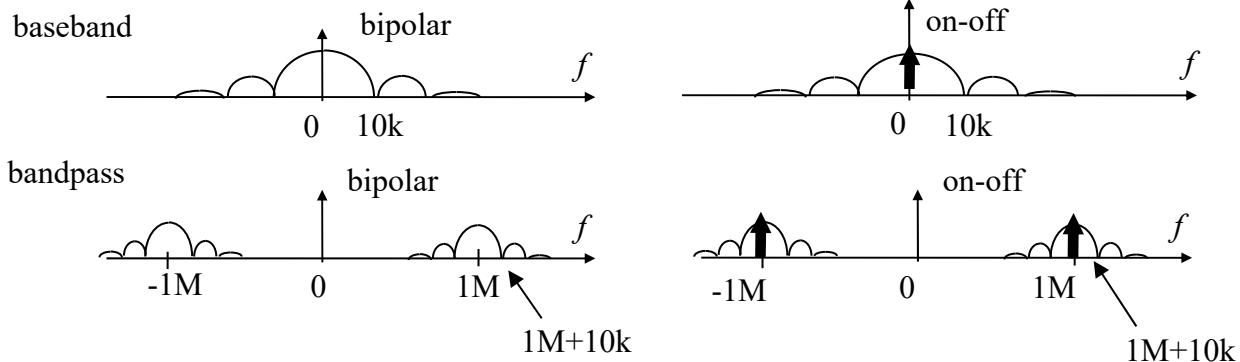


EE 3008 Chapter 5 Tutorial Solutions

Q1) Note that $1/\tau = 10\text{k}$.



Q2) (a) For $b=+1$, the integrator output is

$$B = \int_0^{10^{-4}} A \cos^2(2\pi 10^9 t) dt = 0.5A \int_0^{10^{-4}} (1 + \cos(4\pi 10^9 t)) dt \approx 0.5 \times 10^{-2} \times 10^{-4} = 5 \times 10^{-7}$$

For $b=-1$, the integrator output is $-B$.

(b) Refer to lecture notes. For a bandpass signal, $E_b = \frac{A^2 \tau}{2} = \frac{(10^{-2})^2 10^{-4}}{2} = 0.5 \times 10^{-8}$

$$BER = Q(\sqrt{2E_b / N_0}) = Q(\sqrt{5}) \approx 0.013.$$

(c) If we reduce the transmission power by half, then E_b is halved and so

$$BER = Q(\sqrt{2.5}) \approx 0.057.$$

Q3) (a) The integrator outputs are ($b=-3, -1, +1, +3$)

$$\int_0^{\tau} Ab \cos^2(2\pi f_0 t) dt = 0.5Ab \int_0^{\tau} (1 + \cos(2 \times 2\pi f_0 t)) dt \approx 0.5Ab\tau = bB \quad (\text{with } B \equiv 0.5A\tau)$$

(You may verify that $\int_0^{\tau} \cos(2 \times 2\pi f_0 t) dt \approx 0$ if $f_0 \gg 1$.)

(b) Each symbol energy $= 0.5A^2 b^2 \tau$.

(c) This is the average of the results in (b) over four values of b

$$E_s = 0.5A^2 \tau ((-3)^2 + (-1)^2 + 1^2 + 3^2) / 4 = 2.5A^2 \tau$$

Since each symbol now carries 2 bits, the average bit energy is

$$E_b = E_s / 2 = 1.25A^2 \tau.$$

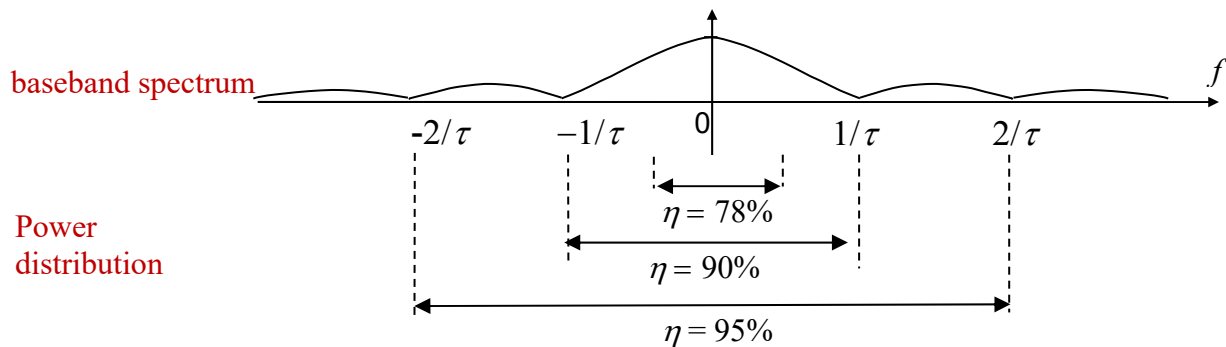
Comparing E_b with B defined in (a), we can write $E_b = 5B^2/\tau$. This relationship will be used later.

Q4) Suppose the rule is same $\Rightarrow 1$ and different $\Rightarrow 0$. The transmitted signal is,

+1, +1 +1 -1 +1, +1 +1 -1 +1, +1 +1 +1 +1, -1 +1 +1 -1, +1

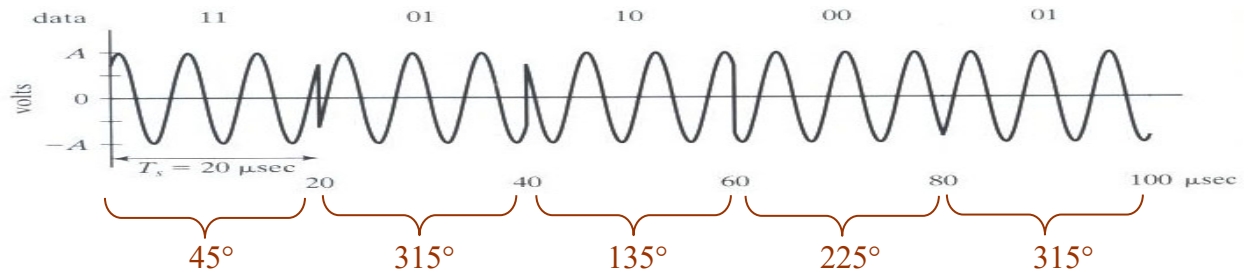
Q5) For a baseband system, a bipolar signal of bandwidth B can support a rate of B with 90% energy preservation (since bandwidth $= 1/\tau$ in this case).

We first consider BPSK. (**Note:** Although the question does not ask BPSK, it is still helpful to start with BPSK as a reference.) In this case, signal of bandwidth of B can support a rate of $B/2$, i.e., 1200 bps. This is because, after modulation, the bandwidth is $2/\tau$. See the figure below.



For QPSK, rate can be doubled using the same bandwidth. Therefore the supported rate is 2400bps.

Q6) (i) As each symbol carries 2 bits, the symbol rate is 50,000 symbols/second. The duration per symbol is $1/50,000 \text{ sec} = 20 \mu\text{sec}$.

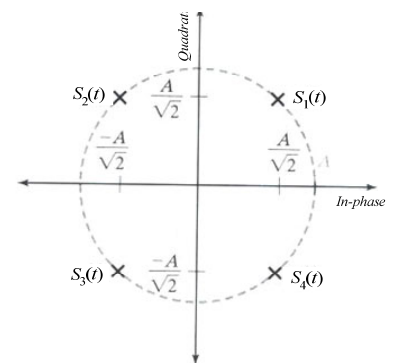


(ii) $s_1(t) = A \sin(2\pi f_c t + 45^\circ) = \frac{A}{\sqrt{2}} \sin(2\pi f_c t) + \frac{A}{\sqrt{2}} \cos(2\pi f_c t)$ $\leftarrow \sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$
 $\sin(45^\circ) = \frac{1}{\sqrt{2}}$

$s_2(t) = A \sin(2\pi f_c t + 135^\circ) = -\frac{A}{\sqrt{2}} \sin(2\pi f_c t) + \frac{A}{\sqrt{2}} \cos(2\pi f_c t)$

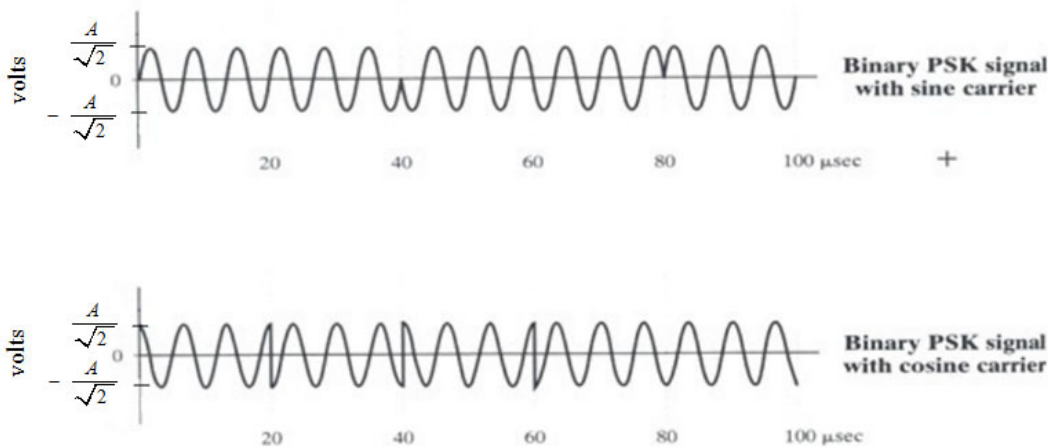
$s_3(t) = A \sin(2\pi f_c t + 225^\circ) = -\frac{A}{\sqrt{2}} \sin(2\pi f_c t) - \frac{A}{\sqrt{2}} \cos(2\pi f_c t)$

$s_4(t) = A \sin(2\pi f_c t + 315^\circ) = \frac{A}{\sqrt{2}} \sin(2\pi f_c t) - \frac{A}{\sqrt{2}} \cos(2\pi f_c t)$



The constellation is given on the right.

(iii)



Q7) The BPSK time duration is denoted by T . Then we have $T = 1/R(\text{sec})$.

Therefore, the energy for the binary signal is

power = $A^2/2$ and energy
= power $\times \tau$

$$E_b = A^2 \int_0^T \cos^2(2\pi f_c t) dt \times 10^{-4}$$

$$= \frac{A^2}{2} \int_0^T [1 + \cos(4\pi f_c t)] dt = \frac{A^2 T}{2} = \frac{A^2}{2R}$$

The SNR can be obtained as $E_b = \frac{A^2}{2} \cdot \tau = \frac{A^2}{2} \cdot \frac{1}{R}$. The BER performance is

$$Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{A^2}{RN_0}}\right) = Q\left(\sqrt{\frac{(10^{-2})^2}{1 \times 10^4 \times 2.5 \times 10^{-9}}}\right) = Q(2)$$

Q8) Keep in mind that the bandwidth after modulation is twice of that before modulation.

- For BPSK, the duration of each pulse is $\tau = 1/100\text{M}$ second. For the baseband, it requires 100MHz of bandwidth. After modulation (for bandpass), bandwidth = $2 \times 100\text{MHz} = 200\text{MHz}$.
- For QPSK, the symbol rate is 100M/2 symbol/s (since each symbol carries 2 bits). Symbol duration $\tau = 2/100\text{M}$ s. After modulation, bandwidth = $2 \times (100\text{M}/2)$ Hz = 100MHz.
- For 8PSK, the symbol rate is 100M/3 symbol/s (since each symbol carries 3 bits). Symbol duration $\tau = 3/100\text{M}$ s. After modulation, bandwidth = $2 \times (100\text{M}/3)$ Hz = 66.67MHz.
- For 16QAM, the symbol rate is 100M/4 symbol/s (since each symbol carries 4 bits). Symbol duration $\tau = 4/100\text{M}$ s. After modulation, bandwidth = $2 \times (100\text{M}/4)$ Hz = 50MHz.

Q9) a) With an integration receiver, the integration output will always be zero when either of the two pulses above is transmitted, so it is not applicable. For such a system, we should use a correlation filter receiver with the first pulse in the figure as the reference. If the impact of noise is ignored, the correlation output y should be 0.2 and -0.2 for the two pulses in the figure, respectively. Let z be our estimation of the transmitted signal. The decision rule is letting $z = 1$ if $y > 0$ and letting $z = 0$ if $y < 0$.

b) The BER performance of this system is the same as BPSK. We have $E_b = 10^2 \times 10 \times 10^{-3} \times 2 = 2J$. The error probability is

$$Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q(2) = 0.0228.$$

c) For the pulse representing “1”, the Fourier transform is given by

$$\begin{aligned} X(f) &= AT \text{sinc}(\pi f T) \left((e^{+j2\pi f \times 0.005} - e^{-j2\pi f \times 0.005}) \right) \\ &= 10 \times 0.01 \times \text{sinc}(\pi f \times 0.01) \left(2 \times 0.5 \times (e^{+j\pi f \times 0.01} - e^{-j\pi f \times 0.01}) \right) \\ &= 0.2 \times \text{sinc}(0.01\pi f) \sin(0.01\pi f) \end{aligned}$$

The energy spectrum is

$$|X(f)|^2 = |0.2 \times \text{sinc}(0.01\pi f) \sin(0.01\pi f)|^2$$

Note that the two pulses are just negative to each other. Therefore the spectrum of the pulse representing -1 is the same as above.

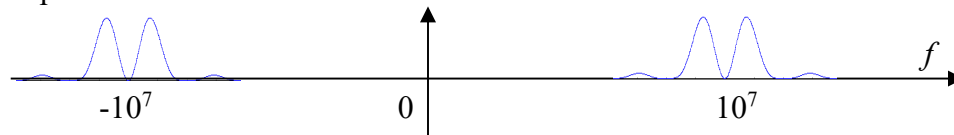
d) Maximum rate = $1/20\text{ms} = 50\text{kbps}$. According to the modulation property, the spectrum of the modulated signal is given by

$$\begin{aligned} X_{\text{after modulation}}(f) &= 0.5(X(f - f_0) + X(f + f_0)) \\ &= 0.5 \left(\left| 0.2 \times \text{sinc}(0.01\pi (f - f_0)) \sin(0.01\pi (f - f_0)) \right|^2 + \left| 0.2 \times \text{sinc}(0.01\pi (f + f_0)) \sin(0.01\pi (f + f_0)) \right|^2 \right) \end{aligned}$$

Based on the discussions in Chapter 3, the average power spectrum is given by.

$$\frac{|X_{\text{after modulation}}(f)|^2}{\tau} = \frac{|X_{\text{after modulation}}(f)|^2}{2 \times 0.01}.$$

A plot of power spectrum is shown below.



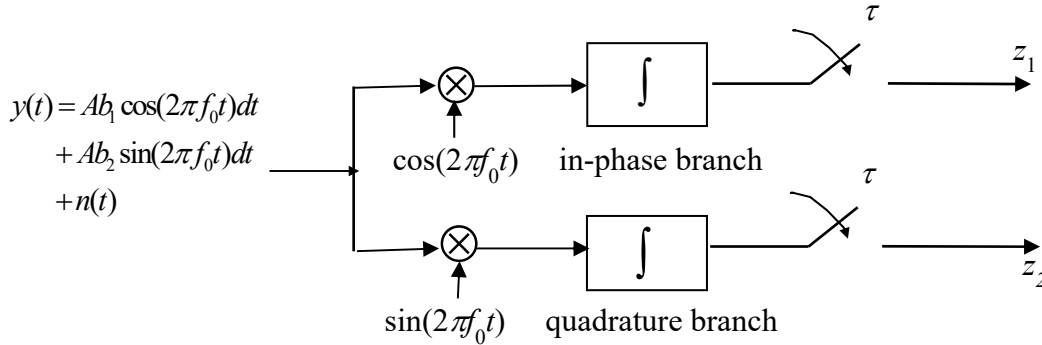
Q10 (1) If the modulation rule is: there is “no change” in the output state if a “1” is present, while there is a “change” in the output state if a “0” is present. Then the corresponding information bits are: 0 0 1 0 0 1

(2) If the modulation rule is: there is “no change” in the output state if a “0” is present, while there is a “change” in the output state if a “1” is present. Then the corresponding information bits are: 1 1 0 1 1 0

Q11 (i) (1) FSK (2) BPSK or ASK with both signs (3) On-off ASK

(ii) 1 0 0 0 1 1 0 0 0 1 0 or 0 1 1 1 0 0 1 1 1 0 1

Q12) A receiver structure is shown below. The solution steps are similar to those for Q3), except that we now need to consider two branches: in phase and quadrature.



(a) The integrator outputs are ($b_1 = -3, -1, +1, +3$, and $b_2 = -3, -1, +1, +3$.)

$$z_1 = \int_0^{\tau} Ab_1 \cos^2(2\pi f_0 t) dt \approx 0.5 Ab_1 \tau = b_1 B \quad (\text{with } B \equiv 0.5 A \tau),$$

$$z_2 = \int_0^{\tau} Ab_2 \sin^2(2\pi f_0 t) dt \approx 0.5 Ab_2 \tau = b_2 B.$$

(b) Each symbol energy $= 0.5 A^2 b_1^2 \tau + 0.5 A^2 b_2^2 \tau$.

The above is the sum of in-phase power and quadrature power. See property 3 in the lecture note regarding amplitude adding and power adding.

(c) This is the average of the results in (b) over four values of b_1 and four values of b_2 :

$$E_s = 2 \times (0.5 A^2 \tau ((-3)^2 + (-1)^2 + 1^2 + 3^2) / 4) = 5 A^2 \tau.$$

Since each symbol now carries 4 bits, the average bit energy is

$$E_b = E_s / 4 = 1.25 A^2 \tau, \quad (\text{same as for Q3}).$$

(d) Since $\tau = 10^{-9}$ s, the baseband bandwidth is 10^9 Hz for 90% in-band energy preservation. After modulation, the bandwidth is 2×10^9 Hz.

Each symbol is with duration $\tau = 10^{-9}$ s and can carry 4 bits of information. So data rate is 4×10^9 bits per second.

Q13) Note: This question may be difficult. You need to put some effort. If you find it too difficult, you may skip it.

(a) Now the received signal is $Ab \cos(2\pi f_0 t) + n(t)$ where $n(t)$ is an AWGN. The integrator output is (using $B \equiv 0.5 A \tau$ as defined in Q3)).

$$y = 0.5 Ab \tau + \int_0^{\tau} n(t) \cos(2\pi f_0 t) dt \approx bB + \eta$$

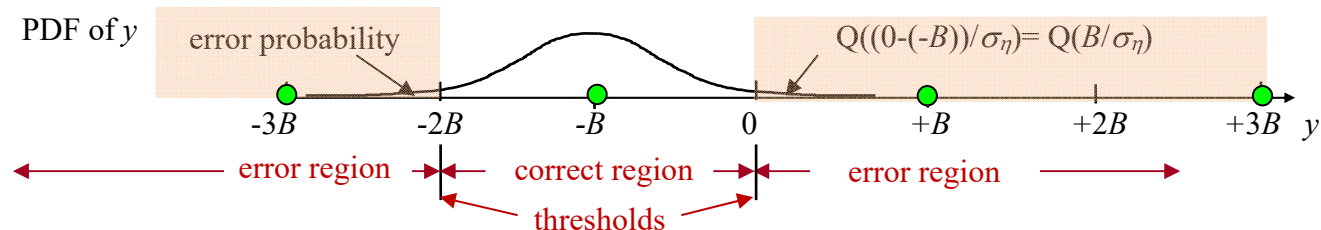
It can be shown that η is a Gaussian random variable with mean $= 0$ and $\sigma_{\eta}^2 = \tau \sigma^2 / 2 = \tau N_0 / 4$. (We will not give the detailed derivation for this relationship.)

The situation of $b = -1$ is illustrated below. The correlator output is $y = -B + \eta$.

We use $-2B$ and 0 as the decision thresholds. There are two types of error events.

- If the noise is too negative, we may make a wrong decision of $-3B$.
- If the noise is too positive, we may make a wrong decision of $+B$ or $+3B$.

The total error probability is $2Q(B/\sigma_\eta)$ for $b = -1$. (The error for $b = +1$ is similar.)



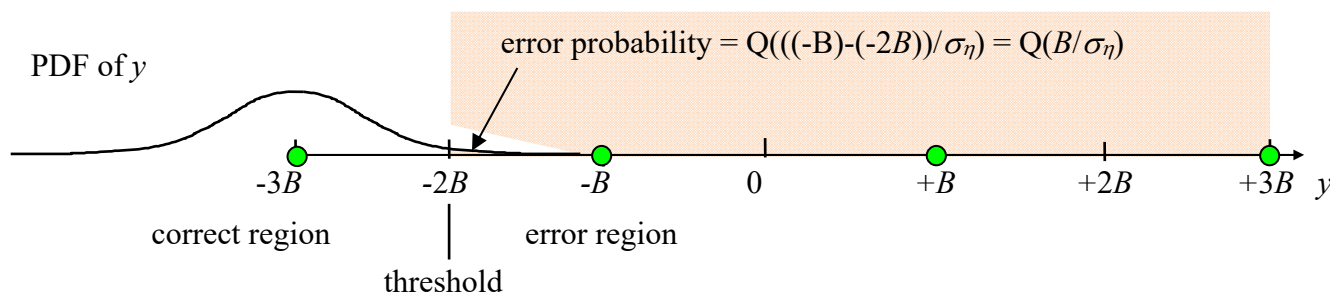
The situation of $b = -3$ is illustrated below. (That for $b = +3$ is similar.) The correlator output is

$$y = -3B + \eta$$

We use $-2B$ as the decision threshold. There is only one type of error events.

- If the noise is too positive, we may make a wrong decision of $-B, B$ or $3B$.

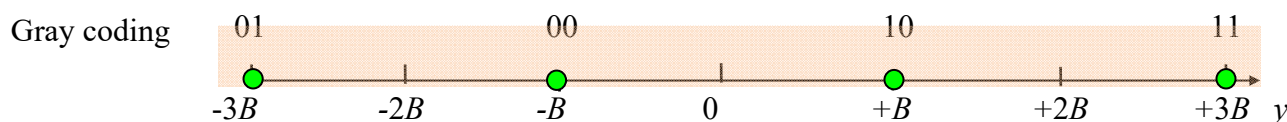
The error probability is $Q(B/\sigma_\eta)$.



(b) Combining the above discussions, we can see that the symbol error rate is

$$SER \text{ (symbol error rate)} = \frac{1}{4} \times (2Q(B/\sigma_\eta) + 2Q(B/\sigma_\eta) + Q(B/\sigma_\eta) + Q(B/\sigma_\eta)) = 1.5Q(B/\sigma_\eta)$$

(c) With Gray coding shown below, a common event may result in one bit error, such as $(01) \rightarrow (00)$. (**Note:** Sometimes an erroneous symbol may result two erroneous bits such as $(01) \rightarrow (10)$, but the probability is small.)



Therefore

$$BER \approx SER/2 = 0.75Q(B/\sigma_\eta) = 0.75Q\left(\sqrt{B^2/\sigma_\eta^2}\right). \quad (1)$$

Substituting $E_b = 5B^2/\tau$ (as derived in Q3)) and $\sigma_\eta^2 = \tau N_0/4$ (see (a)) into (1), we have

$$BER = 0.75Q\left(\sqrt{\frac{\tau E_b}{5\sigma_\eta^2}}\right) = 0.75Q\left(\sqrt{\frac{\tau E_b}{5\tau N_0/4}}\right) = 0.75Q\left(\sqrt{\frac{E_b}{1.25N_0}}\right).$$

(Comments: With the same E_b , this 4-ary scheme has worse BER than BPSK and QPSK.)