \sqrt{3} |V| |I| \cos \phi \end{aligned}$$



Impedance angle



$$P_1 - P_2 = |V||I|(\cos(\phi + 30^\circ) - \cos(30^\circ - \phi))$$

$$= -2|V||I|\sin 30^\circ \sin \phi = -|V||I|\sin \phi$$

$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{-\tan \phi}{\sqrt{3}}$$

$$\tan \phi = -\sqrt{3} \frac{P_1 - P_2}{P_1 + P_2} = \sqrt{3} \frac{P_2 - P_1}{P_1 + P_2}$$

IMPEDANCE ANGLE CAN ALSO BE FOUND

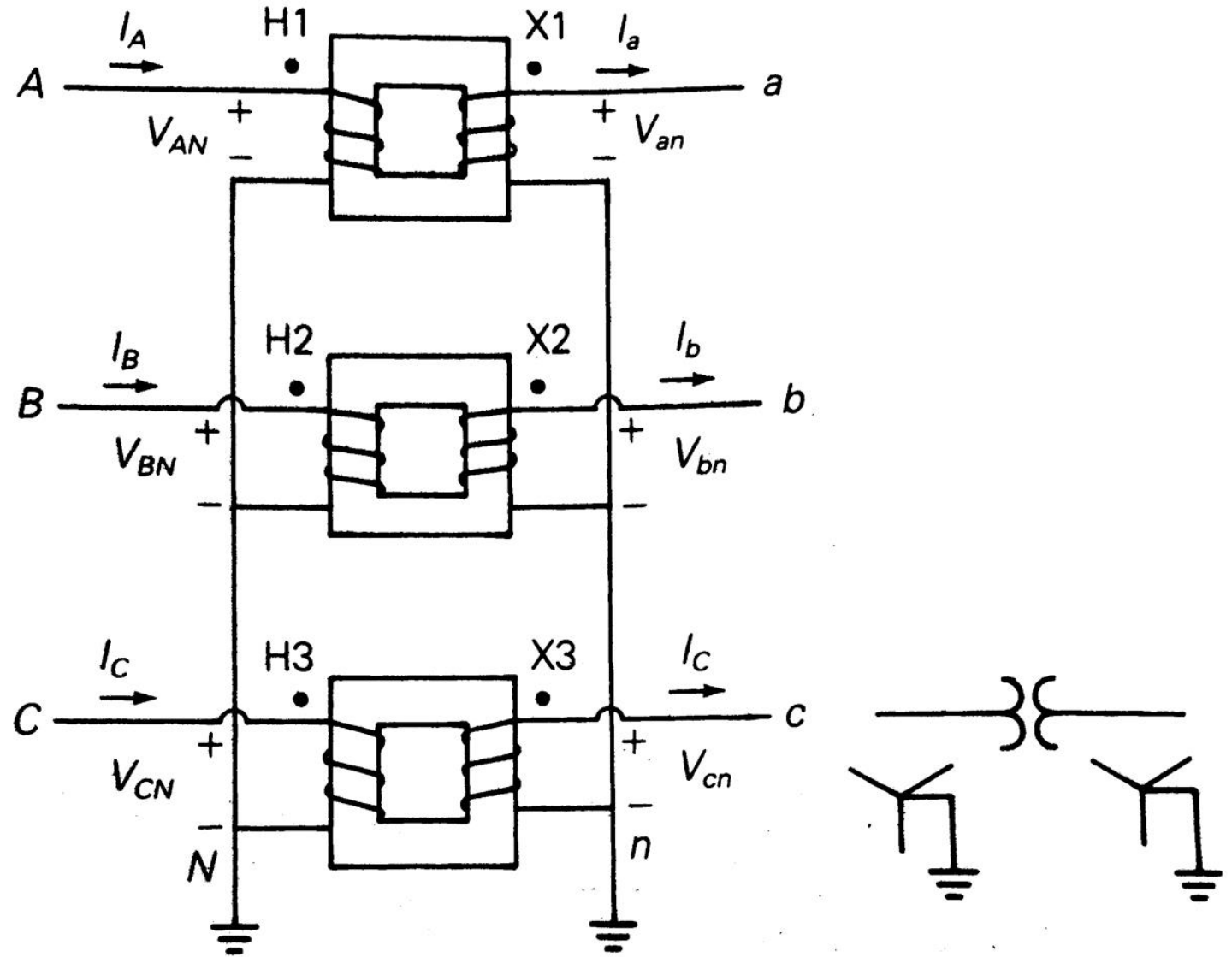
$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Three-phase transformer arrangement

Y-Y connection

Primary and secondary voltages are in phase

Y-connection provides a neutral line which is good for transmission as it allows unbalanced current to flow out and simplify insulation.



Three-phase transformer arrangement

Y-Δ connection

- Y side to N is leading Δ side to n by 30° in this case.
- Δ-connection suppresses third harmonic currents that are generated due to the nonlinear core characteristics (hysteresis) by trapping the harmonic currents in the Δ loop.
- Single transformer: $\frac{V_{AN}}{V_{ab}} = \frac{N_{H1}}{N_{X1}}$

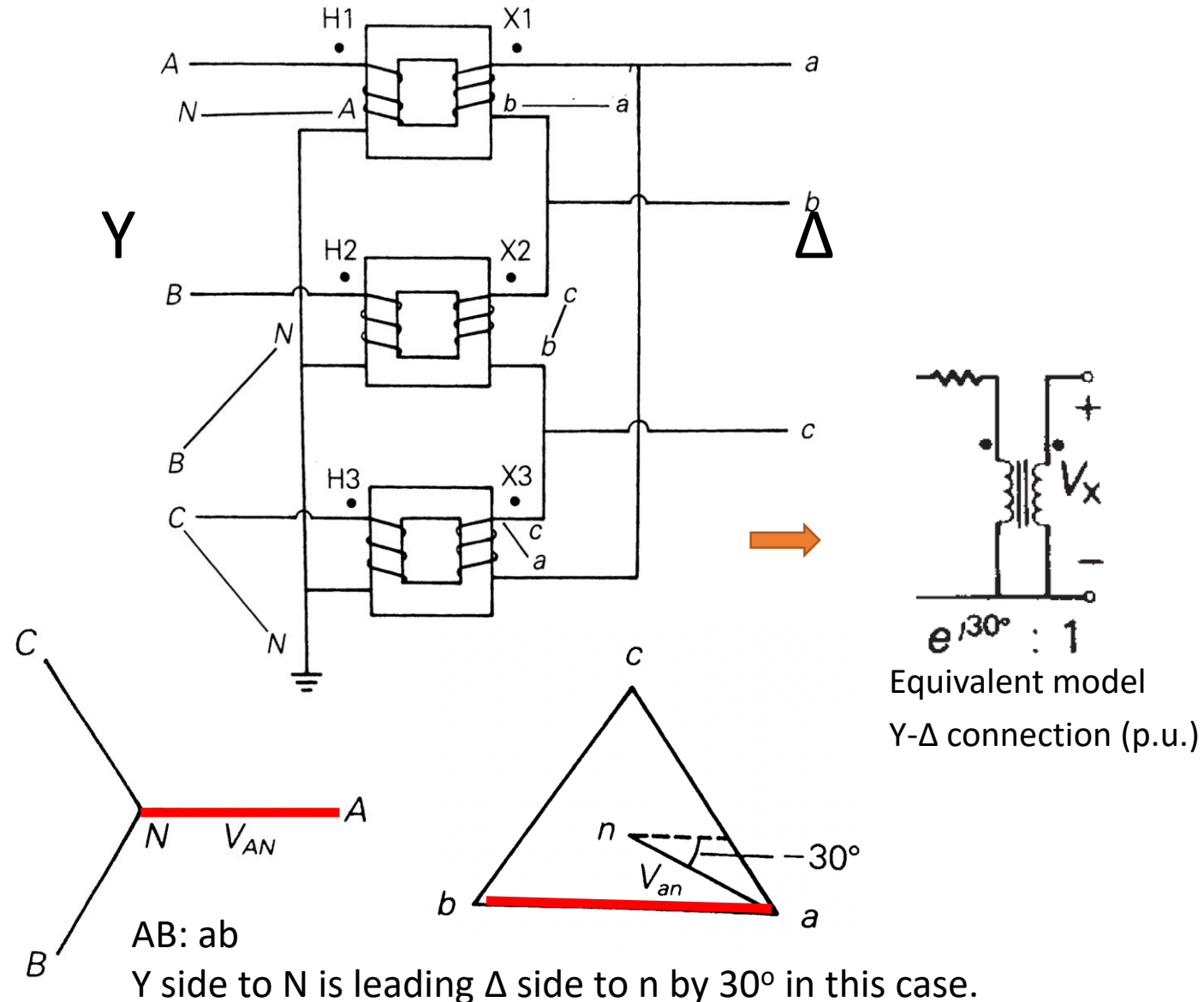
In single phase equivalent circuit, the Δ side quantities

$$V_{an} = \frac{V_{ab}}{\sqrt{3}} \angle -30^\circ$$

or

$$I_{an} = \sqrt{3} I_{ab} \angle -30^\circ$$

Thus, we have: $V_{AN} = \sqrt{3} \frac{N_{H1}}{N_{X1}} V_{an} \angle 30^\circ$



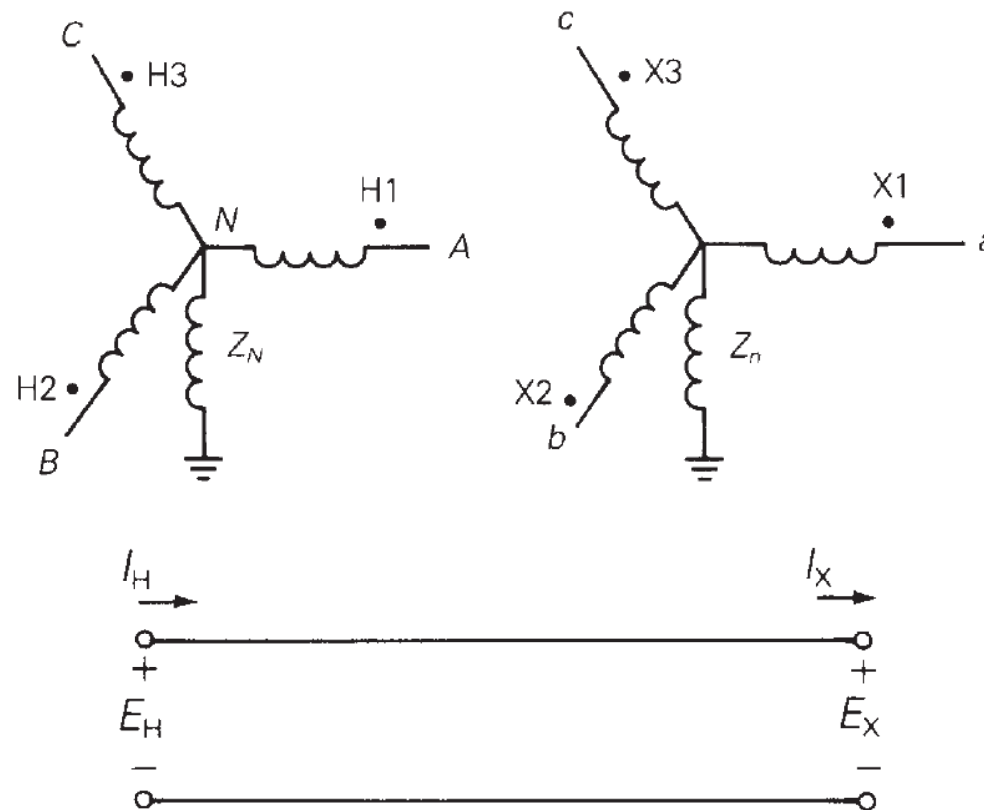
Per unit for three-phase transformer

Y-Y connection

The following are two conventional rules for selecting base quantities:

- A common S_{base} is selected for both the H and X terminals.
- The ratio of the voltage bases $V_{\text{baseH}}/V_{\text{baseX}}$ is selected to be equal to the ratio of the rated line-to-line voltages $V_{\text{ratedHLL}}/V_{\text{ratedXLL}}$.

For the American standard, the positive-sequence (A-B-C) voltages and currents on the high-voltage side of the Y- Δ transformer lead the corresponding quantities on the low-voltage side by 30° .



Example: Voltage calculations

- ❖ Three single-phase two-winding transformers, each rated 400 MVA, 13.8/199.2 kV, with leakage reactance $X_{eq}=0.10$ per unit, are connected to form a three-phase bank. Winding resistances and exciting current are neglected. The high-voltage windings are connected in Y. A three-phase load operating under balanced positive- sequence conditions on the high-voltage side absorbs 1000 MVA at 0.90 p.f. lagging, with $V_{AN}=199.2\angle 0^\circ$ kV. Determine the voltage V_{an} at the low-voltage bus if the low-voltage windings are connected (a) in Y and (b) in Δ .

SOLUTION for (a)

Using the transformer bank ratings as base quantities,

$$S_{base3\phi}=1200 \text{ MVA}, V_{baseHLL}=345 \text{ kV}, \text{ and } I_{baseH}=1200/(345\sqrt{3})=2.008 \text{ kA}.$$

The per-unit load voltage and load current are then

$$V_{AN} = 1.0\angle 0^\circ \text{ per unit}$$

$$I_A = \frac{1000/(345\sqrt{3})}{2.008} \angle -\cos^{-1}0.9 = 0.8333\angle -25.84^\circ \text{ per unit}$$

$$I_a = I_A = 0.8333\angle -25.84^\circ \text{ per unit}$$

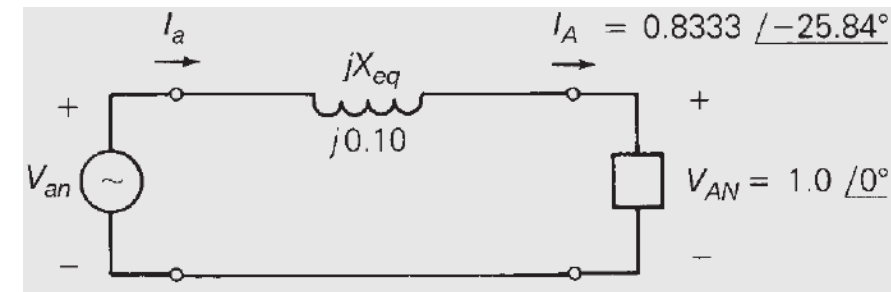
$$V_{an} = V_{AN} + (jX_{eq})I_A$$

$$= 1.0\angle 0^\circ + (j0.10)(0.8333\angle -25.84^\circ)$$

$$= 1.0 + 0.08333\angle 64.16^\circ = 1.0363 + j0.0750 = 1.039\angle 4.139^\circ$$

$$= 1.039\angle 4.139^\circ \text{ per unit}$$

Further, since $V_{baseXLN}=13.8$ kV for the low-voltage Y windings, $V_{an} = 1.039(13.8) = 14.34$ kV, and



Y connected winding

➔ $V_{an} = 14.34\angle 4.139^\circ \text{ kV}$

Example: Voltage calculations

- ❖ Three single-phase two-winding transformers, each rated 400 MVA, 13.8/199.2 kV, with leakage reactance $X_{eq}=0.10$ per unit, are connected to form a three-phase bank. Winding resistances and exciting current are neglected. The high-voltage windings are connected in Y. A three-phase load operating under balanced positive- sequence conditions on the high-voltage side absorbs 1000 MVA at 0.90 p.f. lagging, with $V_{AN}=199.2/\underline{0^\circ}$ kV. Determine the voltage V_{an} at the low-voltage bus if the low-voltage windings are connected (a) in Y and (b) in Δ .

SOLUTION for (b)

* Y side to N is leading Δ side to n by 30° in this case.

$$E_{an} = 1.0 \underline{-30^\circ} \text{ per unit}$$

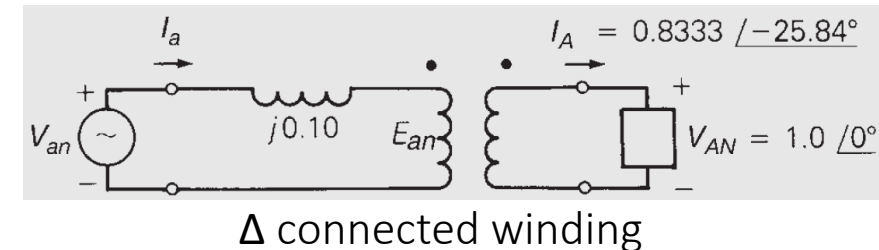
$$I_a = 0.8333 \underline{-25.84^\circ} - 30^\circ = 0.8333 \underline{-55.84^\circ} \text{ per unit}$$

$$V_{an} = E_{an} + (jX_{eq})I_a = 1.0 \underline{-30^\circ} + (j0.10)(0.8333 \underline{-55.84^\circ})$$

$$V_{an} = 1.039 \underline{-25.861^\circ} \text{ per unit}$$

Further, since $V_{baseXLN} = 13.8/\sqrt{3} = 7.967$ kV for the low-voltage Δ windings,
 $V_{an} = (1.039)(7.967) = 8.278$ kV, and

$$V_{an} = 8.278 \underline{-25.861^\circ} \text{ kV}$$



Summary

- Three-phase voltage source and load
- Y and Δ connections
- Phase and line values
- Power in three-phase loads
- Wattmeter and measurements
- 3ϕ transformer connections