

EE3114 HW-1 solutions

1-1 Find the Laplace Transforms of $x(t)u(t)$, where $u(t)$ is the unit step function and

(a) $x(t) = \cos(2t) - 2e^t$ [2 marks]

(b) $x(t) = 1 + 2t^2$ [2 marks]

(c) $x(t) = t - 2t^2e^{-2t}$ [2 marks]

Show your work.

Solutions :

(a) Since

$$Ae^{-at}u(t) \xleftrightarrow{L} \frac{A}{(a+s)}$$

$$\cos(n\omega t)u(t) \xleftrightarrow{L} \frac{s}{s^2 + (n\omega)^2}$$

So

$$\begin{aligned} L(u(t)x(t)) &= L(\cos(2t)u(t)) - L(2e^t u(t)) \\ &= \frac{s}{s^2 + 4} - \frac{2}{s-1} \end{aligned}$$

(b)

Since

$$L(t^n u(t)) = L\left(n \int_0^t \tau^{n-1} u(\tau) d\tau\right) = \frac{1}{s} L(n(n-1) \int_0^t \tau^{n-2} u(\tau) d\tau) = \dots = \frac{1}{s^n} L(n! \int_0^t u(\tau) d\tau) = \frac{n!}{s^{n+1}}$$

So

$$L(u(t)x(t)) = \frac{1}{s} + \frac{4}{s^3}$$

(c)

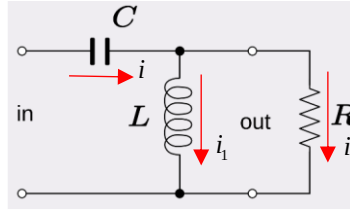
Since

$$Ae^{-at}u(t) \xleftrightarrow{L} \frac{A}{(a+s)}$$

So

$$L(u(t)x(t)) = \frac{1}{s^2} - \frac{4}{(s+2)^3}$$

1-2 Find the Transfer Function $V_o(s)/V_i(s)$ of the following circuit in terms of R, C, L : [2 marks]



$$\left\{ \begin{array}{l} \frac{1}{C} \int i dt + L \frac{di_1}{dt} = v_{in} \\ \frac{1}{C} \int i dt + Ri_2 = v_{in} \\ i = i_1 + i_2 \\ Ri_2 = v_{out} \end{array} \right. \quad \Rightarrow \quad \left\{ \begin{array}{l} \frac{1}{Cs} I(s) + LsI_1(s) = V_{in}(s) \\ \frac{1}{Cs} I(s) + RI_2(s) = V_{in}(s) \\ I(s) = I_1(s) + I_2(s) \\ RI_2(s) = V_{out}(s) \end{array} \right.$$

Solve the Laplace equations :

$$\left\{ \begin{array}{l} \frac{1}{Cs} [I_1(s) + I_2(s)] + LsI_1(s) = V_{in}(s) \\ \frac{1}{Cs} [I_1(s) + I_2(s)] + RI_2(s) = V_{in}(s) \\ RI_2(s) = V_{out}(s) \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{1}{Cs} \times \left[\left(\frac{1}{Cs} + Ls \right) I_1(s) + \frac{1}{Cs} I_2(s) \right] = \frac{1}{Cs} \times V_{in}(s) \\ \left(\frac{1}{Cs} + Ls \right) \times \left[\frac{1}{Cs} I_1(s) + \left(\frac{1}{Cs} + R \right) I_2(s) \right] = \left(\frac{1}{Cs} + Ls \right) \times V_{in}(s) \\ RI_2(s) = V_{out}(s) \end{array} \right.$$

$$\left\{ \begin{array}{l} \left(\frac{1}{Cs} + Ls \right) \left(\frac{1}{Cs} + R \right) I_2(s) - \left(\frac{1}{Cs} \right)^2 I_2(s) = LsV_{in}(s) \\ RI_2(s) = V_{out}(s) \end{array} \right.$$

$$\frac{1}{Ls} \left[\left(\frac{1}{Cs} + Ls \right) \left(\frac{1}{Cs} + R \right) - \left(\frac{1}{Cs} \right)^2 \right] \frac{V_{out}(s)}{R} = V_{in}(s)$$

After Simplifying,

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{RLCs^2}{RLCs^2 + Ls + R}$$

1-3 Find the solution of the following ODE: [2 marks]

$$L\left[\frac{d^2 y(t)}{dt^2}\right] - 3L\left[\frac{dy(t)}{dt}\right] + 2L[y(t)] = L[u(t)]$$

where $u(t)$ is the unit step function, with initial conditions $y(0)=1$, $\frac{dy(0)}{dt}=1$

Solution:

Initial condition: $y(0)=1$, $\frac{dy(0)}{dt}=1$

$$L\left[\frac{d^2 y(t)}{dt^2}\right] - 3L\left[\frac{dy(t)}{dt}\right] + 2L[y(t)] = L[u(t)]$$

$$\left(s^2 Y(s) - sy(0) - \frac{dy(0)}{dt}\right) - 3(sY(s) - y(0)) + 2Y(s) = \frac{1}{s}$$

$$(s^2 Y(s) - s - 1) - 3(sY(s) - 1) + 2Y(s) = \frac{1}{s}$$

$$(s^2 - 3s + 2)Y(s) = \frac{1}{s} + s - 2$$

$$Y(s) = \frac{s^2 - 2s + 1}{s(s^2 - 3s + 2)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s-1}$$

$$\begin{cases} A + B + C = 1 \\ 3A + B + 2C = 2 \\ 2A = 1 \end{cases}$$

$$\Rightarrow A = B = \frac{1}{2}, C = 0.$$

so

$$Y(s) = \frac{s^2 - 2s + 1}{s(s^2 - 3s + 2)} = \frac{1}{2} \left(\frac{1}{s} + \frac{1}{s-2} \right)$$

$$y(t)$$

$$= L^{-1}(Y(s))$$

$$= \frac{1}{2} L^{-1}\left(\frac{1}{s}\right) + \frac{1}{2} L^{-1}\left(\frac{1}{s-2}\right)$$

$$= \frac{1}{2} (1 + e^{2t}) u(t)$$