

MA1502/GE1359 Final Answers to Past Test Papers

Semester A, 2021-22

1. $Dom(F) = \mathbb{R} \setminus \{-1, 3\}$, $Ran(F) = \left(-\infty, -\frac{1}{2}\right] \cup (0, \infty)$
2. (a) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{7-x}}{\left(\frac{x}{x-2}\right)}$, $Dom\left(\frac{f}{g}\right) = (-\infty, 7] \setminus \{0, 2\}$
(b) $(f \circ f)(x) = \sqrt{7 - \sqrt{7-x}}$, $Dom(f \circ f) = [-42, 7]$
(c) $(g \circ f)(x) = \frac{\sqrt{7-x}}{\sqrt{7-x-2}}$, $Dom(g \circ f) = (-\infty, 7] \setminus \{3\}$
3. (a) Let $G(x_1) = G(x_2)$. Then $\sqrt{(x_1 - 3)^2 + 2} = \sqrt{(x_2 - 3)^2 + 2}$
 $\Rightarrow \dots \Rightarrow x_1 = x_2$ is the only solution.
 $\therefore G(x)$ is one-to-one.
 $\therefore G^{-1}(x)$ exists.
(b) $G^{-1}(x) = 3 - \sqrt{x^2 - 2}$
(c) $Dom(G^{-1}) = [\sqrt{6}, \infty)$, $Ran(G^{-1}) = (-\infty, 1]$
4. $\sin(2A + B) = \frac{56}{65}$, $\cos(2A + B) = -\frac{33}{65}$, $2A + B$ lies in Quadrant II.
5. (a) $x = n\pi + \frac{\pi}{6}$, $n \in \mathbb{Z}$.
(b) $x = \frac{n\pi}{3} + (-1)^n \cdot \frac{\pi}{18}$, $n \in \mathbb{Z}$.
6. $\sin \frac{\pi}{24} = \sqrt{\frac{1}{2} \left(1 - \sqrt{\frac{1}{2} \left(1 + \frac{\sqrt{3}}{2}\right)}\right)}$
7. $\frac{x^5 - x^4 + 2x^3 + 15x^2 - 16x + 36}{x^4 - 16} = x - 1 + \frac{3}{x-2} - \frac{2}{x+2} + \frac{x+5}{x^2+4}$

Summer 2021

1. $Dom(f) = [-2, 2]$, $Ran(f) = [1, 3]$
2. (a) $g(x)$ is one-to-one.
(b) $g^{-1}(x) = 1 - \frac{16}{x^2}$
(c) $Dom(g^{-1}) = (0, 2]$, $Ran(g^{-1}) = (-\infty, -3]$
3. (a) $\sin(2\alpha) = -\frac{24}{25}$, $\cos(2\alpha) = \frac{7}{25}$, 2α lies in Quadrant IV
(b) 2

4. (b) $x = n\pi$ or $x = \frac{n\pi}{3} \pm \frac{\pi}{18}$, for $n \in \mathbb{Z}$
5. $\frac{x^3-6x^2+6x+2}{x^4-x^3-x+1} = -\frac{2}{x-1} + \frac{1}{(x-1)^2} + \frac{3x-1}{x^2+x+1}$
6. $(-\infty, -1) \cup (7, \infty)$
7. $p \neq -6$
 $z = \frac{1-2p}{p+6}$, $w = s$, $y = 4 - \frac{p}{2} - 3\left(\frac{1-2p}{p+6}\right) - \frac{3}{2}s$, $x = -8 + \frac{3}{2}p + 7\left(\frac{1-2p}{p+6}\right) + \frac{11}{2}s$,
 where $s \in \mathbb{R}$

Semester A, 2020-21

1. $Dom(f) = \mathbb{R} \setminus \{-3, 3\}$, $Ran(f) = (-\infty, 0) \cup \left[\frac{1}{9}, \infty\right)$
2. (b) $g^{-1}(x) = 2 - \sqrt{3-x}$, $Dom(g^{-1}) = (-\infty, 2]$
3. (b) $\tan\left(\frac{A}{2}\right) = -2$
4. (a) $13 \sin\left(3x + \tan^{-1}\left(\frac{12}{5}\right)\right)$
 (b) $x = \frac{n\pi + (-1)^n \cdot \frac{\pi}{6} - \tan^{-1}\left(\frac{12}{5}\right)}{3}$, for $n \in \mathbb{Z}$
5. $\frac{-5x^2+6x+23}{(x-1)^2(x^2+5)} = -\frac{2}{x-1} + \frac{4}{(x-1)^2} + \frac{2x-7}{x^2+5}$
6. $p = -4$
 $z = s$, $w = t$, $y = 1 + s + 2t$, $x = 1 - s - t$, where $s, t \in \mathbb{R}$
7. (a) $-q^3 + 3rq$
 (b) $x^2 + \left(\frac{q^3-3rq}{r^2} - 4\right)x + \frac{1}{r} + 2\left(\frac{-q^3+3rq}{r^2}\right) + 4 = 0$

Semester A, 2019-20

1. (a) $Dom(f) = \mathbb{R} \setminus \{2\}$, $Ran(f) = (-\infty, 7)$
2. (b) $g^{-1}(x) = 3 + (x-1)^2$
 (c) $Dom(g^{-1}) = [2, \infty)$, $Ran(g^{-1}) = [4, \infty)$
3. (a) $-\frac{63}{16}$ (b) $-\frac{1}{\sqrt{26}}$
4. (b) $x = \frac{2n\pi}{3} \pm \frac{\pi}{6}$ or $x = \frac{n\pi}{3} + (-1)^n\left(-\frac{\pi}{18}\right)$, for $n \in \mathbb{Z}$
5. $\left(-1, \frac{1}{5}\right)$

$$6. \quad (b) \quad \frac{10x^2+9x+19}{2x^3+5x^2+12x+5} = \frac{4}{2x+1} + \frac{3x-1}{x^2+2x+5}$$

$$7. \quad (a) \quad \left(\begin{array}{cccc|c} 1 & 2 & 2 & -1 & 3 \\ 0 & 1 & -1 & 3 & -2 \\ 0 & 0 & 0 & p-3 & q-5 \end{array} \right)$$

$$(b) \quad (i) \, p = 3, \, q \neq 5 \qquad (ii) \, p \neq 3, \, q \in \mathbb{R} \qquad (iii) \, p = 3, \, q = 5$$

$$(c) \quad \text{Case (ii): } z = s, \, w = \frac{q-5}{p-3}, \, y = -2 + s - 3\left(\frac{q-5}{p-3}\right), \, x = 7 - 4s + 7\left(\frac{q-5}{p-3}\right),$$

where $s \in \mathbb{R}$

Case (iii): $z = s, \, w = t, \, y = -2 + s - 3t, \, x = 7 - 4s + 7t$, where $s, t \in \mathbb{R}$