

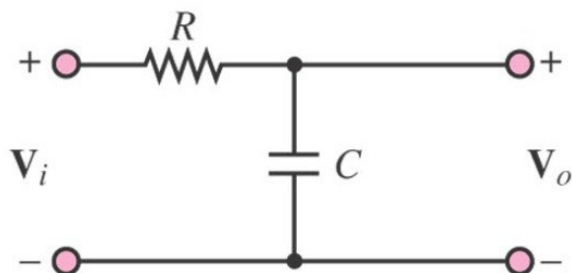
Block B Unit 3: Outline of Contents

- > Passive filters:
 - Review of 1st order passive filters
 - Passive 2nd order filters
- > Active filters
 - 1st order active filters
 - Active band pass filter

Passive 1st order low pass filter

$$\frac{V_o}{V_i}(j\omega) = \frac{1}{1 + j(\omega/\omega_c)}$$

$$\omega_c = \frac{1}{RC}$$



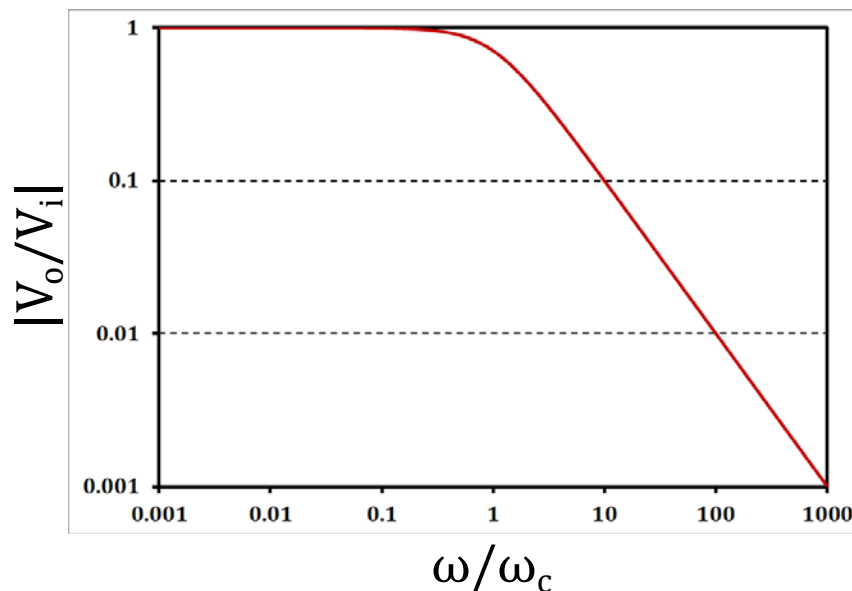
Note that it is not possible to get a gain larger than 1

Recall that ω_c is a constant that is determined by the circuit components, referred to as the corner frequency.

For an input at a frequency of ω_c , $V_o = V_i/\sqrt{2}$

For frequencies $< \omega_c$, $|V_o| \approx |V_i|$ (i.e. nearly constant with frequency)

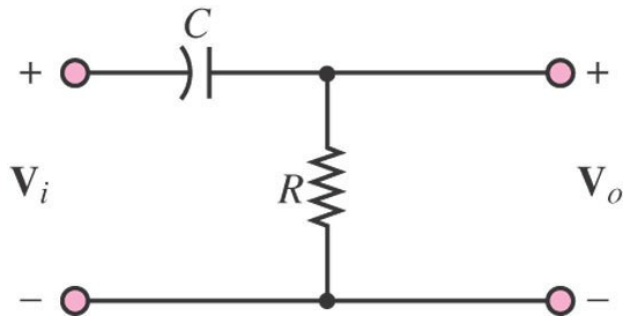
For frequencies $> \omega_c$, $|V_o| \approx |V_i| \omega_c/\omega$ (i.e. inversely proportional to the frequency)



Passive 1st order high pass filter

$$\frac{V_o}{V_i}(j\omega) = \frac{1}{1 + (\omega_c/j\omega)}$$

$$\omega_c = \frac{1}{RC}$$

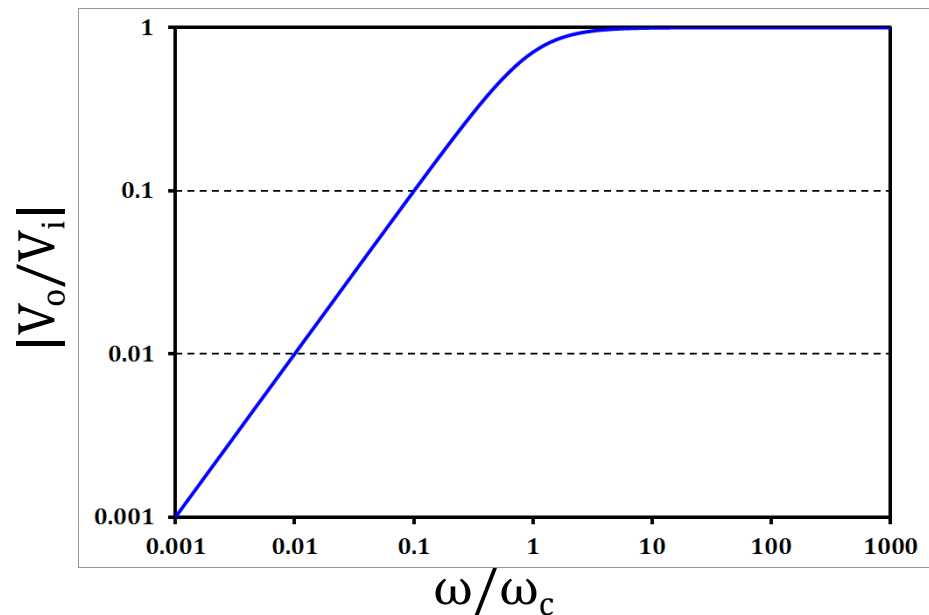


Note that it is not possible to get a gain larger than 1

For an input at a frequency of ω_c , $V_o = V_i/\sqrt{2}$

For frequencies $> \omega_c$, $|V_o| \approx |V_i|$ (i.e. nearly constant with frequency)

For frequencies $< \omega_c$, $|V_o| \approx |V_i| \omega/\omega_c$ (i.e. proportional to the frequency)



Decibels (dB)

$$\text{Gain in decibels (dB)} = 10 \log \frac{P_{out}}{P_{in}}$$

$$\text{Gain in decibels (dB)} = 20 \log \frac{V_{out}}{V_{in}}$$

$$\text{Gain} = 1 \Rightarrow 0\text{dB}$$

$$\text{Gain} = 1/\sqrt{2} \Rightarrow -3\text{dB}$$

$$\text{Gain} = 0.1 \Rightarrow -20\text{dB}$$

$$\text{Gain} = 0.01 \Rightarrow -40\text{dB}$$

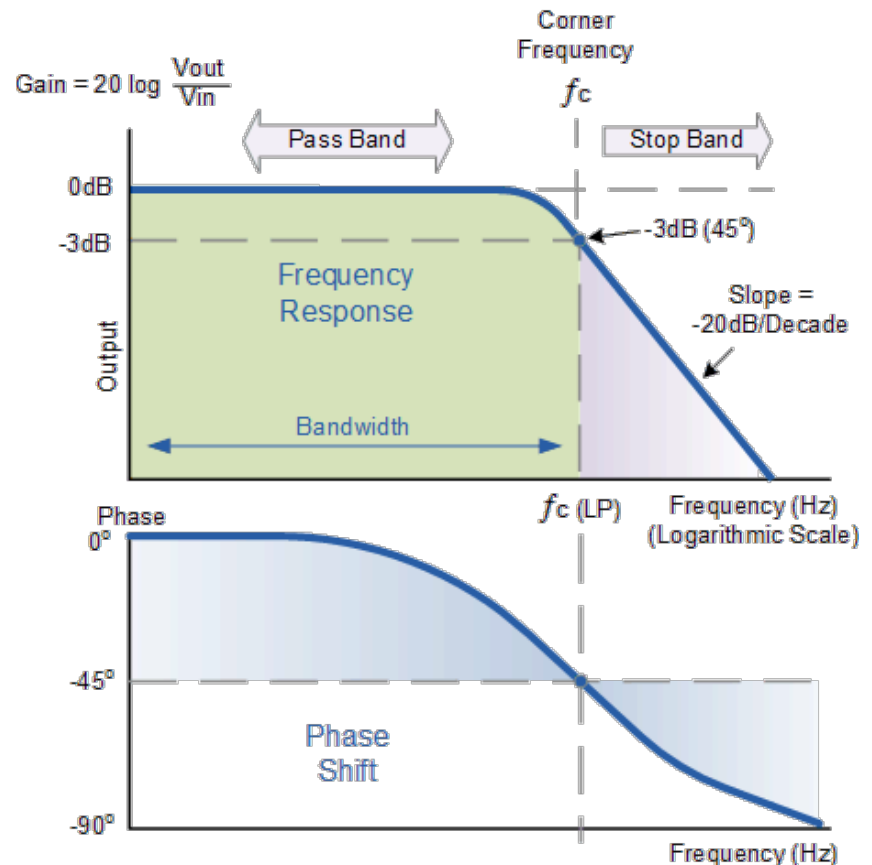
Every factor of 10 corresponds to a change of 20dB

$$\text{Times 10} \Rightarrow +20\text{dB}$$

$$\text{Divide by 10} \Rightarrow -20\text{dB}$$

Recall: $P = V^2/R$

At the corner frequency (f_c), gain drops by 3dB relative to the passband

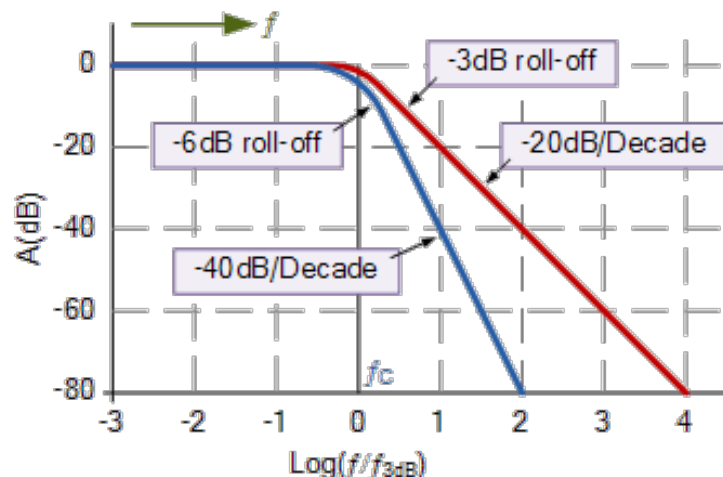
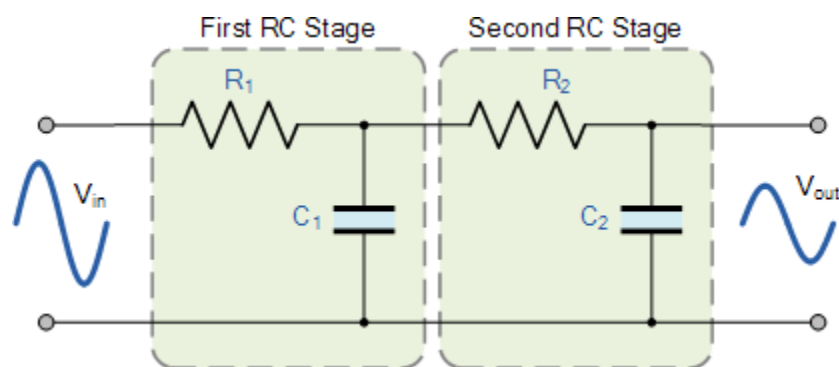


Ideal Filter Specification

Passive 2nd order low pass filter

We can increase the slope of the roll off beyond the cut-off frequency by cascading several stages of low pass filters.

The example here shows a two stage passive low pass filter.



By choice of values, if $(R_2 + 1/j\omega C_2) \gg 1/j\omega C_1$:

$$\frac{V_o}{V_i}(j\omega) = \left(\frac{1}{1 + j(\omega/\omega_{c1})} \right) \left(\frac{1}{1 + j(\omega/\omega_{c2})} \right)$$

$$\omega_{c1} = \frac{1}{R_1 C_1} \quad \omega_{c2} = \frac{1}{R_2 C_2}$$

If we chose the values of components so that $\omega_{c1} = \omega_{c2} = \omega_c$:

$$\frac{V_o}{V_i}(j\omega) = \frac{1}{[1 + j(\omega/\omega_c)]^2}$$

Passive 2nd order low pass filter

- > Slope:
 - 2nd order filter: -40dB/decade ($|V_o/V_i|$ drops by 100 times for 10 times increase in frequency)
 - 1st order filter: -20dB/decade ($|V_o/V_i|$ drops by 10 times for 10 times increase in frequency)
- > At cut-off frequency (f_c),
 - 2nd order filter: gain drops by -6dB
 - 1st order filter: gain drops by -3dB
- > Coupling issues: Impedance of stage 2 chosen to be much larger than load impedance of stage 1
 - What happens as we increase the number of stages in the cascade for higher orders?
- > Passband gain: Passband gain is still less than 1

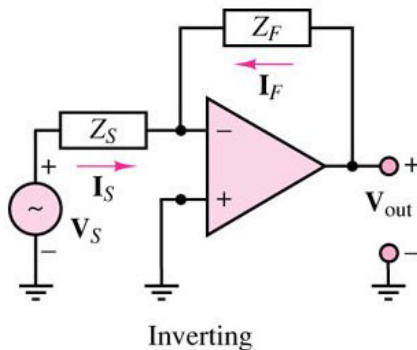
Active filters

Range of applications is greatly expanded if reactive components are used

Addition of reactive components allows us to shape the frequency response

Active filters: Op-amp provides amplification (gain) in addition to filtering effects

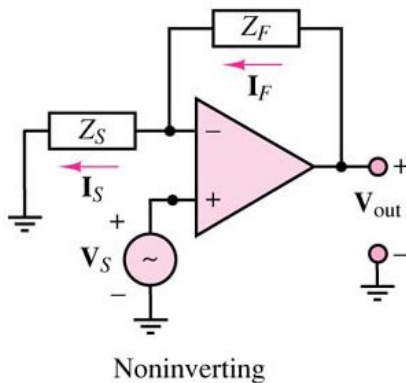
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Substitute R with Z now for our analysis:

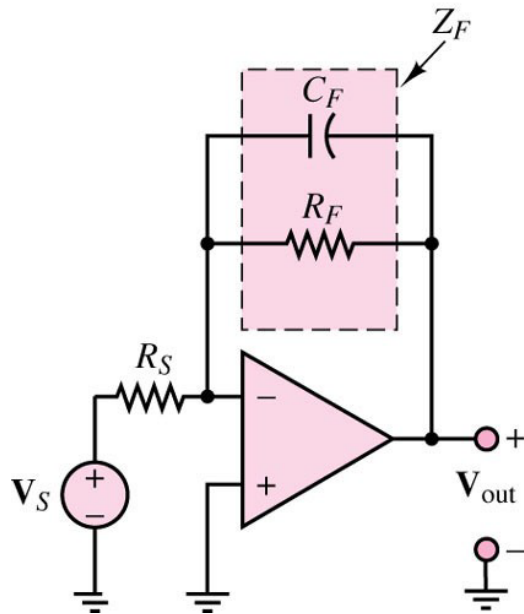
$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

Z_F and Z_S can be arbitrary (i.e. any) complex impedance



Active 1st order low pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F \parallel C_F = \frac{R_F}{1 + j\omega C_F R_F}$$

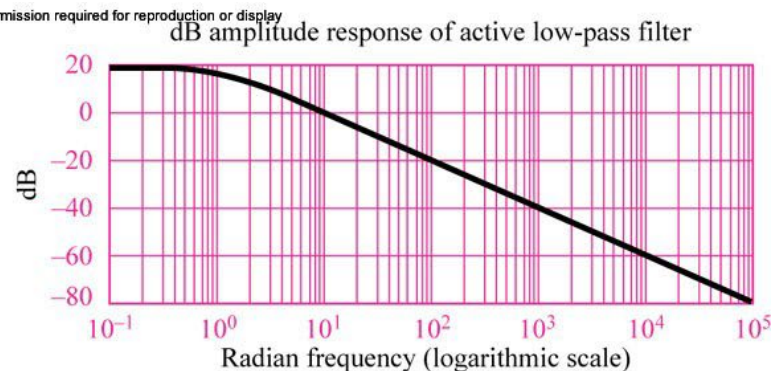
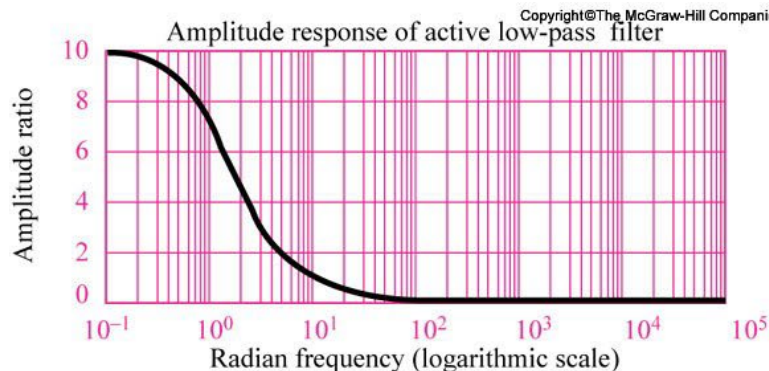
$$Z_S = R_S$$

$$\frac{V_{out}}{V_S}(j\omega) = -\frac{R_F / R_S}{1 + j\omega C_F R_F}$$

Passband gain is R_F/R_S

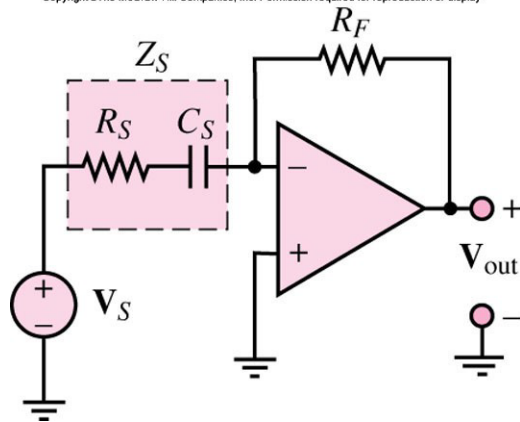
$$\omega_c = \frac{1}{R_F C_F}$$

If we choose $R_F/R_S = 10$, frequency response becomes:



Active 1st order high pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F$$

$$Z_S = R_S + 1/j\omega C_S$$

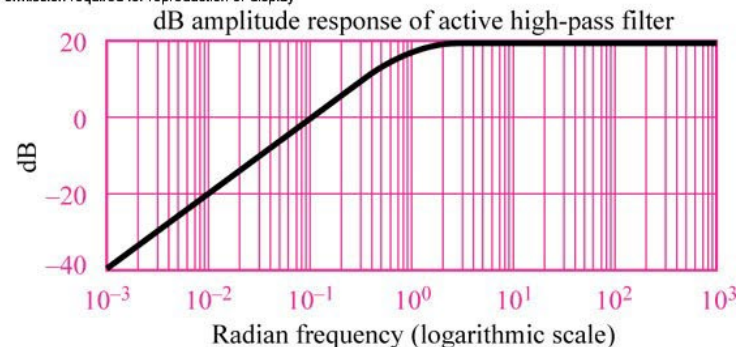
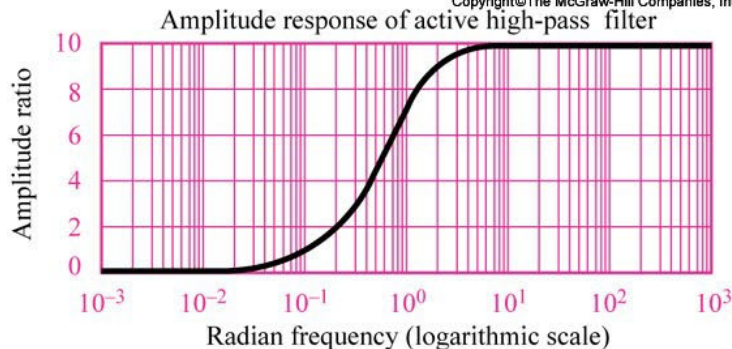
$$\frac{V_{out}}{V_S}(j\omega) = -\frac{R_F/R_S}{1 + (1/j\omega C_S R_S)}$$

Passband gain is R_F/R_S

$$\omega_c = \frac{1}{R_S C_S}$$

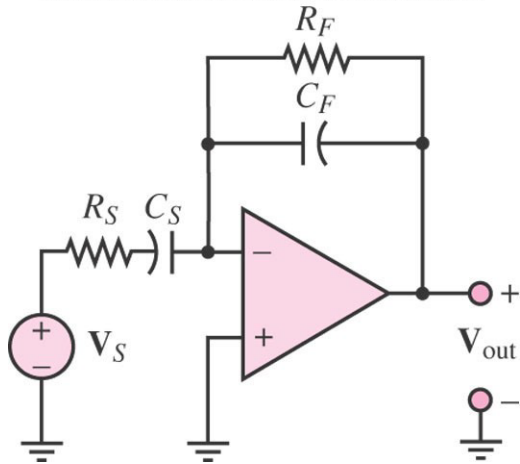
If we choose $R_F/R_S = 10$, frequency response becomes:

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Active band pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_S = R_S + \frac{1}{j\omega C_S}$$

$$Z_F = R_F \parallel C_F = \frac{R_F}{1 + j\omega C_F R_F}$$

$$\frac{V_{out}}{V_S}(j\omega) = \frac{-R_F / R_S}{(1 + 1/j\omega C_S R_S)(1 + j\omega C_F R_F)}$$

Z_S : C_S blocks low frequency inputs but lets high frequency inputs through

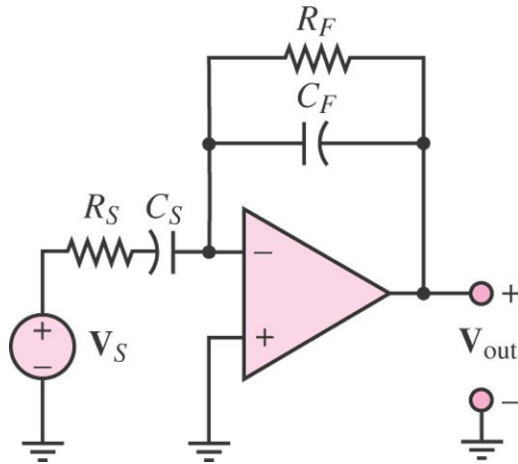
⇒ High pass filter

Z_F : C_F shorts R_F at high frequency (reducing the gain), but otherwise looks just like a low pass filter at lower frequencies

⇒ Low pass filter

Active band pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = \frac{-R_F/R_S}{(1 + \omega_{HP}/j\omega)(1 + j\omega/\omega_{LP})}$$

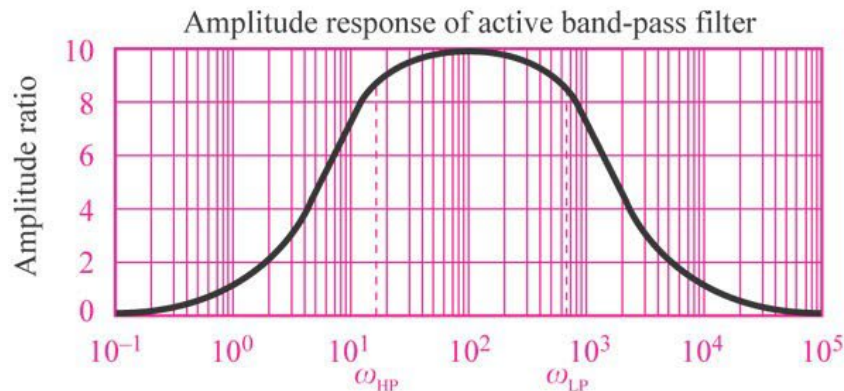
$\omega_{HP} = 1/(C_S R_S)$ – Lower cut-off frequency

$\omega_{LP} = 1/(C_F R_F)$ – Upper cut-off frequency

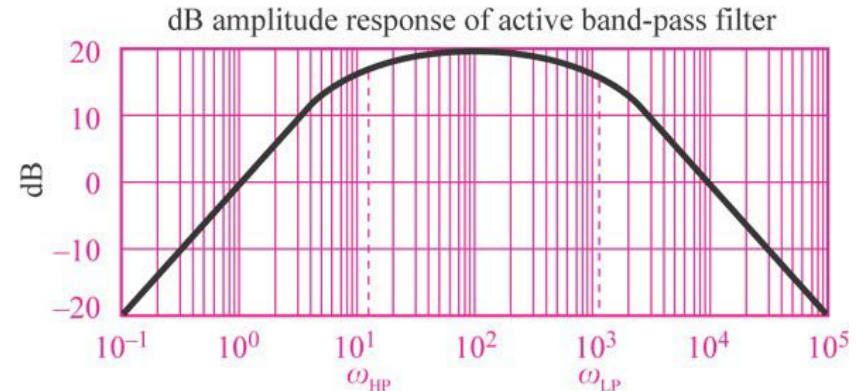
Passband gain is R_F/R_S

For $\omega_{HP} < \omega_{LP}$ and $R_F/R_S = 10$: Frequency response curve is shown below

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Radian frequency (logarithmic scale)



Radian frequency (logarithmic scale)

High Pass Passive Filter Versus Active Filter

Low Pass Passive Filter Versus Active Filter
