

EE3101 Communication Engineering

Assignment

Please hand in your assignment to **Dr W K Wong @ G6508** on or before 5:30pm, 29 April (Wed).

Total Marks: 100

Question 1 [17 marks]

Show that the power spectral density (psd) of the line coding in Fig.Q1 can be expressed as:

$$S(\omega) = a \frac{\sin^b(\omega T / 8)}{(\omega T / 8)^2} \sin^c(\omega T / 4)$$

where a , b , and c are constants. Determine their values.

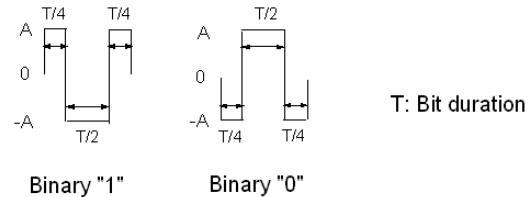


Fig.Q1 Line Coding

Question 2 [24 marks]

(a) A Frequency Shift Keying (FSK) signal is expressed as $s_i(t) = A \cos(\omega_i t + \phi_i)$. Assume that orthogonal transmission is used and M independent frequency sources are used for a M -FSK system.

- Find the minimum frequency separation (in Hz) in terms of the transmission **bit rate** R and M ; and hence
- Show that

$$\frac{\text{transmission bandwidth (between two } \sim 3\text{dB points) of a } M\text{-FSK system}}{\text{minimum transmission bandwidth of a } M\text{-QAM system without ISI}} \sim M \quad \text{if } M \gg 2 \quad [12 \text{ marks}]$$

(b) Suggest a suitable modulation scheme; determine the number of bits per symbol and the symbol duration for a 1.5Mb/s VCD quality video signal in a wireless link. The maximum permissible passband bandwidth is 400kHz and an appropriate raised cosine filter with 35% excess bandwidth (i.e. $\alpha = 0.35$) is used at the transmitter. Assume that the modulation scheme should be as simple as possible and zero ISI is required.

[4 marks]

(c) A PLL with Lag-Lead loop filter is used to track the phase of a sinusoid. The PLL locks to the incoming sinusoid with zero phase error at time $t < 0$. Assume that the gain of the VCO is K_0 ($K_0 \neq 0$) and the transfer function of the Lag-Lead loop filter is:

$$F(\omega) = \frac{j\omega\tau_2 + 1}{j\omega\tau_1 + 1} \quad \text{where } \tau_1 \text{ and } \tau_2 \text{ are constants, and } \tau_1 > \tau_2.$$

- Determine the phase error at equilibrium state if the incoming sinusoid has a phase step of $\Delta\theta$ at $t = 0$.

$$\{ \text{Note: A phase step of } \Delta\theta \text{ occurs at } t = 0 \implies \Theta(\omega) = \frac{\Delta\theta}{j\omega} \}$$

- Can this PLL be applied to the carrier frequency recovery for BPSK signal directly, as shown in Fig.Q2? Why?

[8 marks]

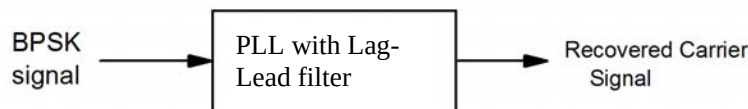


Fig.Q2 Carrier Recovery for BPSK signal

Question 3 [24 marks]

In a binary transmission system, the line coding in Fig.Q3 (a) is used to represent binary “1” and “0”. A receiver in Fig.Q3(b) is used for signal detection.

Fig.3(a) Waveforms for Binary “1” and “0”
(T_b is the bit duration)

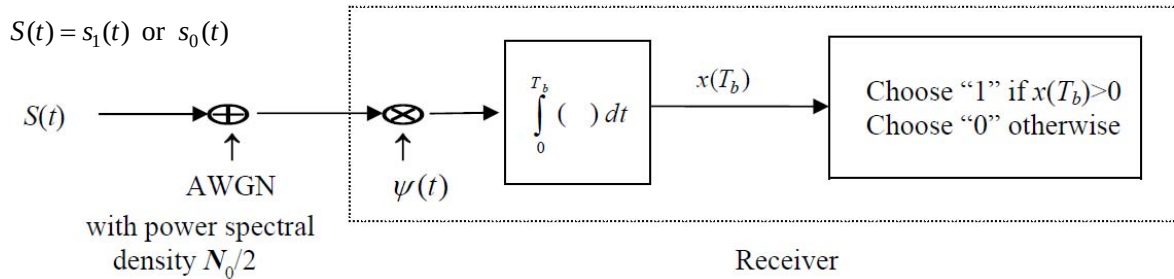
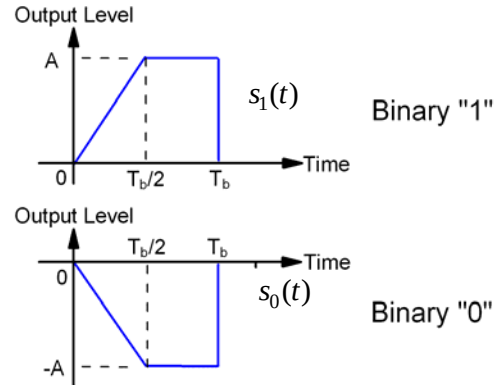


Fig.3(b) The receiver of a Binary Transmission System

(a) If the reference signal $\psi(t) = B$, where B is a constant,

- (i) show that the average bit energy E_b at the receiver input is $E_b = \frac{2A^2T_b}{3}$
- (ii) show that the average bit error probability of the system is $P_e = Q\left[\sqrt{\frac{27E_b}{16N_0}}\right]$
- (iii) estimate P_e when $\frac{E_b}{N_0} = 9\text{dB}$.

[14 marks]

(b) Determine the optimum reference signal $\psi(t)$ such that the maximum signal-to-noise (power) ratio can be achieved at $x(T_b)$. Hence show that the optimum average bit error probability is:

$$P_{e,\text{optimum}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad [10 \text{ marks}]$$

Note: $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{u^2}{2}\right) du$

$$Q(x) \approx \frac{\exp(-x^2/2)}{x\sqrt{2\pi}} \quad \text{for } x > 3.2$$

Question 4 [24 marks]

Consider a communication system transmits N BPSK signals with the same bit rate $1/T$ but different carrier frequencies simultaneously, i.e. $S(t) = \sum_{i=1}^N s_i(t)$, where $s_i(t) = E_i(t) \sqrt{\frac{2}{T}} \cos(i\omega_0 t)$, $E_i(t) \in \{-E, +E\}$, $\omega_0 = \frac{2\pi}{T}$.

Assume that the system operates on an Additive White Gaussian Noise (AWGN) channel with double-sided noise power spectral density of $N_0/2$ and individual ideal BPSK coherent detectors are used for data detection. Fig.Q4 shows the coherent detector for the i^{th} BPSK signal.

- (a) Show that there is an overlap of the power spectral density of neighboring BPSK signals if a raised cosine filter with excess bandwidth greater than zero (i.e. $\alpha > 0$) is used for each BPSK signal. Express the overlapped region between the i^{th} BPSK ($i < N$) signal and the $(i+1)^{\text{th}}$ BPSK signal in terms of i, ω_0, T and α . [8 marks]

- (b) Assume that there exists a small amount of timing offset Δ at the reference signal for the i^{th} BPSK demodulator, i.e. $\psi_i(t) = \sqrt{\frac{2}{T}} \cos[i\omega_0(t - \Delta)]$.

- (i) Show that signal $x_i(T)$ at the integrator output can be expressed as:

$$E_i \cos(i\omega_0\Delta) + \sum_{\substack{j=1 \\ i \neq j}}^N E_j \left\{ \frac{\sin[(i-j)\omega_0 T - i\omega_0\Delta] + \sin(i\omega_0\Delta)}{T(i-j)\omega_0} \right\} + \sum_{j=1}^N E_j \left\{ \frac{\sin[(i+j)\omega_0 T - i\omega_0\Delta] + \sin(i\omega_0\Delta)}{T(i+j)\omega_0} \right\} + \text{noise}$$

$$\text{where } \text{noise} = \sqrt{\frac{2}{T}} \int_0^T n(t) \cos[i\omega_0(t - \Delta)] dt$$

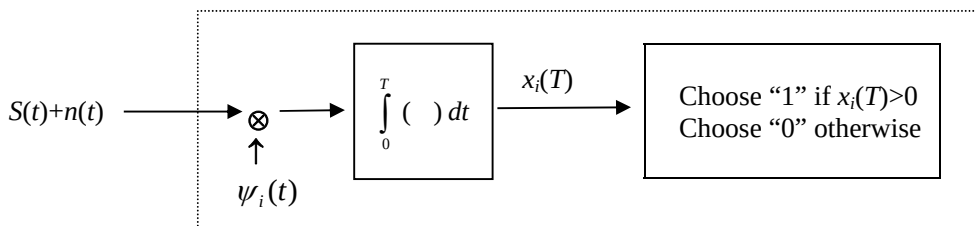
{Note: You need to consider ALL frequency components, i.e. both low and high frequency components}

- (ii) Show that the probability of error P_e at the i^{th} BPSK demodulator is:

$$P_e = Q\left[\sqrt{\frac{2}{N_0}} E \cos(i\omega_0\Delta)\right]$$

- (iii) If the P_e without timing offset (i.e. $\Delta=0$) is 10^{-8} , estimate the P_e when $i=6$ and $\omega_0\Delta=0.12 \text{ rad}$.

[16 marks]



$n(t)$: AWGN with double-sided power spectral density of $N_0/2$

Coherent BPSK Demodulation

$$\psi_i(t) = \sqrt{\frac{2}{T}} \cos(i\omega_0 t)$$

Fig.Q4 Coherent Detector for the i^{th} BPSK Signal

Note: $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{u^2}{2}) du$

$$Q(x) \approx \frac{\exp(-x^2/2)}{x\sqrt{2\pi}} \quad \text{for } x > 3.2, \quad \text{and } Q(5.61) \approx 1 \times 10^{-8}$$

Question 5 [11 marks]

Consider a system operates with a (8,4) systematic block code.

- (a) What is the code rate of the system? [2 marks]
- (b) What is the Hamming distance between 00011101 and 11100010? [2 marks]
- (c) Assume that the minimum distance $d_{\min}$ of the above code is 4, find the error-correction capability of the code. [2 marks]
- (d) Assume that an ideal BPSK system is used to transmit the data and it cannot correct any error patterns with more than that as specified in (c). The probability of the channel bit error is 10^{-4} . Estimate the probability of un-correctable message error. [5 marks]