

EE3101 Tutorial 1 Solutions

(1) Consider the followings:

Bandwidth efficiency, able to provide multiple services within 1 channel, cost, performance, transmission power, ...etc.

$$(2) R_o = \frac{18A^2}{13}$$

$$R_1 = \frac{A^2}{13} [(1)(-1) + (-1)(-2) + (-2)0 + 0(2) + 2(-1) + (-1)0 + (0)1 + (1)0 + 0(-2) + (-2)(1) + 1(0) + 0(1)] = -\frac{3A^2}{13}$$

$$R_2 = \frac{A^2}{13} [1(-2) + (-1)0 + (-2)2 + 0(-1) + (2)0 + (-1)1 + (0)0 + (1)(-2) + 0(1) + (-2)0 + 1(1)] = -\frac{8A^2}{13}$$

$$(3) R_{4n} = \frac{1}{4} [(2A)^2 + (A)^2 + (-A)^2 + (-2A)^2] = \frac{5A^2}{2} \quad n = 0, 1, 2, \dots$$

$$R_{4n+1} = \frac{1}{4} [(2A)(A) + (A)(-A) + (-A)(-2A) + (-2A)(2A)] = \frac{-A^2}{4} \quad n = 0, 1, 2, \dots$$

$$R_{4n+2} = \frac{1}{4} [(2A)(-A) + (A)(-2A) + (-A)(2A) + (-2A)(A)] = -2A^2 \quad n = 0, 1, 2, \dots$$

$$R_{4n+3} = \frac{1}{4} [(2A)(-2A) + (A)(2A) + (-A)(A) + (-2A)(-A)] = \frac{-A^2}{4} \quad n = 0, 1, 2, \dots$$

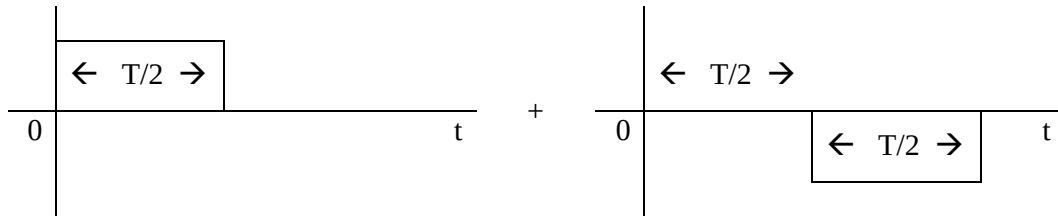
$$R_r = R_{-r}$$

(4) Method 1

Consider the construction of $P(\omega)$ --- “1” and “0” are represented by discrete impulses with amplitude A and $-A$ respectively.

$P(\omega)$ will be the same as that in the section 1.2.2 of the lecture notes, i.e. $P(\omega) = A^2/T$.

Consider the construction of $H(\omega)$ --- for each + impulse input, there will be a positive rectangular pulse (with the same amplitude as impulse and width $T/2$) followed by a negative rectangular pulse (with the same amplitude as impulse and width $T/2$), as shown in the following (similarly for the negative impulse input):



$H(\omega)$ is thus equal to $\exp(-j\omega T/4) [T/2 \text{Sa}(\omega T/4)] + \exp(-j\omega 3T/4) [-T/2 \text{Sa}(\omega T/4)]$
 { a normal $T/2 \text{Sa}(\omega T/4)$ function plus a negative function with an additional $T/2$ delay }

$$H(\omega) = T/2 \text{Sa}(\omega T/4) \exp(-j\omega T/4) \{ 1 - \exp(-j\omega T/2) \}$$

$$|H(\omega)|^2 = H(\omega)H^*(\omega) \quad *: \text{complex conjugate}$$

$$= (T/2)^2 \text{Sa}^2(\omega T/4) [\exp(-j\omega T/4)][\exp(+j\omega T/4)][1 - \exp(-j\omega T/2)][1 - \exp(j\omega T/2)]$$

$$|H(\omega)|^2 = (T/2)^2 \text{Sa}^2(\omega T/4) [2 - \exp(-j\omega T/2) - \exp(j\omega T/2)]$$

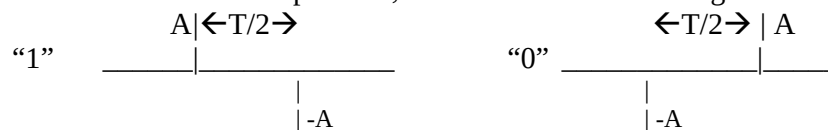
$$|H(\omega)|^2 = (T/2)^2 \text{Sa}^2(\omega T/4) [2 - 2\cos(\omega T/2)]$$

$$|H(\omega)|^2 = T^2 \text{Sa}^2(\omega T/4) \{ [1 - \cos(\omega T/2)]/2 \} = T^2 \text{Sa}^2(\omega T/4) \sin^2(\omega T/4)$$

$$\text{i.e. } S(\omega) = |H(\omega)|^2 P(\omega) = A^2 T \text{Sa}^2(\omega T/4) \sin^2(\omega T/4)$$

Method 2

Consider the construction of $P(\omega)$ --- binary “1” corresponds to two discrete impulses with amplitude A and $-A$ and with $T/2$ separation, as shown in the following:



similarly, each binary “0” corresponds to two discrete impulses with amplitude $-A$ and A and with $T/2$ separation.

$$R_0 = (A)(A)1/4 + (-A)(-A)1/4 + (-A)(-A)1/4 + (A)(A)1/4 = A^2$$

$$R_1 = (A)(-A)1/4 + (-A)(A)1/4 + (-A)(A)1/4 + (A)(-A)1/4 = -A^2/2$$

$$\begin{array}{cccc} \wedge & \wedge & \wedge & \wedge \\ | & | & | & | \\ a & b & c & d \end{array}$$

where a & b are the 1st and 2nd terms of binary “1” resp.

c & d are the 1st and 2nd terms of binary “0” resp.

Hint:

In order to give you a better understanding on R_1 calculation, you can consider the construction of $P(\omega)$ --- binary "1" corresponds to two discrete impulses with amplitude p and $-q$ and with $T/2$ separation, and binary "0" corresponds to two discrete impulses with amplitude $-r$ and s and with $T/2$ separation. as shown in the following:



$$R_1 = (p)(-q)1/4 + (-q)[p/2 + (-r)/2]/4 + (-r)s/4 + (s)[(-r)/2 + p/2]/4 = -A^2/2$$

[Note: $p=q=r=s=A$]

$R_r = 0$ for $|r| \neq 0, 1$ (pls verify it yourself), and with the similar technique as the example in section 1.2.4 of the lecture notes,

$$P(\omega) = 4 A^2/T \sin^2(\omega T/4)$$

Consider the construction of $H(\omega)$ --- it is the same as that in examples 1.2.3. and 1.2.4 ---

$$H(\omega) = T/2 \text{Sa}(\omega T/4)$$

$$\text{i.e. } S(\omega) = |H(\omega)|^2 P(\omega) = A^2 T \text{Sa}^2(\omega T/4) \sin^2(\omega T/4)$$