

MA1200 CALCULUS AND BASIC LINEAR ALGEBRA I

LECTURE: CG1

Chapter 2 Sets and Functions

Set Notation

A **set** is a collection of distinct objects. Each object in a set is called an **element** or a **member** of that set. A set may contain a finite number of elements, infinitely many elements, or even no elements.

For example,

- $V = \{a, e, i, o, u\}$ is the set of all vowels of the English alphabets.
- $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is the set of all integers from 1 to 10.
- $B = \{2, 4, 6, 8, 10, \dots\}$ is the set of all positive even numbers.

They are all sets, and their elements are listed inside the brackets “{ }”.

A set is a collection, but not a list. The order in which the elements are written is not important.

For example, $S = \{a, b, c\} = \{b, a, c\} = \{c, b, a\}$.

In general, we use the notation

$$\{x|x \text{ processes certain properties}\}$$

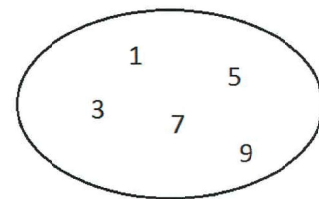
to denote a set of objects that share some common properties. The vertical line “|” means “such that”.

Example 1

- $C = \{x|x \text{ is an odd number and } 0 < x < 10\} = \{1, 3, 5, 7, 9\}$.
- $D = \{x|x \text{ is negative and } x \text{ is a multiple of } 5\} = \{-5, -10, -15, -20, -25, \dots\}$.

A set can be represented using **Venn diagram**.

For example, a Venn diagram for the set $C = \{1, 3, 5, 7, 9\}$ is shown on the right.



Some notations:

- “ $\in$ ” means “belongs to” or “is an element of”.

If “ a belongs to S ” or “ a is an element of S ”, we write $a \in S$.

element $\downarrow$ set



- “ $\notin$ ” means “does not belong to” or “is not an element of”.

If “ b does not belong to S ” or “ b is not an element of S ”, we write $b \notin S$.

- “ $\subset$ ” means “is a **subset** of”.

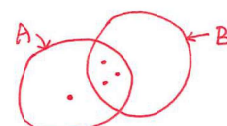
If every element in set A also belongs to set B , we say that “ A is a **subset** of B ” and we write $A \subset B$.

set $\uparrow$ set



- “ $\not\subset$ ” means “is not a subset of”.

If there is at least one element which belongs to set A but does not belong to set B , we say that “ A is not a subset of B ” and we write $A \not\subset B$.

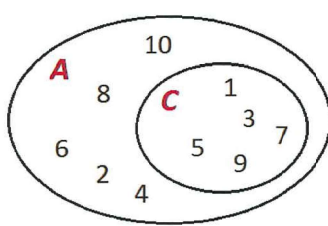
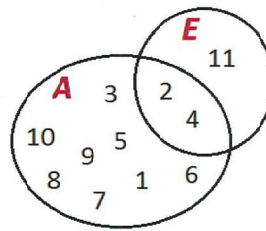


Remarks:

1. Some authors use " $\subseteq$ " to denote "is a subset of", and " $\not\subseteq$ " to denote "is not a subset of".
2. By the definition of subsets, any set is a subset of itself.

Example 2

Given the sets $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $C = \{1, 3, 5, 7, 9\}$ and $E = \{2, 4, 11\}$. Then C is a subset of A , denoted by $C \subset A$, since every element in C also belongs to A . The set E is not a subset of A , because $11 \in E$ but $11 \notin A$. We write $E \not\subset A$.

 $C \subset A$  $E \not\subset A$ **Equality of sets**

Two sets A and B are equal (written as $A = B$) if they contain the same elements. For example, if $A = \{1, 2, 3\}$, $B = \{3, 2, 1\}$ and $C = \{1, 2\}$, then we have $A = B$ but $A \neq C$.

Some commonly used sets in Mathematics include:

- $\emptyset = \{\}$ is called an "empty set", which contains no elements.
- $\mathbb{N} = \{1, 2, 3, 4, \dots\}$ is the set of all natural numbers (positive integers).
- $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots\}$ is the set of all integers.
- $\mathbb{Q} = \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\}$ is the set of all rational numbers.
- $\mathbb{R}$ is the set of all real numbers.
- $\mathbb{C}$ is the set of all complex numbers (*will be discussed in MA1201*)

Using notations of subsets, we have

$$\emptyset \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}.$$

Caution!! Be careful when using the notations “ $\in$ ” and “ $\subset$ ”. For example, $1 \in \mathbb{Z}$ (which means “1 is an element of $\mathbb{Z}$ ”) and $\{1\} \subset \mathbb{Z}$ (which means “the set containing the number 1 is a subset of $\mathbb{Z}$ ”), but **never write** $1 \subset \mathbb{Z}$. (This doesn’t make sense!!)

Example 3

Use set notations to represent each of the following sets.

- (a) The set of integers which are smaller than -6 and greater than -13 .
 (b) The set of integers which are greater than 2 but less than or equal to 15 .

Solution

- (a) $\{-12, -11, -10, -9, -8, -7\}$ or $\{x | x \in \mathbb{Z} \text{ and } -13 < x < -6\}$
 or $\{x \in \mathbb{Z} | -13 < x < -6\}$
 (b) $\{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ or $\{x | x \in \mathbb{Z} \text{ and } 2 < x \leq 15\}$
 or $\{x \in \mathbb{Z} | 2 < x \leq 15\}$

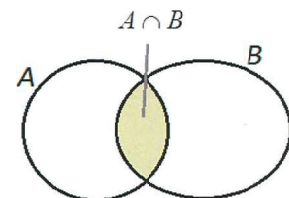
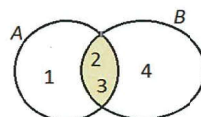
Operations of sets

Given two sets A and B . We can combine the two sets to form new sets by using set operations:

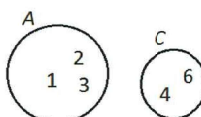
- Intersection**

The **intersection of sets A and B** , written as $A \cap B$, is a set whose elements belong to **both A and B** . That is, $A \cap B = \{x | x \in A \text{ and } x \in B\}$.

E.g. If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then $A \cap B = \{2, 3\}$.

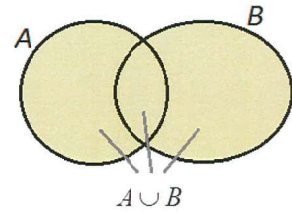


E.g. If $A = \{1, 2, 3\}$ and $C = \{4, 6\}$, then $A \cap C = \emptyset$. That is, A and C are disjoint sets.

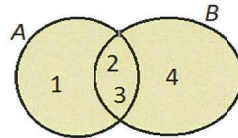


- Union**

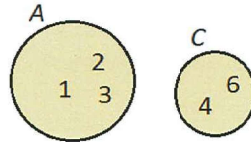
The **union of sets A and B** , written as $A \cup B$, is a set whose elements belong to either A or B or both of them. That is, $A \cup B = \{x | x \in A \text{ or } x \in B\}$.



E.g. If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then $A \cup B = \{1, 2, 3, 4\}$.

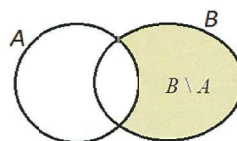


E.g. If $A = \{1, 2, 3\}$ and $C = \{4, 6\}$, then $A \cup C = \{1, 2, 3, 4, 6\}$.

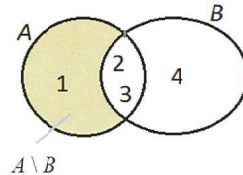
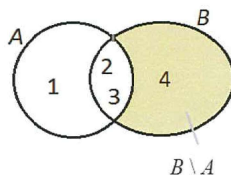


- Complement**

The **complement of A with respect to B** , written as $B \setminus A$, is a set whose elements belong to B but not belong to A . That is, $B \setminus A = \{x | x \in B \text{ but } x \notin A\}$. The line “ $\setminus$ ” means “exclude”.



E.g. If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then $B \setminus A = \{4\}$ and $A \setminus B = \{1\}$.



E.g. If $A = \{1, 2, 3\}$ and $C = \{4, 6\}$, then $C \setminus A = \{4, 6\} = C$ and $A \setminus C = \{1, 2, 3\} = A$.

E.g. $\mathbb{R} \setminus \{1, 3\}$ is the set of all real numbers except 1 and 3.

E.g. $\mathbb{R} \setminus \mathbb{Q}$ is the set of all irrational numbers. For example, $\pi = 3.14159 \dots$, $e = 2.71828 \dots$ and $\sqrt{2} = 1.4142 \dots$ are irrational numbers.

Example 4

Let $A = \{2, 4, 6, 8\}$ and $B = \{-3, 6, 8, 12.4\}$.

Write the set described by each of the following. List all the elements in the set.

- (a) $A \cup B$ (b) $A \cap B$ (c) $B \cap \mathbb{Z}$ (d) $B \cap \mathbb{R}$

Solution

(a) $A \cup B = \{-3, 2, 4, 6, 8, 12.4\}$

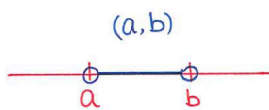
(b) $A \cap B = \{6, 8\}$

(c) $B \cap \mathbb{Z} = \{-3, 6, 8\}$

(d) $B \cap \mathbb{R} = \{-3, 6, 8, 12.4\} = B$

Intervals

Recall that $\mathbb{R}$ is the set of all real numbers. Let a and b be two distinct real numbers where $a < b$. We use the following notations to describe some **subsets of real numbers** (known as **intervals**):



$$(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$$

$$[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$$

$$(a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$$

$$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$$

$$[a, \infty) = \{x \in \mathbb{R} \mid x \geq a\}$$

$$(a, \infty) = \{x \in \mathbb{R} \mid x > a\}$$

$$(-\infty, a] = \{x \in \mathbb{R} \mid x \leq a\}$$

$$(-\infty, a) = \{x \in \mathbb{R} \mid x < a\}$$

$$\mathbb{R} = (-\infty, \infty)$$

Note: Never write $[a, \infty]$, $(a, \infty]$, $[-\infty, a]$ and $[-\infty, a)$.

Note the difference between $\{\}$, $()$, $[\]$.

E.g. $\{1, 2\}$ is a set containing 1 & 2.



E.g. $[1, 2)$ is a set of real numbers greater than or equal to 1 but less than 2.



Example 5

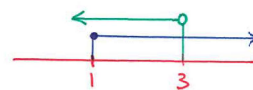
Express each of the following sets as interval notations:

(a) "The set of all real numbers which are smaller than or equal to 6" = $(-\infty, 6]$

(b) $\{x \in \mathbb{R} \mid x > 2\} = (2, \infty)$

(c) $\{x \in \mathbb{R} \mid x < 3 \text{ and } x \geq 1\} = [1, 3)$

(d) $\{x \in \mathbb{R} \mid x < 3 \text{ or } x \geq 1\} = \mathbb{R}$



Example 6

Simplify each of the following:

(a) $(1, 4) \cap [2, 6] = [2, 4)$

(b) $[-2, 0) \cap [0, 3] = \emptyset$

(c) $[-2, 0) \cup [0, 3] = [-2, 3]$

(d) $[-2, 3] \cup (3, \infty) = [-2, \infty)$

(e) $(-\infty, 6] \cap (3, \infty) = (3, 6]$