

Review

21.2 Electric Charge:

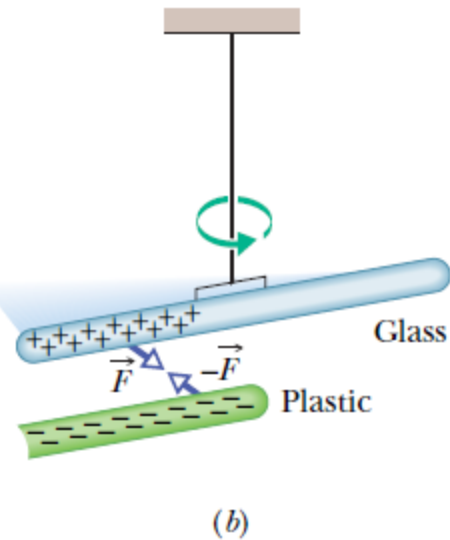
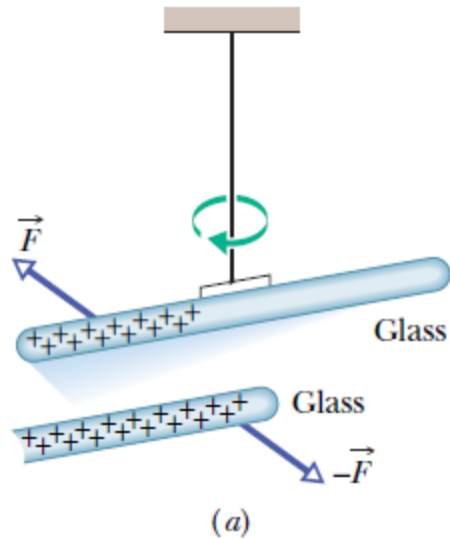


Fig. 21-2 (a) Two charged rods of the same sign repel each other. (b) Two charged rods of opposite signs attract each other. Plus signs indicate a positive net charge, and minus signs indicate a negative net charge.

Electric charge has two different signs.

Charges with the same electrical sign repel each other, and charges with the opposite electrical signs attract each other.

21.4 Coulomb's Law

The equation giving the **electrostatic force** for charged particles is called **Coulomb's law**:

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r} \quad (\text{Coulomb's law}),$$

The SI unit of charge is the **coulomb**.

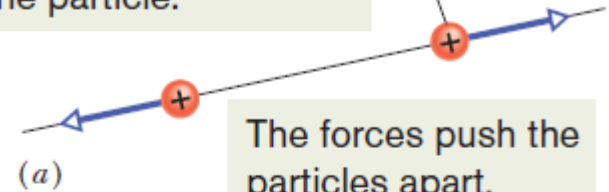
The **electrostatic constant** is

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2.$$

The quantity ϵ_0 is called the **permittivity constant**

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2.$$

Always draw the force vector with the tail on the particle.



The forces push the particles apart.

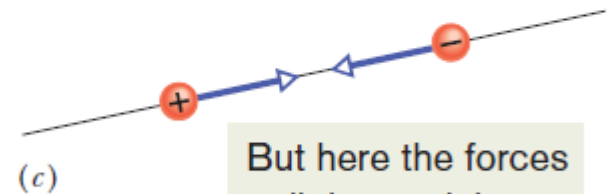
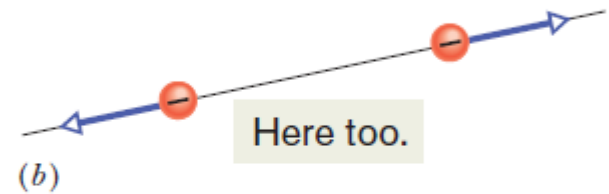


Fig. 21-6 Two charged particles repel each other if they have the same sign of charge, either (a) both positive or (b) both negative. (c) They attract each other if they have opposite signs of charge.

Example, Charge Sharing:

In Fig. 21-10a, two identical, electrically isolated conducting spheres *A* and *B* are separated by a (center-to-center) distance *a* that is large compared to the spheres. Sphere *A* has a positive charge of $+Q$, and sphere *B* is electrically neutral. Initially, there is no electrostatic force between the spheres. (Assume that there is no induced charge on the spheres because of their large separation.)

(a) Suppose the spheres are connected for a moment by a conducting wire. The wire is thin enough so that any net charge on it is negligible. What is the electrostatic force between the spheres after the wire is removed?

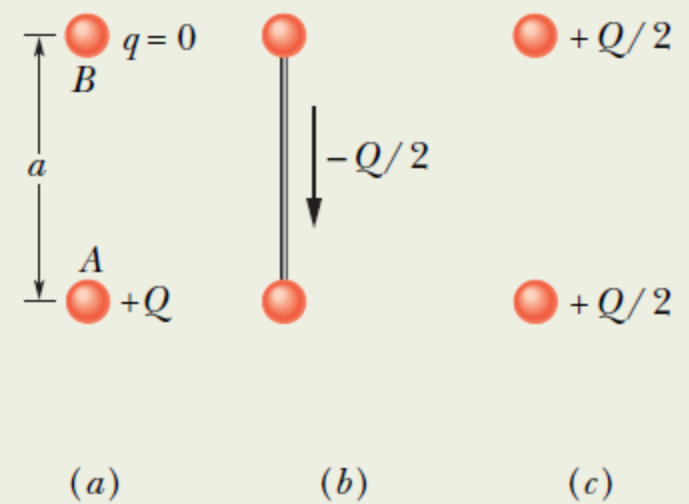


Fig. 21-10 Two small conducting spheres *A* and *B*. (a) To start, sphere *A* is charged positively. (b) Negative charge is transferred from *B* to *A* through a connecting wire. (c) Both spheres are then charged positively.

(1) Since the spheres are identical, connecting them means that they end up with identical charges (same sign and same amount). (2) The initial sum of the charges (including the signs of the charges) must equal the final sum of the charges.

Reasoning: When the spheres are wired together, the (negative) conduction electrons on *B* move away from one another (along the wire to positively charged *A*— Fig. 21-10b.)

As *B* loses negative charge, it becomes positively charged, and as *A* gains negative charge, it becomes less positively charged. The transfer of charge stops when the charge on *B* has increased to $Q/2$ and the charge on *A* has decreased to $Q/2$, which occurs when $Q/2$ has shifted from *B* to *A*.

The spheres, now positively charged, repel each other.

$$F = \frac{1}{4\pi\epsilon_0} \frac{(Q/2)(Q/2)}{a^2} = \frac{1}{16\pi\epsilon_0} \left(\frac{Q}{a}\right)^2.$$

21.5 Charge is Quantized

Table 21-1

The Charges of Three Particles

Particle	Symbol	Charge
Electron	e or e^-	$-e$
Proton	p	$+e$
Neutron	n	0

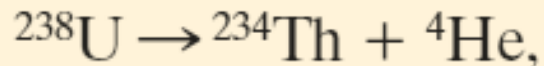
Quarks have charge of $\pm \frac{1}{3} e$ or $\pm \frac{2}{3} e$, but they never appear individually (color confinement).

21.6 Charge is Conserved

The hypothesis of **conservation of charge** has stood up under close examination, both for large-scale charged bodies and for atoms, nuclei, and elementary particles.

Example 1: **Radioactive decay of nuclei**, in which a nucleus transforms into (becomes) a different type of nucleus.

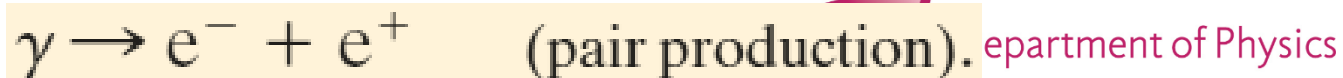
A uranium-238 nucleus (^{238}U) transforms into a thorium-234 nucleus (^{234}Th) by emitting an *alpha particle*. An alpha particle has the same makeup as a helium-4 nucleus, it has the symbol ^4He . Here the net charge is $92 \rightarrow 90 + 2$.



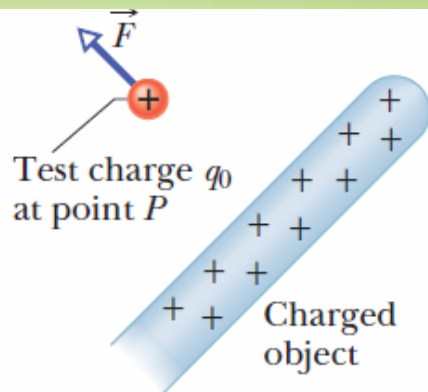
Example 2: An electron e^- (charge $-e$) and its antiparticle, the positron e^+ (charge $+e$), undergo an annihilation process, transforming into two gamma rays (high-energy light):. Here the net charge is $0 \rightarrow 0$.



Example 3: A gamma ray (in a certain environment) transforms into an electron and a positron. Here the net charge is again $0 \rightarrow 0$.

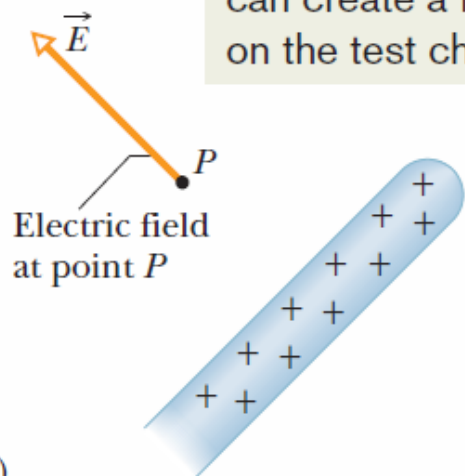


22.2 The Electric Field:



(a)

The rod sets up an electric field, which can create a force on the test charge.



(b)

Fig. 22-1 (a) A positive test charge q_0 placed at point P near a charged object. An electrostatic force $\vec{F}$ acts on the test charge. (b) The electric field $\vec{E}$ at point P produced by the charged object.

The electric field is a *vector field*.

$$\vec{E} = \frac{\vec{F}}{q_0} \quad (\text{electric field}).$$

The SI unit for the electric field is the newton per coulomb (**N/C**).

$$\vec{F} = k \frac{qq_0}{r^2} \hat{r}$$

22.3 Electric Field Lines:

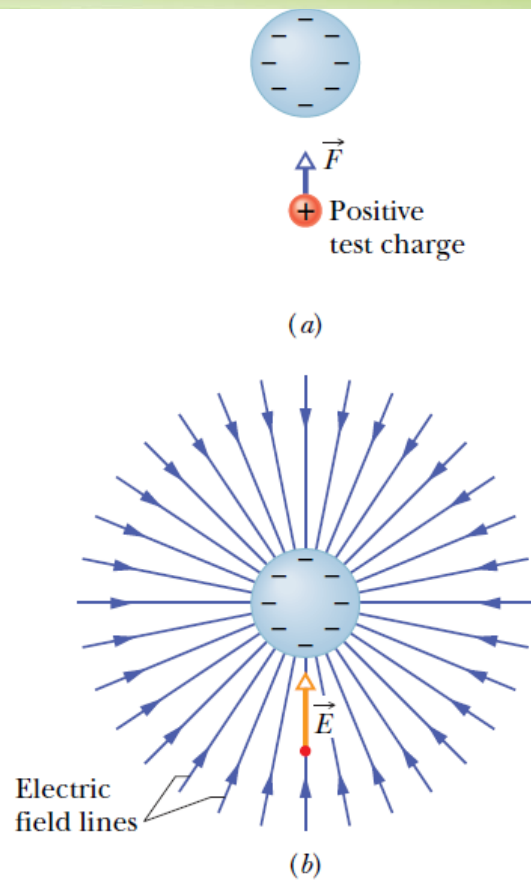


Fig. 22-2 (a) The electrostatic force $\vec{F}$ acting on a positive test charge near a sphere of uniform negative charge. (b) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sphere. The field lines extend *toward* the negatively charged sphere. (They originate on distant positive charges.)

- Electric field lines are imaginary lines which extend away from positive charge (where they originate) and toward negative charge (where they terminate).
- At any point, the direction of the tangent to a curved field line gives the direction of the electric field at that point.
- The field lines are drawn so that the number of lines per unit area, measured in a plane that is perpendicular to the lines, is proportional to the magnitude of E . Thus, E is large where field lines are close together and small where they are far apart.

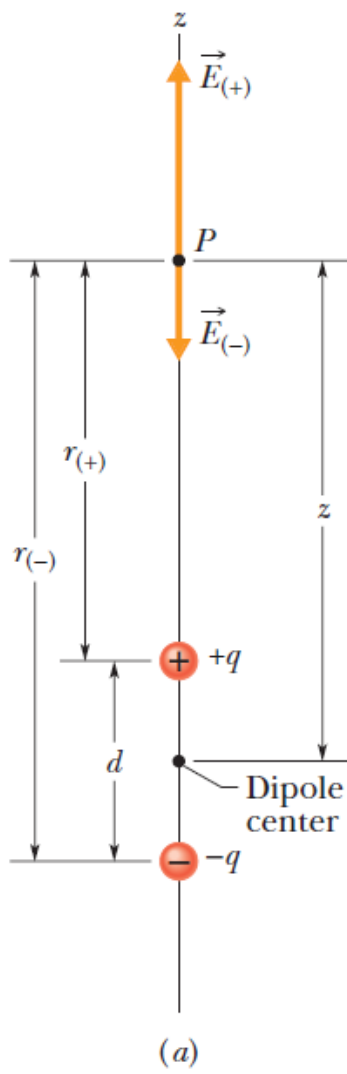
22.4 The Electric Field due to a Point:

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \quad (\text{point charge}).$$

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

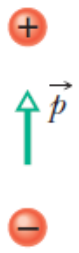
$$\begin{aligned} \vec{E} = \frac{\vec{F}_0}{q_0} &= \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} \\ &= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n. \end{aligned}$$

22.5 The Electric Field due to an Electric Dipole:



$\vec{p} = q\vec{d}$, a vector quantity known as the *electric dipole moment* of the dipole

Up here the +q field dominates.



Down here the -q field dominates.

$$\begin{aligned} E &= E_{(+)} - E_{(-)} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(+)}^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_{(-)}^2} \\ &= \frac{q}{4\pi\epsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\epsilon_0(z + \frac{1}{2}d)^2} \end{aligned}$$

See Supplement

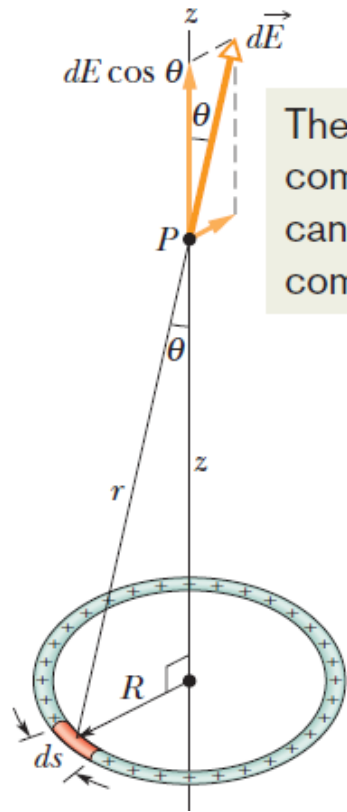
$$\begin{aligned} E &= \frac{q}{4\pi\epsilon_0 z^2} \left(\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2} \right) \\ E &= \frac{q}{4\pi\epsilon_0 z^2} \frac{2d/z}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\epsilon_0 z^3} \frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2} \end{aligned}$$

$d/2z \ll 1$

$$E = \frac{1}{2\pi\epsilon_0} \frac{qd}{z^3}$$

$$E = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3} \quad \text{(electric dipole).}$$

22.6 The Electric Field due to a Line Charge:



The perpendicular components just cancel but the parallel components add.

$$dq = \lambda ds.$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{r^2}.$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda ds}{(z^2 + R^2)}.$$

$$dE \cos \theta = \frac{z\lambda}{4\pi\epsilon_0(z^2 + R^2)^{3/2}} ds.$$

$$E = \int dE \cos \theta = \frac{z\lambda}{4\pi\epsilon_0(z^2 + R^2)^{3/2}} \int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\epsilon_0(z^2 + R^2)^{3/2}}.$$

Fig. 22-10 A ring of uniform positive charge. A differential element of charge occupies a length ds (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point P . The component of $d\vec{E}$ along the central axis of the ring is $dE \cos \theta$.

22.9: A Dipole in an Electric Field: Potential Energy

Potential energy can be associated with the orientation of an electric dipole in an electric field.

The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment $\vec{p}$ is lined up with the field $\vec{E}$.

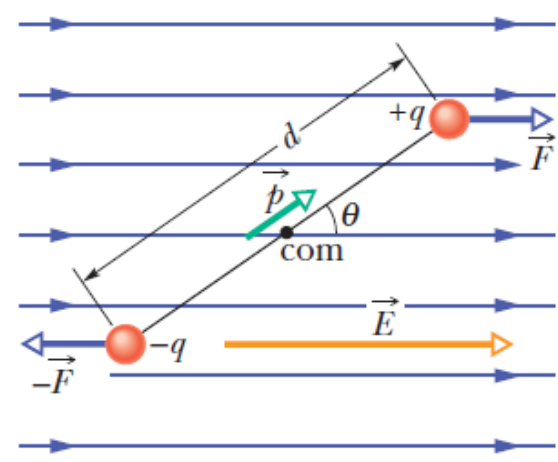
The expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle θ (Fig.22-19) is 90° .

The potential energy U of the dipole at any other value of θ can be found by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90° .

$$U = -W = -\int_{90^\circ}^{\theta} \tau d\theta = \int_{90^\circ}^{\theta} pE \sin \theta d\theta = -pE \cos \theta.$$

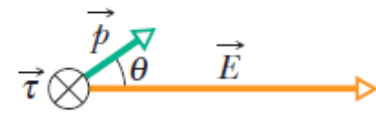


$$U = -\vec{p} \cdot \vec{E} \quad (\text{potential energy of a dipole}).$$



(a)

The dipole is torqued into alignment.



(b)

Fig. 22-19 (a) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance d . The line between them represents their rigid connection. (b) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol $\otimes$.

23.3: Electric Flux

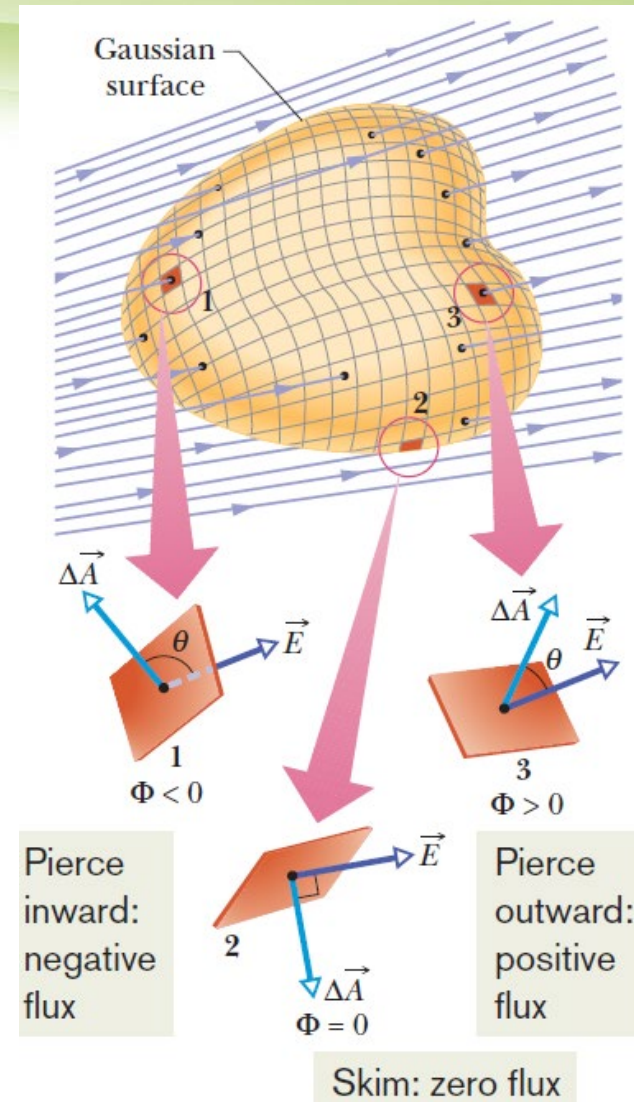
The electric flux through a surface is defined to be the inner product of the electric field and the surface vector:

$$\Phi_E = \vec{E} \bullet \Delta \vec{A}$$

For a closed surface, it is

$$\Phi = \sum \vec{E} \cdot \Delta \vec{A}.$$

$$\Phi = \oiint \vec{E} \cdot d\vec{A} \quad (\text{electric flux through a Gaussian surface})$$



23.5 Gauss' Law and Coulomb's Law:

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = \epsilon_0 \oint E dA = q_{enc}$$

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$$

$$\epsilon_0 E(4\pi r^2) = q$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}.$$

One may also derive Gauss' law from Coulomb's law.

These two laws are equivalent.

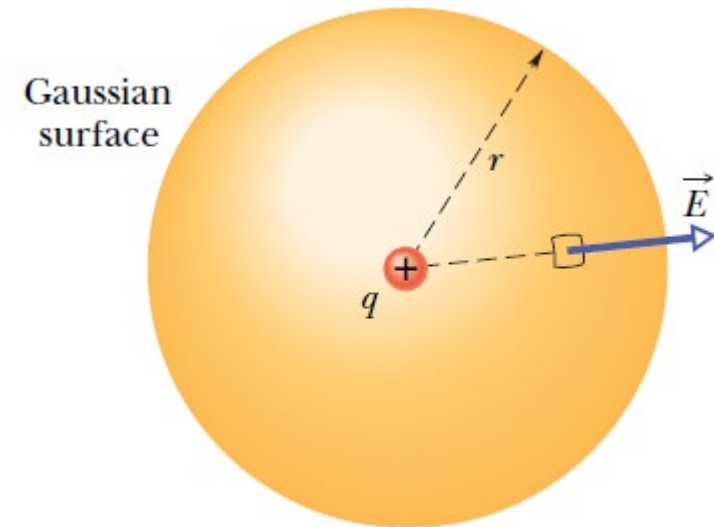


Fig. 23-8 A spherical Gaussian surface centered on a point charge q .

23.6 A Charged Isolated Conductor:

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

Figure 23-9a shows, in cross section, an isolated lump of copper hanging from an insulating thread and having an excess charge q . The Gaussian surface is placed just inside the actual surface of the conductor. The electric field inside this conductor must be zero. Since the excess charge is not inside the Gaussian surface, it must be outside that surface, which means it must lie on the actual surface of the conductor.

Figure 23-9b shows the same hanging conductor, but now with a cavity that is totally within the conductor. A Gaussian surface is drawn surrounding the cavity, close to its surface but inside the conducting body. Inside the conductor, there can be no flux through this new Gaussian surface. Therefore, there is no net charge on the cavity walls; **all the excess charge remains on the outer surface of the conductor.**

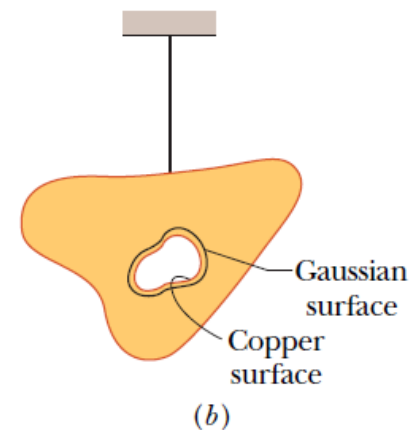
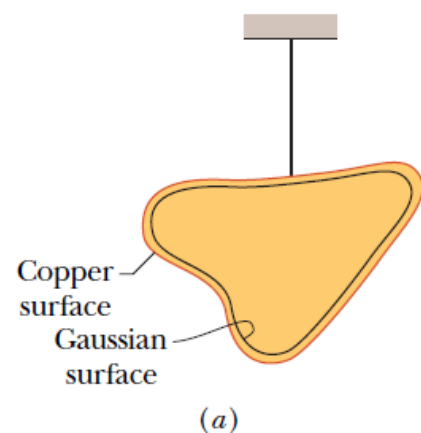


Fig. 23-9 (a) A lump of copper with a charge q hangs from an insulating thread. A Gaussian surface is placed within the metal, just inside the actual surface. (b) The lump of copper now has a cavity within it. A Gaussian surface lies within the metal, close to the cavity surface.

23.9 Applying Gauss' Law, Spherical Symmetry:

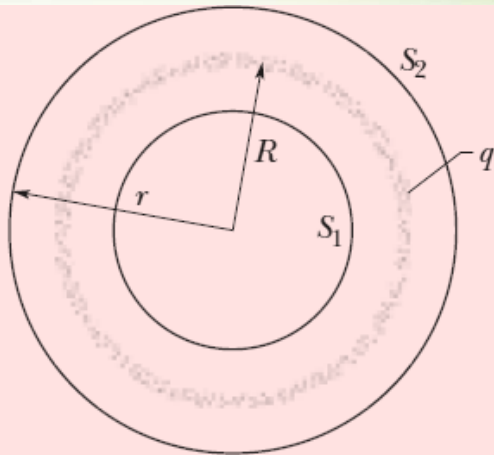
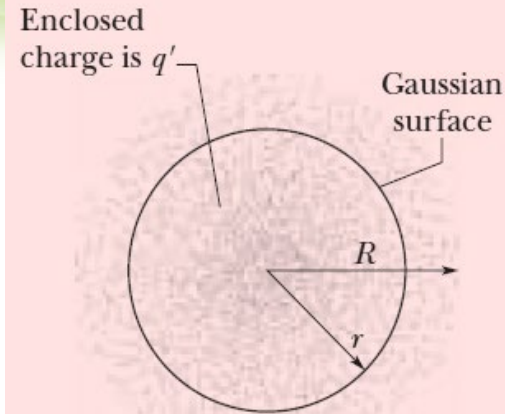


Fig. 23-18 A thin, uniformly charged, spherical shell with total charge q , in cross section. Two Gaussian surfaces S_1 and S_2 are also shown in cross section. Surface S_2 encloses the shell, and S_1 encloses only the empty interior of the shell.

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad (\text{spherical shell, field at } r \geq R).$$

$$E = 0 \quad (\text{spherical shell, field at } r < R),$$

Recall the case for gravitation.



$$r \leq R; \quad q' = \frac{4\pi r^3}{3} \rho$$

$$r > R; \quad q' = \frac{4\pi R^3}{3} \rho$$

Fig. 23-19 The dots represent a spherically symmetric distribution of charge of radius R , whose volume charge density ρ is a function only of distance from the center. The charged object is not a conductor, and therefore the charge is assumed to be fixed in position. A concentric spherical Gaussian surface with $r < R$ is shown.

$$r \leq R; \quad E = \frac{1}{4\pi\epsilon_0} \frac{q'}{r^2} = \frac{1}{4\pi\epsilon_0 r^2} \frac{4\pi r^3}{3} \rho = \frac{\rho}{3\epsilon_0} r$$

$$r > R; \quad E = \frac{1}{4\pi\epsilon_0} \frac{q'}{r^2} = \frac{1}{4\pi\epsilon_0 r^2} \frac{4\pi R^3}{3} \rho = \frac{R^3 \rho}{3\epsilon_0 r^2}$$

Definition of Potential Energy

- Potential energy is defined as the *stored energy of position*.
- It can be viewed as energy that is *'stored'* by the physical system, (e.g. arrangement of objects in a force field).
- It is called *potential* because there is no work being done at its current form, it does not cause any change in its surroundings.
- However, it has the *potential* to be converted to other forms of energy, such as kinetic energy.
- The standard unit for measuring potential energy is joule.

Work and Potential Energy

- Let's take a closer look at the gravitational potential energy formulation (P.E. is often represented by U):

$$P.E._{grav} = U_{grav} = \underbrace{mg}_{\text{Gravitational Force } F_{grav}} \underbrace{h}_{\text{Displacement from ground } d}$$

- We can express the potential energy as

$$U_{grav} = \vec{F}_{grav} \cdot \vec{d}$$

Potential energy has the same SI unit J Joule as Work Done W

Account for the Work Done

- Consider the potential energy of the rock on slide 4 when it is located on the ground and when it is at a distance h above ground, the corresponding potential energies are:

$$U_{d=0} = mg \cdot 0 = 0 \qquad U_{d=h} = mg \cdot h = mgh$$

- Case 1: moving the rock from $d=0$ to $d=h$

$$U_{\text{gain/loss}} = U_{\text{after}} - U_{\text{before}}$$

$$\Delta U = U_{d=h} - U_{d=0} = mgh - 0 = mgh = W_{\text{you}}$$

You contributed work
(energy) into the
system.

- Case 2: dropping the rock from $d=h$ to $d=0$

$$\Delta U = U_{d=0} - U_{d=h} = 0 - mgh = -mgh = -W_{\text{you}} = W_{\text{system}}$$

The system did the
work and released
the energy.

Conservative and Non-Conservative Forces

- Consider a system consists of multiple objects where there are forces acting between any object with all other objects in the system.
- If one of the objects in the system is moved from A to B, work done is required to apply the force on the object to make it move, lets call this work done W_1 . Here the kinetic energy used to move the object is conversed into some other type of energy.
- When the configuration is reversed, where the object is moved from B to A. The force reverses the energy transfer, doing work W_2 in the process.

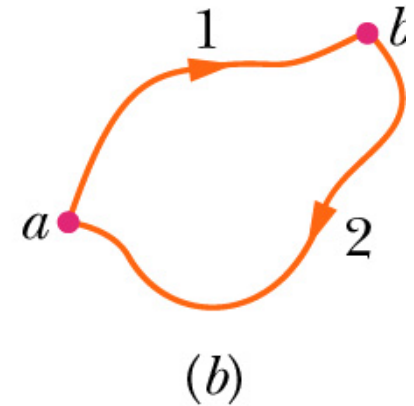
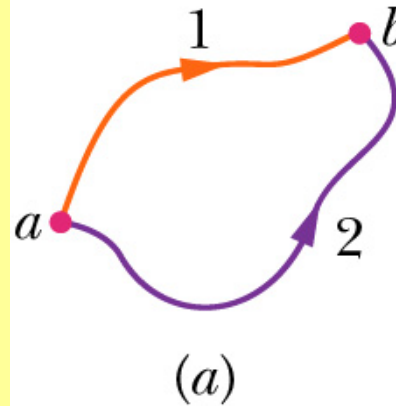
When $W_1 = -W_2$ is always true and the other type of energy is a potential energy, then the force is said to be a conservative force.

A force that is not conservative is called a non-conservative force. For example, the kinetic frictional force and drag force are non-conservative.



Path Independence of Conservative Forces

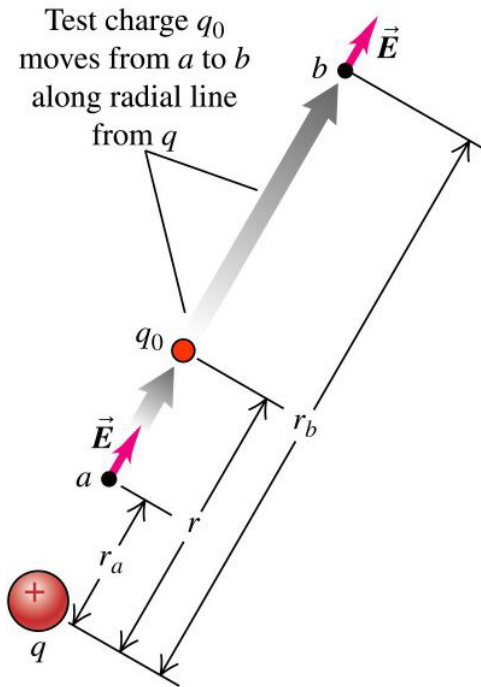
- (a) As a conservative force acts on it, a particle can move from point a to point b along either path 1 or path 2.
- (b) The particle moves in a round trip, from point a to point b along path 1 and then back to point a along path 2.



a) The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

b) The net work done by a conservative force on a particle moving around every closed path is zero.

Potential Energy of Two Point Charges



Suppose we have two charges q and q_0 that are separated by a distance r .

The force between them is given by Coulomb's Law

$$F = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2}$$

We now displace the charge q_0 along a radial line from point a to point b .

Since the force is not constant and is a function of the displacement, we need to integrate

$$W_{a \rightarrow b} = \int_{r_a}^{r_b} F_r dr = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} dr = -\frac{qq_0}{4\pi\epsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a} \right)$$

Electric Potential Energy

Let's rearrange the result and we can see that the right hand side represent the same function at points a and b that we are taking the difference.

$$W_{a \rightarrow b} = \frac{qq_0}{4\pi\epsilon_0} \left(\frac{1}{r_a} - \frac{1}{r_b} \right) = \frac{qq_0}{4\pi\epsilon_0 r_a} - \frac{qq_0}{4\pi\epsilon_0 r_b}$$

We define this function to be the potential energy

$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r}$$

$$W_{a \rightarrow b} = U(r_a) - U(r_b)$$

$$\Delta U = U(r_b) - U(r_a) = -W_{a \rightarrow b}$$

The usual convention is when $r = \infty$ $U(r) = 0$

The potential energy is negative when the charges are of opposite sign (attractive force). This means that the system did the work to bring the electron at infinity next to the electron that has the opposite charge.

When an electrostatic force acts between two or more charged particles within a system of particles, we can assign an **electric potential energy** U to the system. If the system changes its configuration from an initial state i to a different final state f , the electrostatic force does work W on the particles. If the resulting change is ΔU , then

$$\Delta U = U_f - U_i = -W.$$

As with other conservative forces, the work done by the electrostatic force is *path independent*.

Usually the reference configuration of a system of charged particles is taken to be that in which the particles are all infinitely separated from one another. The corresponding reference potential energy is usually set be zero $U_i = 0$. Therefore,

$$U_f = U = -W_\infty$$

Potential energy equals to the negative of the work done by the system required to bring the particle from infinity to f .

Potential Energy *increases* if the particle moves in the direction opposite to the force on it and *decreases* if the particle moves in the same direction as the force on it.

24.3 Electric Potential:

Electric potential V (or simply the *potential*) is the potential energy per unit charge at a point. This is a scalar quantity that is a function of the location in an electric field.

Thus,

$$V = \frac{\text{Electric Potential Energy}}{\text{Unit Charge}} = \frac{U}{q}$$

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}.$$

$$= -\frac{W}{q} \quad (\text{potential difference defined}).$$

If we set $U_i = 0$ at infinity as our reference potential energy, then the electric potential V must also be zero there. Therefore, the electric potential at any point in an electric field can be defined to be

$$V = -\frac{W_\infty}{q} \quad (\text{potential defined})$$

Here W_∞ is the work done by the electric field on a charged particle as that particle moves in from infinity to point f .

The SI unit for electric potential is the joule per coulomb. This combination is called the **volt (abbreviated V)**. $1 \text{ volt} = 1 \text{ joule per coulomb}.$



This unit of volt allows us to adopt a more conventional unit for the electric field, E , which is expressed in Newton per Coulomb.

$$P.E. (J) = \text{Electric Potential (V)} \times \text{Charge (C)}$$

$$1 \text{ N/C} = \left(1 \frac{\text{N}}{\text{C}}\right) \left(\frac{1 \text{ V} \cdot \text{C}}{1 \text{ J}}\right) \left(\frac{1 \text{ J}}{1 \text{ N} \cdot \text{m}}\right) = 1 \text{ V/m.}$$

$$\text{Work (J)} = \text{Force (N)} \times \text{Displacement (m)}$$

We can now define an energy unit that is a convenient one for energy measurements in the atomic/subatomic domain: One *electron-volt (eV)* is the energy equal to the work required to move a single elementary charge e , such as that of the electron or the proton, through a potential difference of exactly one volt. The magnitude of this work is $q\Delta V$, and

$$1 \text{ eV} = e(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.60 \times 10^{-19} \text{ J.}$$

The net potential at a point due to a group of point charges can be found with the help of the superposition principle. First the individual potential resulting from each charge is considered at the given point. Then we sum the potentials.

For n charges, the net potential is

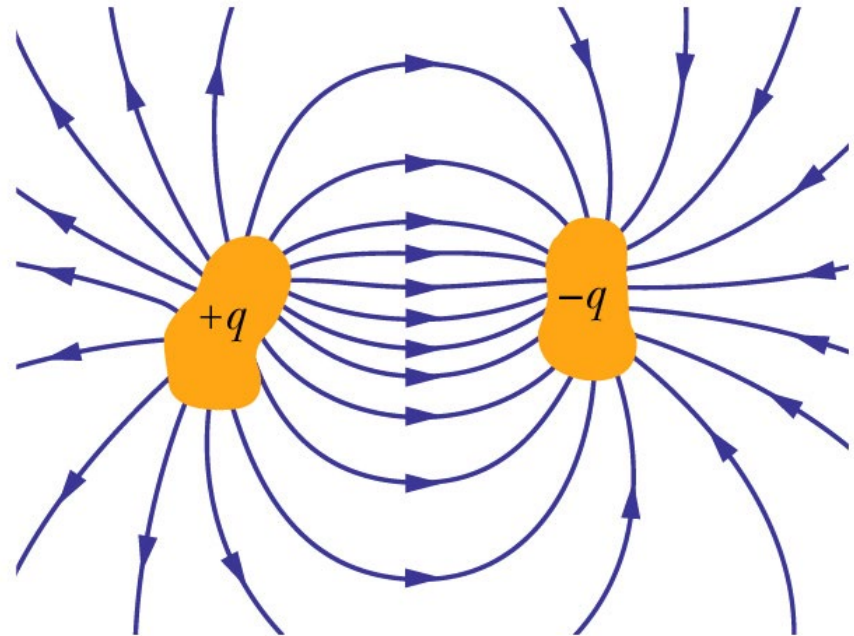
$$V = \sum_{i=1}^n V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i} \quad (n \text{ point charges}).$$

We see that the electric field points in the direction of *decreasing* potential

25.2: Capacitance:

A capacitor can be constructed by simply placing two conductors of **any** shape in isolation. No matter what their geometry, these conductors are usually called conductor ‘plates’.

Two conductors, isolated electrically from each other and from their surroundings, form a *capacitor*. When the capacitor is charged, the conductors, or *plates* as they are called, have equal but opposite charges of magnitude q .



25.2: Capacitance:

When a capacitor is charged, its plates have charges of equal magnitudes but opposite signs: $q+$ and $q-$. However, we refer to the charge of a capacitor as being q , the absolute value of these charges on the plates.

The charge q and the potential difference V for a capacitor are proportional to each other:

$$q = CV.$$

The proportionality constant C is called the **capacitance** of the capacitor. Its value depends only on the geometry of the plates and not on their charge or potential difference.

Capacitance is a measure of the capacitor's ability to store electric charge.

The SI unit is called the *farad* (F): **1 farad (1 F) = 1 coulomb per volt = 1 C/V.**

25.3: Calculating the Capacitance (parallel plate capacitor):

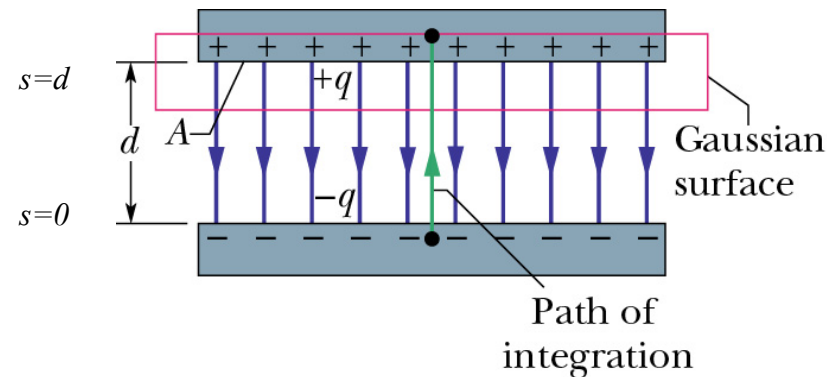
Gauss' Law

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \quad q = \epsilon_0 EA$$

Integrate E to get the potential

Eq. 25-25

$$V = V_f - V_i = - \int_{-}^{+} \vec{E} \cdot d\vec{s} = E \int_0^d ds = Ed$$



Integrate E from the -ve plate to the +ve plate.
The E field is in opposite direction with the integration path ds

Relates q and V

$$q = CV.$$



$$C = \frac{\epsilon_0 A}{d} \quad (\text{parallel-plate capacitor}).$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2.$$

For the air filled parallel plate capacitor, if the region between the plate is filled with dielectric with permittivity constant ϵ , then $C = \frac{\epsilon A}{d}$

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25.4: Capacitors in Parallel:

$$q_1 = C_1 V, \quad q_2 = C_2 V, \quad \text{and} \quad q_3 = C_3 V.$$



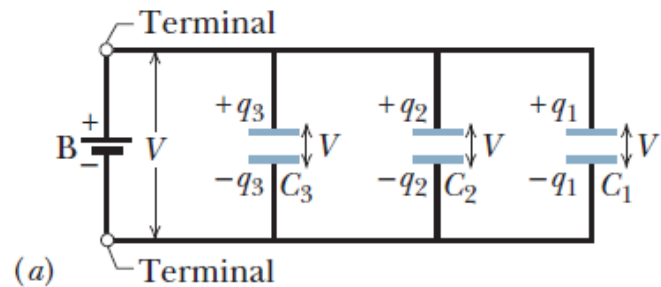
$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$



$$C_{\text{eq}} = \frac{q}{V} = C_1 + C_2 + C_3,$$



$$C_{\text{eq}} = \sum_{j=1}^n C_j \quad (n \text{ capacitors in parallel}).$$



Parallel capacitors and their equivalent have the same V ("par- V ").

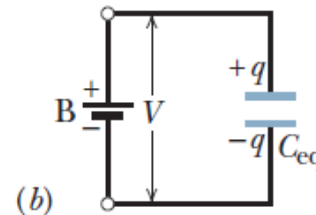


Fig. 25-8 (a) Three capacitors connected in parallel to battery B. The battery maintains potential difference V across its terminals and thus across *each* capacitor. (b) The equivalent capacitor, with capacitance C_{eq} , replaces the parallel combination.

25.4: Capacitors in Series:

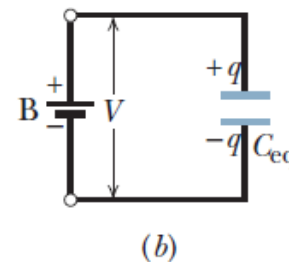
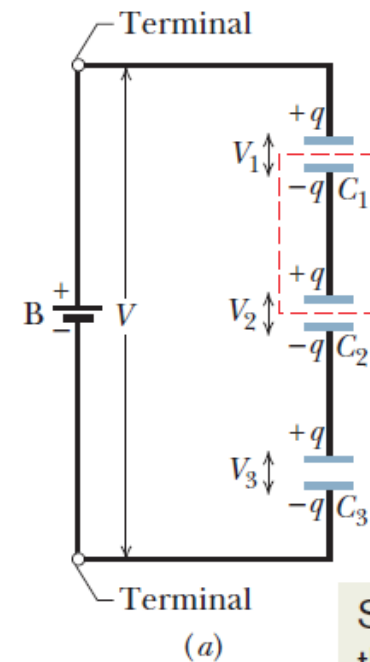
$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

$$V = V_1 + V_2 + V_3 = q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right).$$

$$C_{\text{eq}} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{\text{eq}}} = \sum_{j=1}^n \frac{1}{C_j} \quad (n \text{ capacitors in series}).$$



Series capacitors and their equivalent have the same q ("seri- q ").

Fig. 25-9 (a) Three capacitors connected in series to battery B . The battery maintains potential difference V between the top and bottom plates of the series combination. (b) The equivalent capacitor, with capacitance C_{eq} , replaces the series combination.

25.5: Energy Stored in an Electric Field:

Consider that we are charging up a capacitor by moving a small amount of charge dq from one plate to the other plate. The amount of work dW we do moving the charge dq across the plates with potential V' , is

$$dW = V' dq' = \frac{q'}{C} dq'.$$

The total W needed to move all the charge q is

$$W = \int dW = \frac{1}{C} \int_0^q q' dq' = \frac{q^2}{2C}.$$

This is the energy that is stored in the charged capacitor

$$U = \frac{q^2}{2C} \quad (\text{potential energy}).$$

$$U = \frac{1}{2} CV^2 \quad (\text{potential energy}).$$



The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

25.5: Energy Density:

We define the energy density u (J/m³) in the parallel plate capacitor by dividing the potential energy U with the volume of the capacitor.

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}.$$

$$u = \frac{1}{2} \epsilon_0 \left(\frac{V}{d} \right)^2. \quad \text{From slide 12}$$

$$V = Ed \quad \text{Eq. 25-25}$$

$$u = \frac{1}{2} \epsilon_0 E^2 \quad (\text{energy density}).$$

Note that the energy density u is a function of the electric field E only.

Dielectric

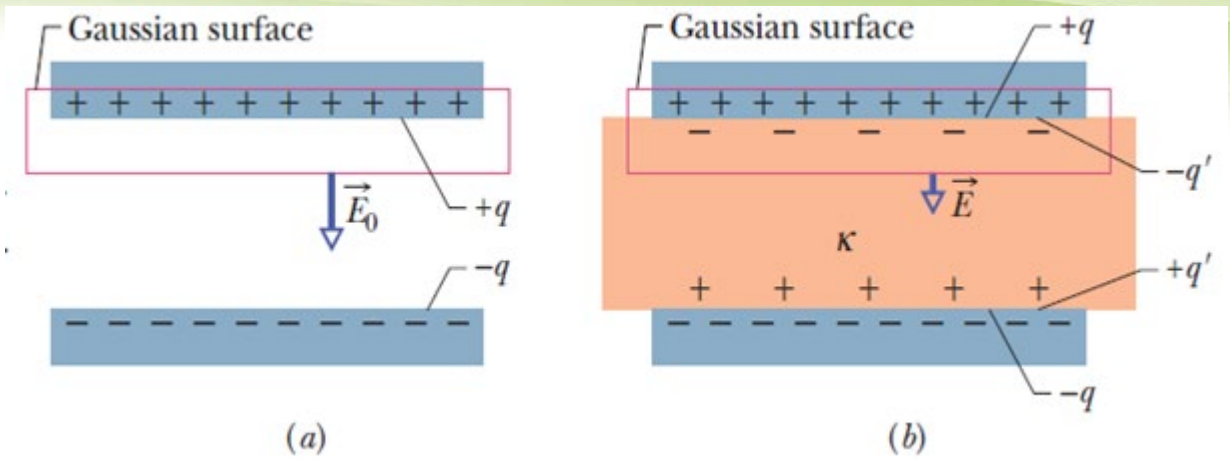
In a region completely filled by a dielectric material of dielectric constant κ , all electrostatic equations containing the permittivity constant ϵ_0 are to be modified by replacing ϵ_0 with $\kappa\epsilon_0$

e.g. Electric field due to a point charge inside a dielectric will be modified by

$$E = \frac{1}{4\pi\kappa\epsilon_0} \frac{q}{r^2}$$

For a **fixed** distribution of charges in the dielectric, the electric field will be **weaken** by a factor of κ compares to in free space (air).

25.8: Dielectrics and Gauss' Law:



$$\epsilon_0 \oiint \vec{E}_0 \cdot d\vec{A} = \epsilon_0 E_0 A = q$$



$$E_0 = \frac{q}{\epsilon_0 A}$$

$$\epsilon_0 \oiint \vec{E} \cdot d\vec{A} = \epsilon_0 E A = q - q'$$



$$E = \frac{q - q'}{\epsilon_0 A}$$

Let $q - q' = \frac{q}{\kappa}$. then $E = \frac{q}{\kappa \epsilon_0 A} = \frac{q}{\epsilon A}$ where $\epsilon = \kappa \epsilon_0$ ϵ dielectric permittivity

$\epsilon_0 \oiint \kappa \vec{E} \cdot d\vec{A} = \epsilon \oiint \vec{E} \cdot d\vec{A} = q$ (Gauss's Law with dielectric) Eq. 25-36

26.2: Electric Current:

Electric current i is defined as the time rate of flow of electric charges

$$i = \frac{dq}{dt}$$

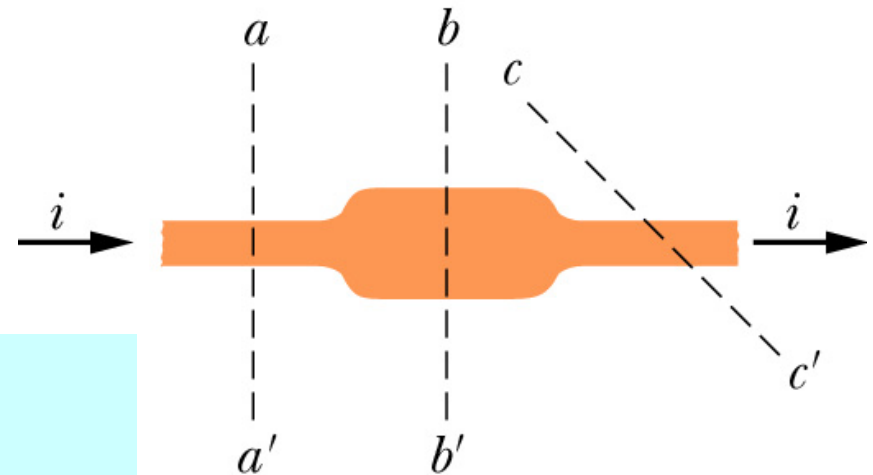
If $i(t)$ is known, we can calculate the total amount of charges that passes through a cross-section in a time interval from 0 to t :

$$q = \int dq = \int_0^t i dt$$

Unit for electric current:

1 ampere = 1 A = 1 coulomb per second = 1 C/s

The current i through the conductor has the same value at planes aa' , bb' , and cc' because the same amount of charges flow through these planes over the same time interval.



26.3: Current Density:

The magnitude of **current density**, $\mathbf{J}$, is equal to **the current per unit area** through any element of cross section. It provides the localized view of the current flow, the total current flow is.

$$i = \int \vec{J} \cdot d\vec{A}.$$

Current density, $\mathbf{J}$, is a vector quantity, if the current is uniform across the surface and parallel to $d\mathbf{A}$, then $\mathbf{J}$ is also uniform and parallel to $d\mathbf{A}$, and then

$$i = \int J dA = J \int dA = JA$$

$$J = \frac{i}{A},$$

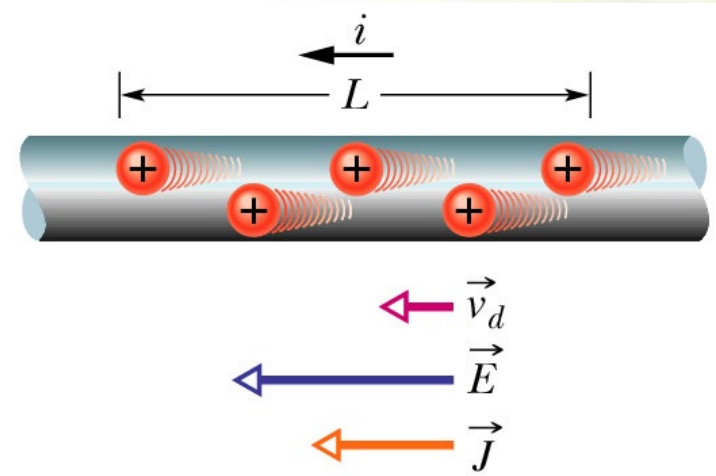
Here, A is the total area of the surface.

The SI unit for current density is the ampere per square meter (A/m^2).

26.3: Current Density, Drift Speed:

What really happens with an electric current flowing in a conducting wire?

- Thermal agitation from heat will cause movement of charges in a conductor even in the absence of current flow, however, this type of movements is random and without a prefer direction.
- When a potential difference is setup in the conductor so there is a current through it, these charges will migrate or drift in the direction of the electric field. Although there is still random motion within their movement, but there is a net *drift* with a **drift speed** v_d in the direction of the electric field.



Positive charge carriers drift at speed v_d in the direction of the applied electric field $\mathbf{E}$. By convention, the direction of the current density $\mathbf{J}$ and the sense of the current arrow are drawn in that same direction.

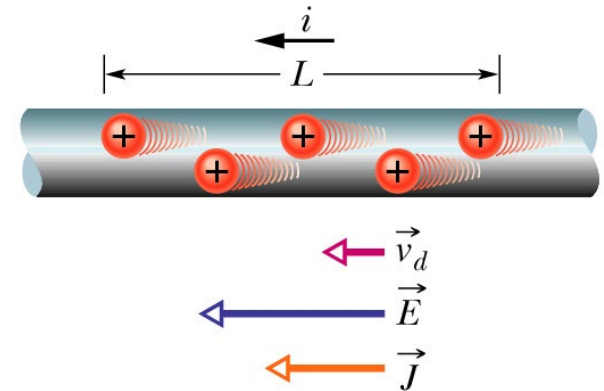
26.3: Current Density, Drift Speed:

Let us assume that these charge carriers all move with the same drift speed v_d and that the current density J is uniform across the wire's cross-sectional area A . The number of charge carriers in a length L of the wire is nAL , where n is the number of carriers per unit volume. The total charge of the carriers in the length L , each with charge e , is then

$$\Delta q = (nAL)e$$

Because the charge carriers all move along the wire at the same speed v_d , the time required to move all these charges over the length of wire L is

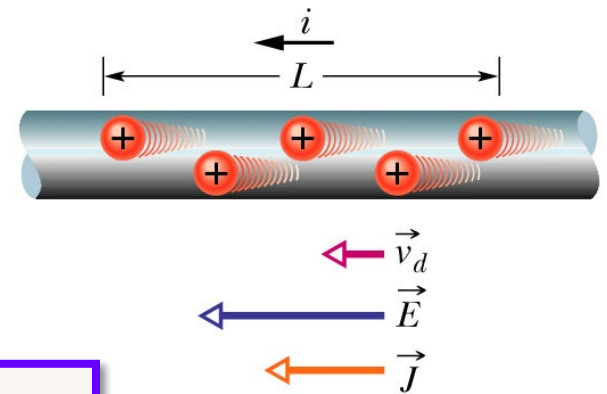
$$\Delta t = \frac{L}{v_d}$$



26.3: Current Density, Drift Speed:

We can relate the drift velocity to the electric current density from the definition of current flow

$$i = \frac{dq}{dt} = \frac{\Delta q}{\Delta t} = \frac{(nAL)e}{L/v_d} = nAev_d$$



$$\Rightarrow v_d = \frac{i}{nAe} = \frac{J}{ne} \Rightarrow \boxed{\vec{J} = (ne)\vec{v}_d}$$

$$\vec{J} \left(\frac{\text{A}}{\text{m}^2} \right) = n \left(\frac{\text{\# of carriers}}{\text{volume m}^3} \right) e(\text{C}) \vec{v}_d \left(\frac{\text{m}}{\text{s}} \right) = \left(\frac{\text{C}}{\text{s}} \cdot \frac{1}{\text{m}^2} \right) = \frac{\text{A}}{\text{m}^2}$$

26.4: Resistance and Resistivity:

How fast can the charge carriers move down a block of material depends on the geometry and the material properties.

Resistance is a measure used to determine the ease of flow of charges. Similar to capacitance, it's value is associated with the object.

We determine the **resistance** between any two points of a conductor by applying a potential difference V between those points and measuring the current i that results. The resistance R is then

$$1 \text{ ohm} = 1 \, \Omega = 1 \text{ volt per ampere} \\ = 1 \text{ V/A}.$$

The SI unit for resistance that follows from Eq. 26-8 is the volt per ampere. This has a special name, the **ohm** (symbol Ω):

$$R = \frac{V}{i} \quad (\text{definition of } R).$$

In a circuit diagram, we represent a resistor and a resistance with the symbol



26.4: Resistance and Resistivity:

The **resistivity**, ρ , of a resistor is defined as:

$$\rho = \frac{E}{J} \quad \Rightarrow \quad \vec{E} = \rho \vec{J}.$$

This is a **local** property.

The SI unit for ρ is Ωm .

The **conductivity** σ of a material is the reciprocal of its resistivity:

$$\sigma = \frac{1}{\rho} \quad \Rightarrow \quad \vec{J} = \sigma \vec{E}.$$

Similar to resistance R , which relates voltage V and current i , resistivity ρ is used to relate electric field $\mathbf{E}$ and current density $\mathbf{J}$.

Table 26-1

Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot \text{m}$)	Temperature Coefficient of Resistivity, α (K^{-1})
<i>Typical Metals</i>		
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin ^a	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
<i>Typical Semiconductors</i>		
Silicon, pure	2.5×10^3	-70×10^{-3}
Silicon, <i>n</i> -type ^b	8.7×10^{-4}	$\rho = \rho_0(1 + \alpha(T - T_0))$
Silicon, <i>p</i> -type ^c	2.8×10^{-3}	
<i>Typical Insulators</i>		
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

26.7: Power in Electric Circuits:

Charge dq moves through a decrease in potential of magnitude V , and thus its electric potential energy decreases in magnitude by the amount

$$dU = dq V = i dt V.$$

The power P associated with that transfer is the rate of transfer dU/dt , given by

$$P = iV \quad (\text{rate of electrical energy transfer}).$$

$$P = i^2 R \quad (\text{resistive dissipation})$$

$$P = \frac{V^2}{R} \quad (\text{resistive dissipation}).$$

The unit of power is the volt-ampere (V A).

$$\text{Pt} \quad 1 \text{ V} \cdot \text{A} = \left(1 \frac{\text{J}}{\text{C}}\right) \left(1 \frac{\text{C}}{\text{s}}\right) = 1 \frac{\text{J}}{\text{s}} = 1 \text{ W}.$$

The battery at the left supplies energy to the conduction electrons that form the current.

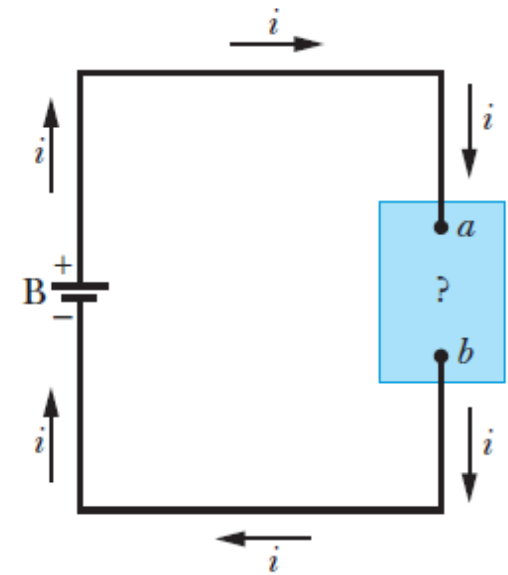


Fig. 26-13 A battery B sets up a current i in a circuit containing an unspecified conducting device.

27.4: Calculating the Current in a Single-Loop Circuit, Potential Method:

LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

In the figure, let us start at point a , whose potential is V_a , and mentally go clockwise around the circuit until we are back at a , keeping track of potential changes as we move.

Our starting point is at the low-potential terminal of the battery. Since the battery is ideal, the potential difference between its terminals is equal to $\mathcal{E}$.

As we go along the top wire to the top end of the resistor, there is no potential change because the wire has negligible resistance.

When we pass through the resistor, however, the potential decreases by iR .

We return to point a by moving along the bottom wire. At point a , the potential is again V_a . The initial potential, as modified for potential changes along the way, must be equal to our final potential; that is,

$$V_a + \mathcal{E} - iR = V_a \quad \longrightarrow \quad \mathcal{E} - iR = 0.$$

The battery drives current through the resistor, from high potential to low potential.

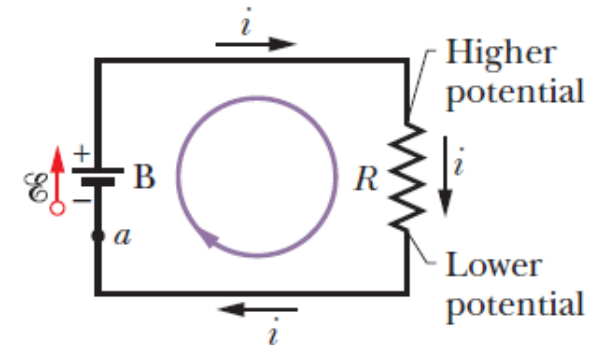


Fig. 27-3 A single-loop circuit in which a resistance R is connected across an ideal battery B with emf $\mathcal{E}$. The resulting current i is the same throughout the circuit.

27.4: Calculating the Current in a Single-Loop Circuit, Potential Method:

For circuits that are more complex than that of the previous figure, two basic rules are usually followed for finding potential differences as we move around a loop:



RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.



EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$; in the opposite direction it is $-\mathcal{E}$.

27.6: Potential between two points:

Going clockwise from a:

$$V_a + \mathcal{E} - ir = V_b,$$
$$V_b - V_a = \mathcal{E} - ir.$$

$$i = \frac{\mathcal{E}}{R + r}.$$

$$V_b - V_a = \mathcal{E} - \frac{\mathcal{E}}{R + r} r$$
$$= \frac{\mathcal{E}}{R + r} R.$$

$$V_b - V_a = \frac{12 \text{ V}}{4.0 \Omega + 2.0 \Omega} 4.0 \Omega = 8.0 \text{ V}.$$

The internal resistance reduces the potential difference between the terminals.

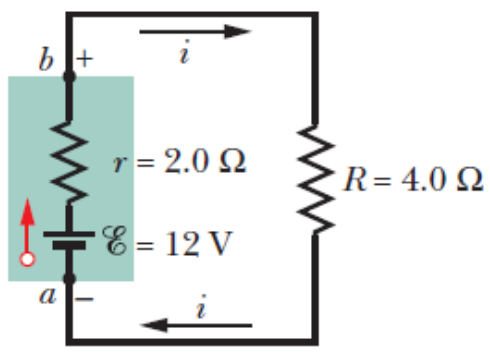


Fig. 27-6 Points *a* and *b*, which are at the terminals of a real battery, differ in potential.

Going counterclockwise from a:

$$V_a + iR = V_b$$
$$V_b - V_a = iR.$$

+

$$i = \frac{\mathcal{E}}{R + r}.$$

→

$$V_b - V_a = 8.0 \text{ V}.$$

To find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter.

27.7: Multi-loop Circuits:



JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

Consider junction d in the circuit. Incoming currents i_1 and i_3 , and it leaves via outgoing current i_2 . Since there is no variation in the charge at the junction, the total incoming current must equal

the total outgoing current: $i_1 + i_3 = i_2$.

This rule is often called *Kirchhoff's junction rule* (or *Kirchhoff's current law*).

For the left-hand loop,

$$\mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0.$$

For the right-hand loop,

$$-i_3 R_3 - i_2 R_2 - \mathcal{E}_2 = 0.$$

And for the entire loop,

$$\mathcal{E}_1 - i_1 R_1 - i_2 R_2 - \mathcal{E}_2 = 0.$$

The current into the junction must equal the current out (charge is conserved).

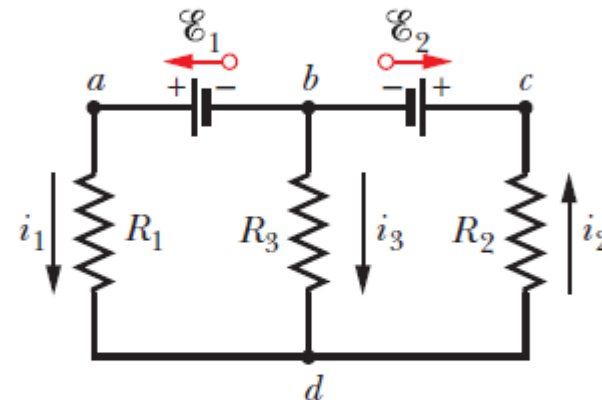


Fig. 27-9 A multiloop circuit consisting of three branches: left-hand branch bad , right-hand branch bcd , and central branch bd . The circuit also consists of three loops: left-hand loop $badb$, right-hand loop $bcdb$, and big loop $badcb$.

27.7: Multi-loop Circuits, Resistors in Parallel:

When a potential difference V is applied across resistances connected in parallel, the resistances all have that same potential difference V .

Resistances connected in parallel can be replaced with an equivalent resistance R_{eq} that has the same potential difference V and the same *total* current i as the actual resistances.

Fig. 27-10 (a) Three resistors connected in parallel across points a and b . (b) An equivalent circuit, with the three resistors replaced with their equivalent resistance R_{eq} .

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad \text{and} \quad i_3 = \frac{V}{R_3},$$

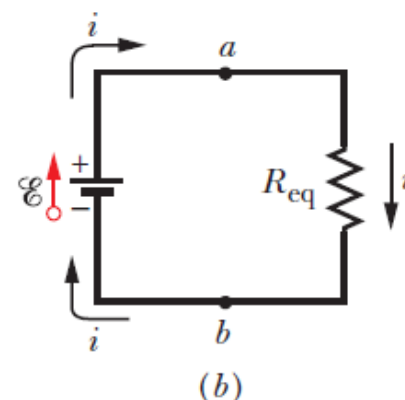
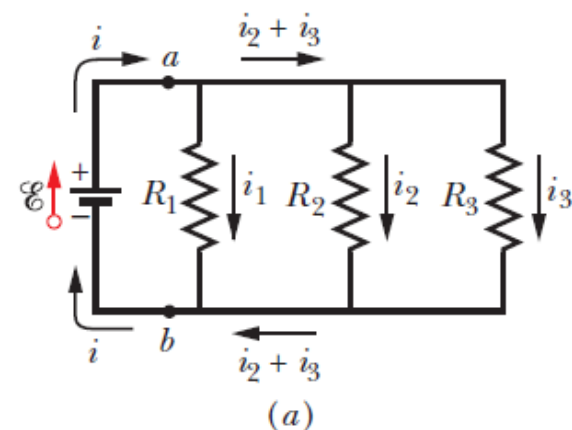
where V is the potential difference between a and b .

From the junction rule, $i = i_1 + i_2 + i_3 = V \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$.

$$\Rightarrow i = \frac{V}{R_{\text{eq}}} \quad \Rightarrow \quad \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

$$\Rightarrow \quad \frac{1}{R_{\text{eq}}} = \sum_{j=1}^n \frac{1}{R_j} \quad (n \text{ resistances in parallel}).$$

Parallel resistors and their equivalent have the same potential difference ("par-V").



27.9: RC Circuits, Charging a Capacitor:

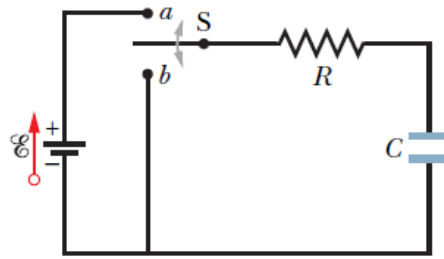
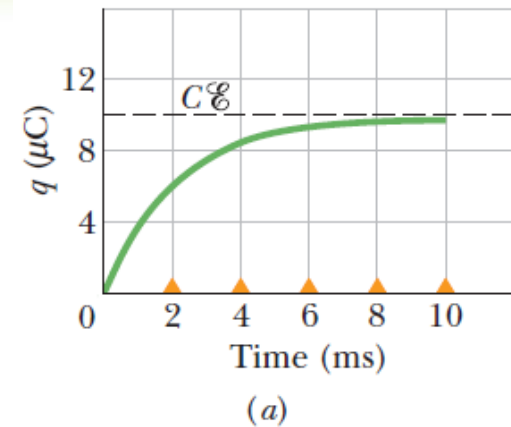


Fig. 27-15 When switch S is closed on a , the capacitor is *charged* through the resistor. When the switch is afterward closed on b , the capacitor *discharges* through the resistor.

The capacitor's charge grows as the resistor's current dies out.



It turns out that: $\mathcal{E} - iR - \frac{q}{C} = 0$.

We know that: $i = \frac{dq}{dt}$. $\Rightarrow R \frac{dq}{dt} + \frac{q}{C} = \mathcal{E}$ (charging equation).

$$q = C\mathcal{E}(1 - e^{-t/RC}) \quad (\text{charging a capacitor}).$$

$$V_C = \frac{q}{C} = \mathcal{E}(1 - e^{-t/RC}) \quad (\text{charging a capacitor}).$$

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

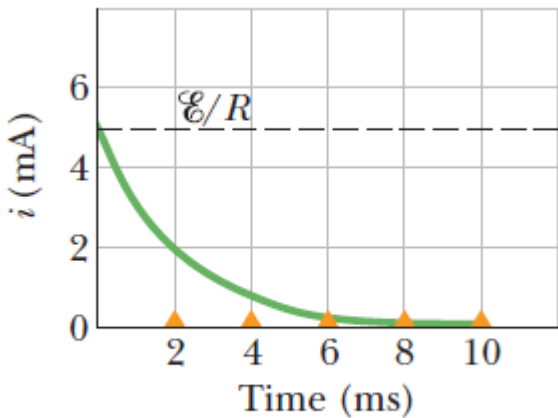
27.9: RC Circuits, Discharging a Capacitor:

Assume that the capacitor of the figure is fully charged to a potential V_0 equal to the emf of the battery $\mathcal{E}$.

At a new time $t = 0$, switch S is thrown from a to b so that the capacitor can discharge through resistance R .

$$R \frac{dq}{dt} + \frac{q}{C} = 0 \quad \text{(discharging equation).}$$

$$q = q_0 e^{-t/RC} \quad \text{(discharging a capacitor),}$$



(b)

Fig. 27-16 (b) This shows the decline of the charging current in the circuit. The curves are plotted for $R = 2000 \, \Omega$, $C = 1 \, \mu\text{F}$, and $\mathcal{E} = 10 \, \text{V}$; the small triangles represent successive intervals of one time constant τ .

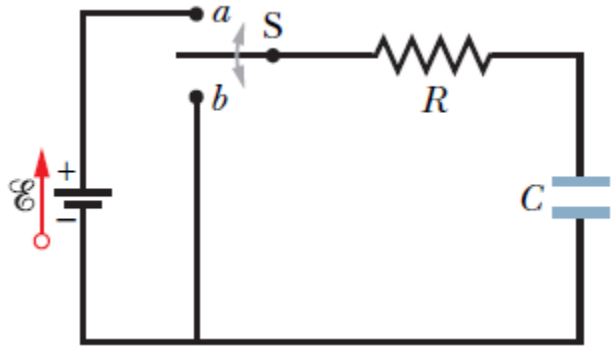


Fig. 27-15 When switch S is closed on a , the capacitor is *charged* through the resistor. When the switch is afterward closed on b , the capacitor *discharges* through the resistor.

Source of the Magnetic Field

- If there are no magnetic charges, then what is the source of the magnetic field?
 - The answer is electric charges in motion.
 - One way that magnetic fields are produced is to use moving electrically charged particles, such as a current in a wire, to make an **electromagnet**. The current produces a magnetic field that is utilizable.
 - Another way to produce a magnetic field is by means of elementary particles such as electrons, because these particles have an *intrinsic magnetic field* around them.

The definition of Magnetic Field $\mathbf{B}$

- Experimentally we find that when a charged particle moving with velocity $\mathbf{v}$ in a magnetic field, it encounters a force $\mathbf{F}_B$ due to the field that acts on the particle.

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

Experimental result

- Since $\mathbf{F}_B$, $\mathbf{v}$ can be measured, this proves to be a convenient way to define and measure $\mathbf{B}$.
- Here q is the charge of the particle, $\mathbf{v}$ is its velocity, and $\mathbf{B}$ the magnetic field in the region. The magnitude of this force is then:

$$F_B = |q|vB \sin \phi$$

- Here ϕ is the angle between vectors $\mathbf{v}$ and $\mathbf{B}$.

28.3: Finding the Magnetic Force on a Particle:

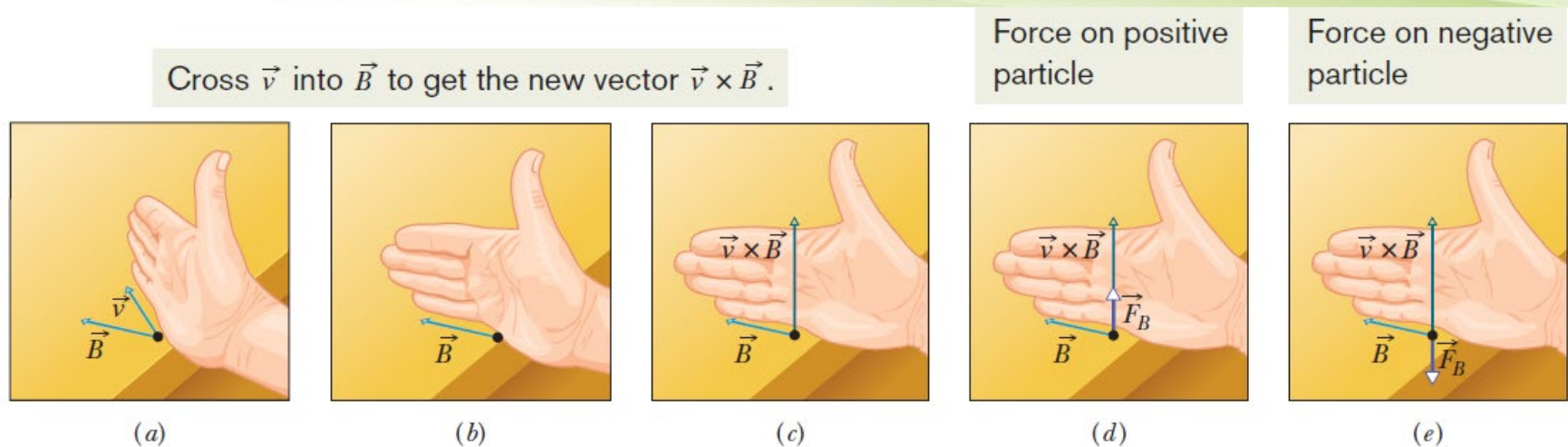


Fig. 28-2 (a) – (c) The right-hand rule (in which $\vec{v}$ is swept into $\vec{B}$ through the smaller angle ϕ between them) gives the direction of $\vec{v} \times \vec{B}$ as the direction of the thumb. (d) If q is positive, then the direction of $\vec{F}_B = q\vec{v} \times \vec{B}$ is in the direction of $\vec{v} \times \vec{B}$. (e) If q is negative, then the direction of $\vec{F}_B$ is opposite that of $\vec{v} \times \vec{B}$.



The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

28.4: Crossed Fields, Discovery of an Electron:

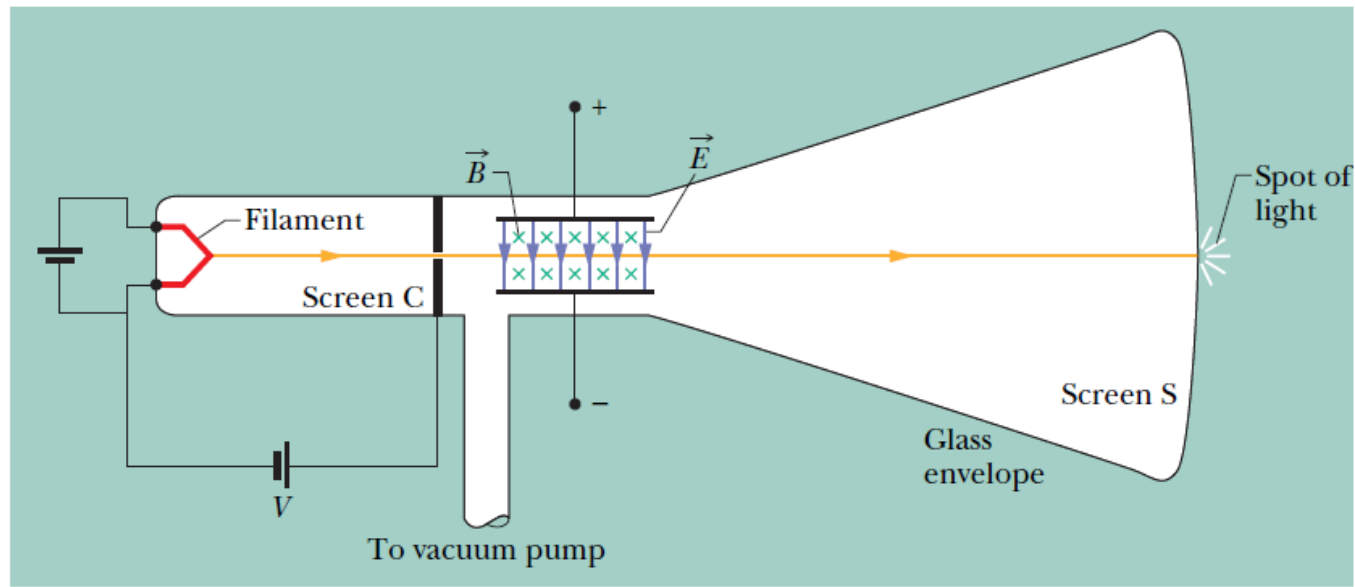


Fig. 28-7 A modern version of J.J. Thomson's apparatus for measuring the ratio of mass to charge for the electron. An electric field $\vec{E}$ is established by connecting a battery across the deflecting-plate terminals. The magnetic field $\vec{B}$ is set up by means of a current in a system of coils. The procedure first determines the velocity of the particle by adjusting the $\vec{E}$ and $\vec{B}$ field ratio, then the B field is turned off and the e/m ratio can then be determined from the amount of deflection measured on the screen.

When the two fields in Fig. 28-7 are adjusted so that the two deflecting forces acting on the charged particle cancel, we have

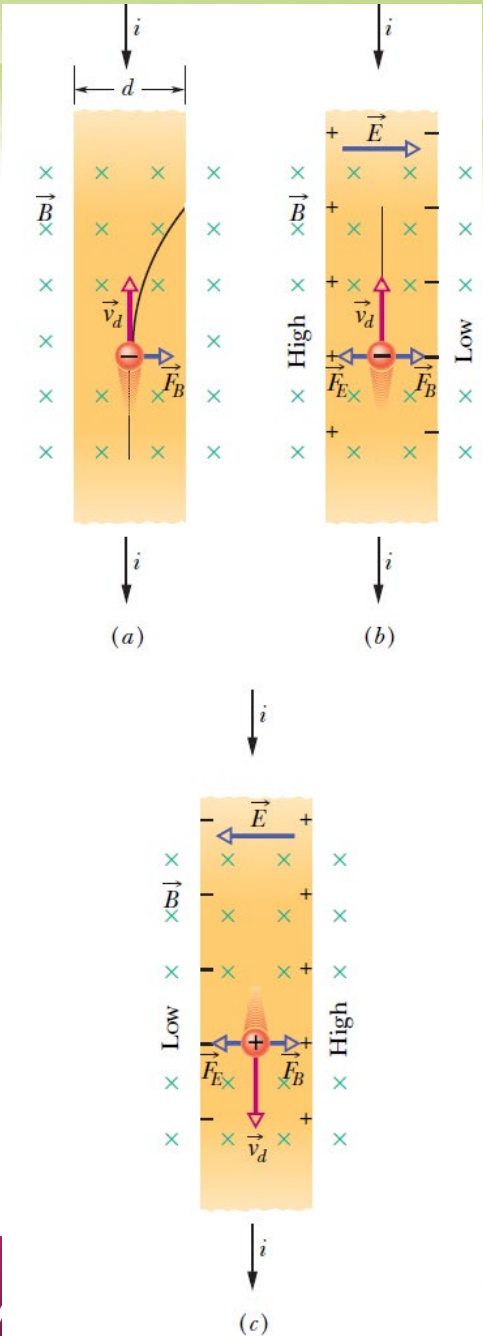
$$|q|E = |q|vB \sin(90^\circ) = |q|vB \quad \Rightarrow \quad v = \frac{E}{B}.$$

Thus, the crossed fields allow us to measure the speed of the charged particles passing through them.

28.5: Crossed Fields, The Hall Effect:

Fig. 28-8 A strip of copper carrying a current i is immersed in a magnetic field .

- (a) The situation immediately after the magnetic field is turned on. The curved path that will then be taken by *an electron* is shown.
- (b) The situation at equilibrium, which quickly follows. Note that negative charges pile up on the right side of the strip, leaving uncompensated positive charges on the left. Thus, the left side is at a higher potential than the right side.
- (c) For the same current direction, if the charge carriers were positively charged, they would pile up on the right side, and the right side would be at the higher potential.



28.5: Crossed Fields, The Hall Effect:

A Hall potential difference V is associated with the electric field across strip width d , and the magnitude of that potential difference is $V = Ed$. When the electric and magnetic forces are in balance (Fig. 28-8b),

$$F_E = F_B \Rightarrow eE = ev_d B$$

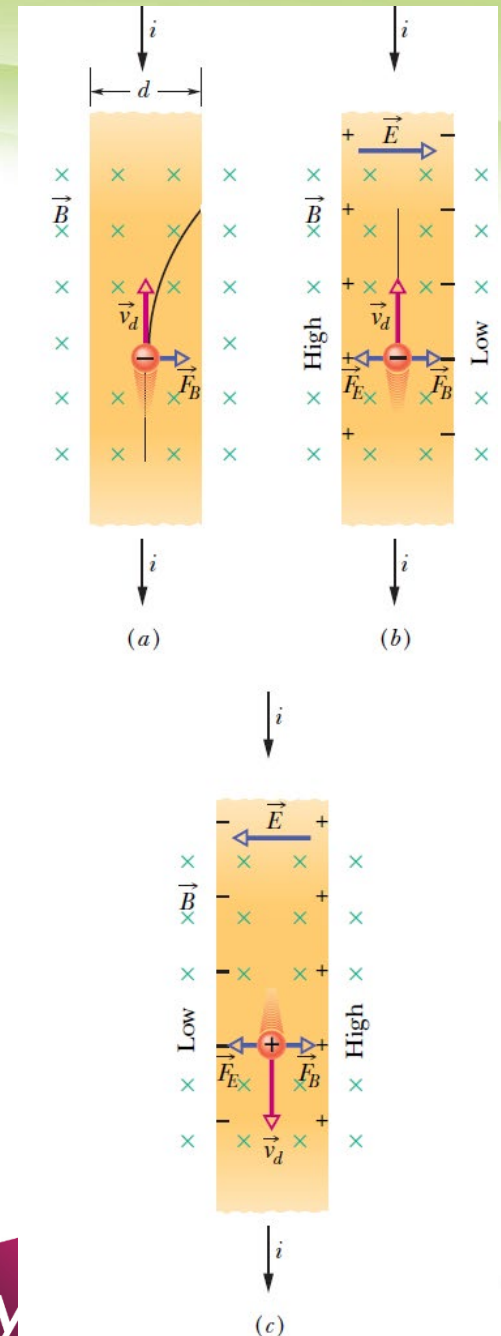
where v_d is the drift speed. But,

$$v_d = \frac{J}{ne} = \frac{i}{neA}$$

Where J is the current density, A the cross-sectional area, e the electronic charge, and n the number of charges per unit volume, or commonly known as the carrier density.

Therefore,
$$n = \frac{Bi}{Vle},$$

Here, $l = (A/d)$, the thickness of the strip.



28.6: A Circulating Charged Particle:

Electrons circulating in a chamber containing gas at low **pressure** (their path is the glowing circle). A uniform **magnetic field** $\mathbf{B}$, pointing directly out of the plane of the page, fills the chamber. Note the radially directed **magnetic** force $\mathbf{F}_B$; for circular motion to occur, $\mathbf{F}_B$ *must* point toward the centre of the circle. Use the right-hand rule for cross products to confirm that $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ gives $\mathbf{F}_B$ the proper direction. (Don't forget the sign of q .)

The magnetic force continuously deflects the particle, and since $\mathbf{B}$ and $\mathbf{v}$ are always perpendicular to each other, this deflection causes the particle to follow **a circular path**.

The magnetic force acting on the particle has a magnitude of $|q|vB$.

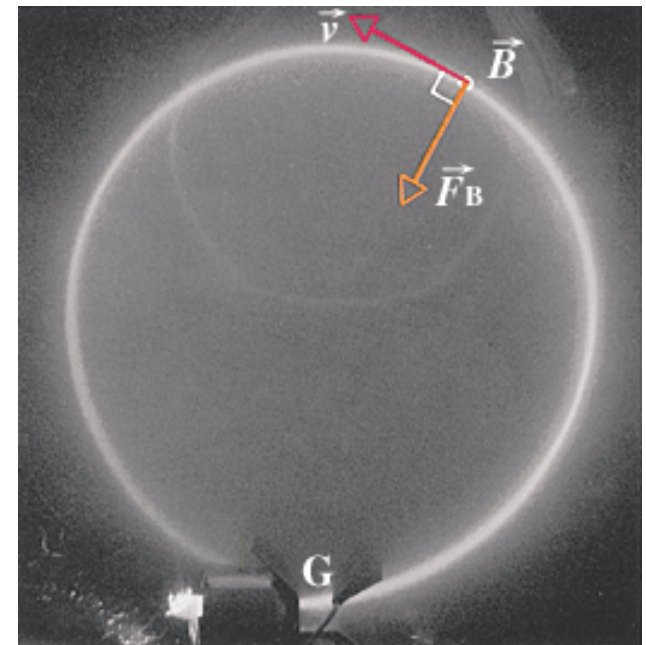
For uniform circular motion $F = m \frac{v^2}{r}$, $\Rightarrow$ $|q|vB = \frac{mv^2}{r}$.

$$r = \frac{mv}{|q|B} \quad (\text{radius}).$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \quad (\text{period}). \quad \text{Eq. 28-17}$$

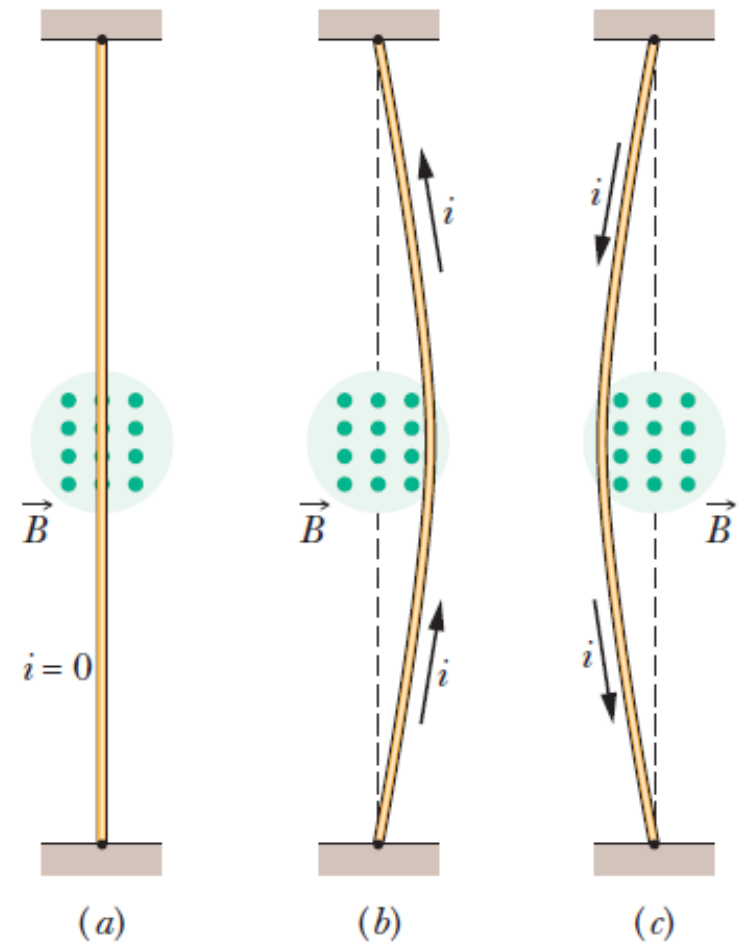
$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \quad (\text{frequency}).$$

$$\omega = 2\pi f = \frac{|q|B}{m} \quad (\text{angular frequency}).$$



28.8: Magnetic Force on a Current-Carrying Wire:

Fig. 28-14 A flexible wire passes between the pole faces of a magnet (only the farther pole face is shown). (a) Without current in the wire, the wire is straight. (b) With upward current, the wire is deflected rightward. (c) With downward current, the deflection is leftward. The connections for getting the current into the wire at one end and out of it at the other end are not shown.



28.8: Magnetic Force on a Current-Carrying Wire:

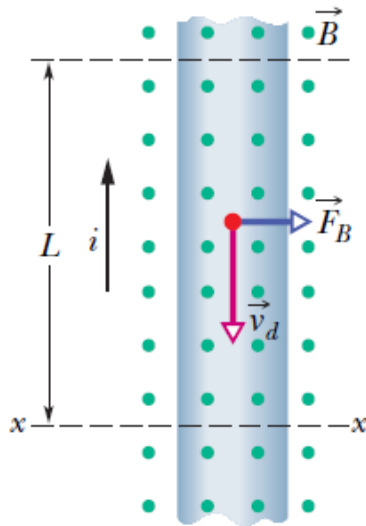


Fig. 28-15 A close-up view of a section of the wire of Fig. 28-14*b*. The current direction is upward, which means that electrons drift downward. A magnetic field that emerges from the plane of the page causes the electrons and the wire to be deflected to the right.

Consider a length L of the wire in the figure. All the conduction electrons in this section of wire will drift past plane xx in a time $t = L/v_d$.

Thus, in that time a charge will pass through that plane that is given by

$$q = it = i \frac{L}{v_d}$$

$$F_B = qv_d B \sin \phi = \frac{iL}{v_d} v_d B \sin 90^\circ$$

$$F_B = iLB.$$

$$\vec{F}_B = i\vec{L} \times \vec{B} \quad (\text{force on a current}).$$

Here $\vec{L}$ is a length vector that has magnitude L and is directed along the wire segment in the direction of the (conventional) current.

If a wire is not straight or the field is not uniform, we can imagine the wire broken up into small straight segments. The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write $d\vec{F}_B = i d\vec{L} \times \vec{B}$, and we can find the resultant force on any given arrangement of currents by integrating Eq. 28-28 over that arrangement.

28.9: Torque on a Current Loop:

For N loops, when $A=ab$, the area of the loop, the total torque is:

$$\tau = N\tau' = NiabB \sin \theta = (NiA)B \sin \theta,$$

Magnetic dipole moment of a coil with N loop:

$$\mu = NiA$$

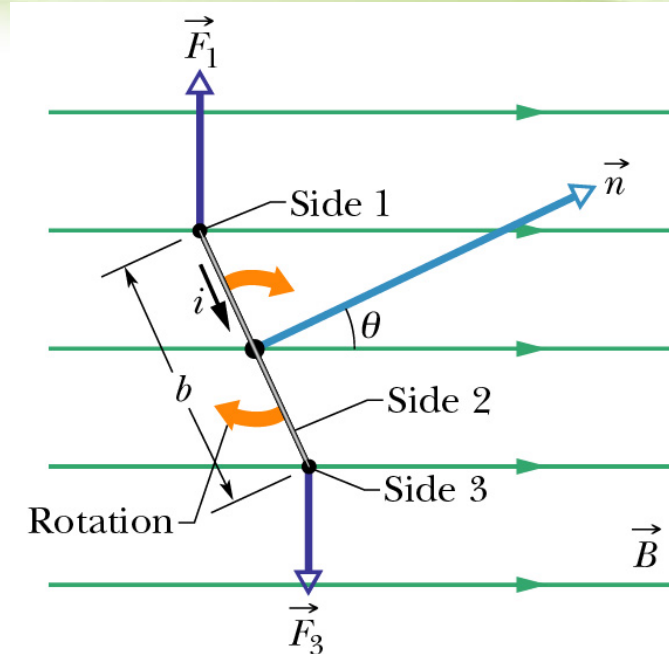
The torque on a magnetic dipole moment due to a magnetic field:

$$\vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow \tau = \mu B \sin \theta$$

Similar to $\vec{\tau} = \vec{p} \times \vec{E}$ for an electric dipole

Potential Energy of the Magnetic Dipole in a Magnetic Field

$$U = -\vec{\mu} \cdot \vec{B}$$



a = length of sides 1 and 3

b = length of sides 2 and 4

$A = a \times b$ = area enclosed by the coil

29.2: Calculating the Magnetic Field due to a Current Biot-Savart Law

The Biot-Savart Law in electromagnetism describes the magnetic field generated by an electric current.

A small current element as shown will generate a circulating magnetic field around the element. The magnitude of the field is proportional to the current and the length of the element. Similar to the electric field it is also inversely proportional to the square of the distance.

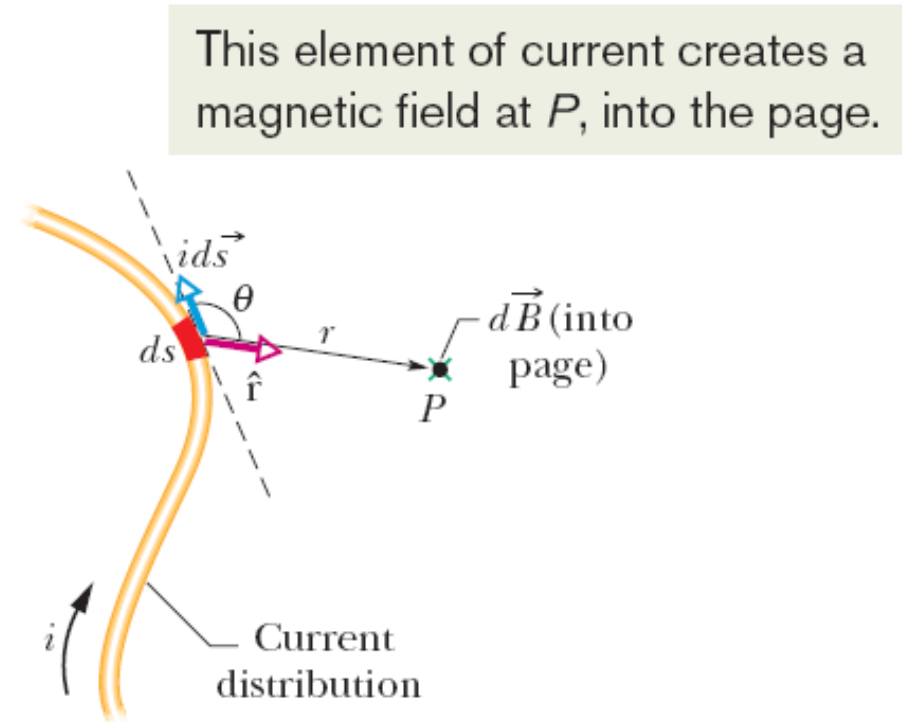


Fig. 29-1 A current-length element $i d\vec{s}$ produces a differential magnetic field $d\vec{B}$ at point P . The green $\times$ (the tail of an arrow) at the dot for point P indicates that $d\vec{B}$ is directed *into* the page there.

29.2: Calculating the Magnetic Field due to a Current

Biot-Savart Law

The magnitude of the field $d\mathbf{B}$ produced at point P at distance r by a current length element $i d\mathbf{s}$ turns out to be

$$dB = \frac{\mu_0}{4\pi} \frac{i ds \sin \theta}{r^2}$$

*This law is
experimentally
deduced*

where θ is the angle between the directions of i and $\hat{r}$, a unit vector that points from ds toward P . Symbol μ_0 is a constant, called the permeability constant, whose value is

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \approx 1.26 \times 10^{-6} \text{ T} \cdot \text{m/A}.$$

Therefore, in vector form

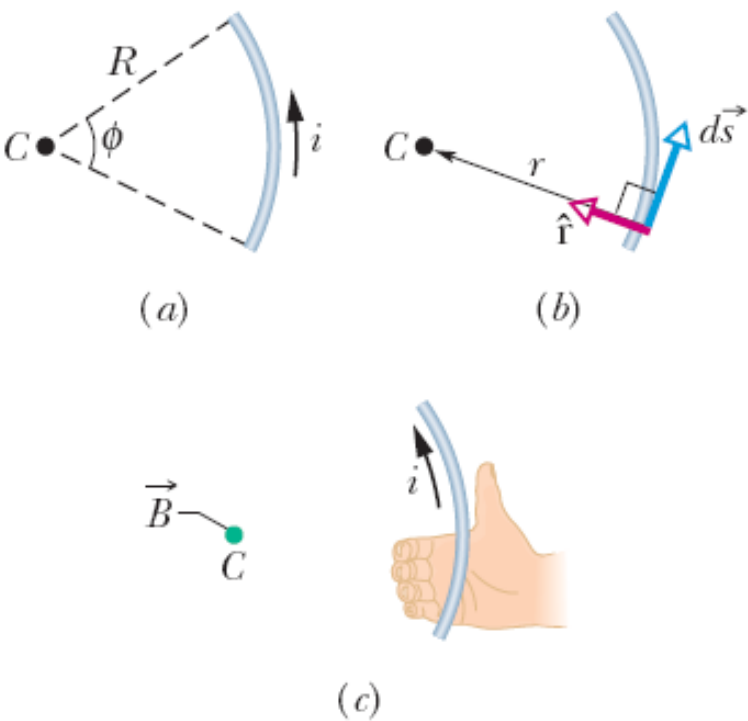
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \hat{r}}{r^2} \quad (\text{Biot-Savart Law})$$



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29.2: Magnetic Field due to a Current in a Circular Arc of Wire:



The right-hand rule reveals the field's direction at the center.

B field at the center of the circle due to the current element *ids* of the circular arc of current:

$$dB = \frac{\mu_0}{4\pi} \frac{i ds \sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi} \frac{i ds}{R^2}$$

$\vec{ds} \perp \hat{r}$

$$B = \int dB = \int_0^\phi \frac{\mu_0}{4\pi} \frac{i R d\phi}{R^2} = \frac{\mu_0 i}{4\pi R} \int_0^\phi d\phi$$

$ds = R d\phi$

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

B due to the circular current arc of angle ϕ

$$B = \frac{\mu_0 i (2\pi)}{4\pi R} = \frac{\mu_0 i}{2R}$$

B at center of a full circle of current.

29.3: Force Between Two Parallel Wires:

B field due to wire *a*
at a distance *d*.

$$B_a = \frac{\mu_0 i_a}{2\pi d}.$$

Force on wire *b* due to
the **B** field of *a*.

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a,$$

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}.$$

The field due to *a*
at the position of *b*
creates a force on *b*.

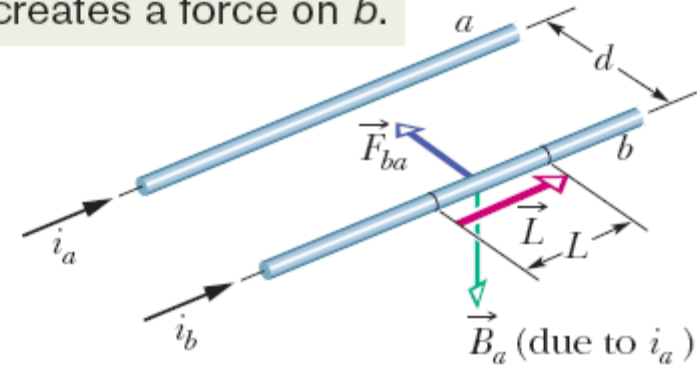


Fig. 29-9 Two parallel wires carrying currents in the same direction attract each other. $\vec{B}_a$ is the magnetic field at wire *b* produced by the current in wire *a*. $\vec{F}_{ba}$ is the resulting force acting on wire *b* because it carries current in $\vec{B}_a$.



To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.



Parallel currents attract each other, and antiparallel currents repel each other.

29.4: Ampere's Law:

Consider the magnetic field at point P from the long straight wire carrying current i .

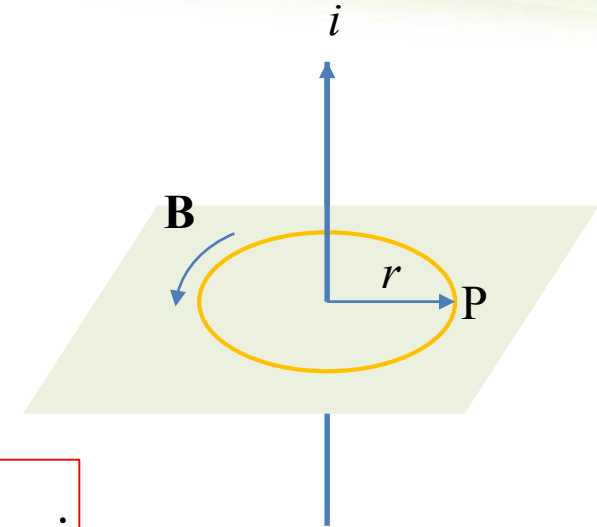
$$B = \frac{\mu_0 i}{2\pi r}$$

Since $\mathbf{B}$ is tangent to the circle the dot product between $\mathbf{B}$ and $d\mathbf{s}$ is simply Bds .

$$\vec{B} \cdot d\vec{s} = Bds$$

If we integrate the dot product over the complete circle, then

$$\oint \vec{B} \cdot d\vec{s} = \frac{\mu_0 i}{2\pi r} \times 2\pi r = \mu_0 i$$



The line integral of the $\mathbf{B}$ field around any closed loop is proportional to the electric current passing through the area enclosed by the loop.

Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

29.4: Ampere's Law:

Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

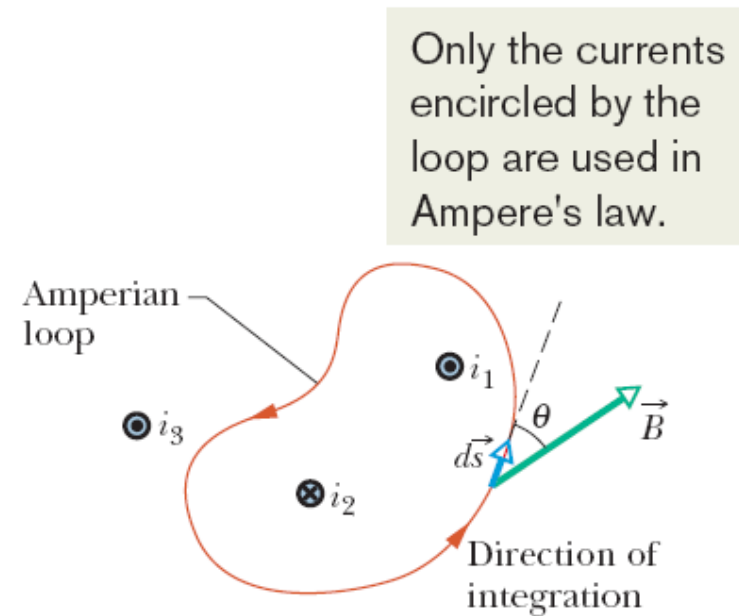


Fig. 29-11 Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

29.4: Ampere's Law, Magnetic Field Outside a Long Straight Wire Carrying Current:

All of the current is encircled and thus all is used in Ampere's law.

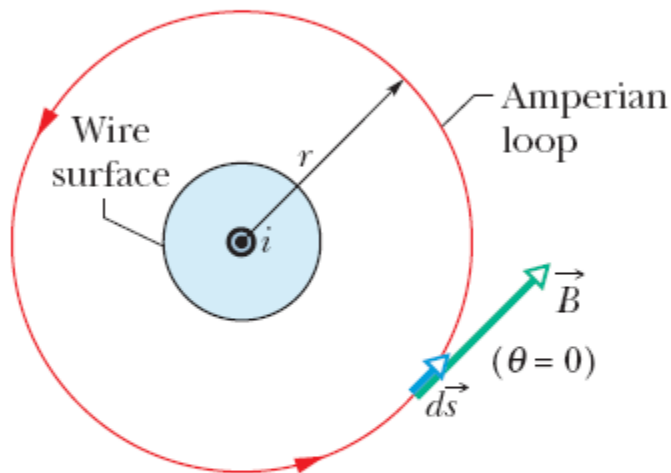


Fig. 29-13 Using Ampere's law to find the magnetic field that a current i produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos \theta ds = B \oint ds = B(2\pi r).$$

$$B(2\pi r) = \mu_0 i$$
$$B = \frac{\mu_0 i}{2\pi r} \quad (\text{outside straight wire}).$$

29.5: Solenoids and Toroids:

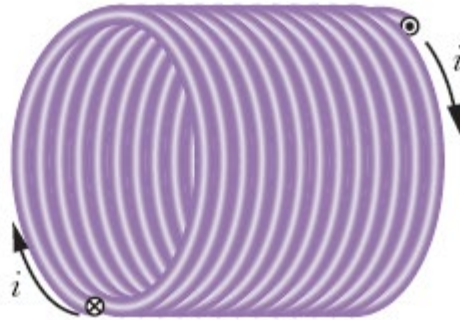


Fig. 29-16 A solenoid carrying current i .

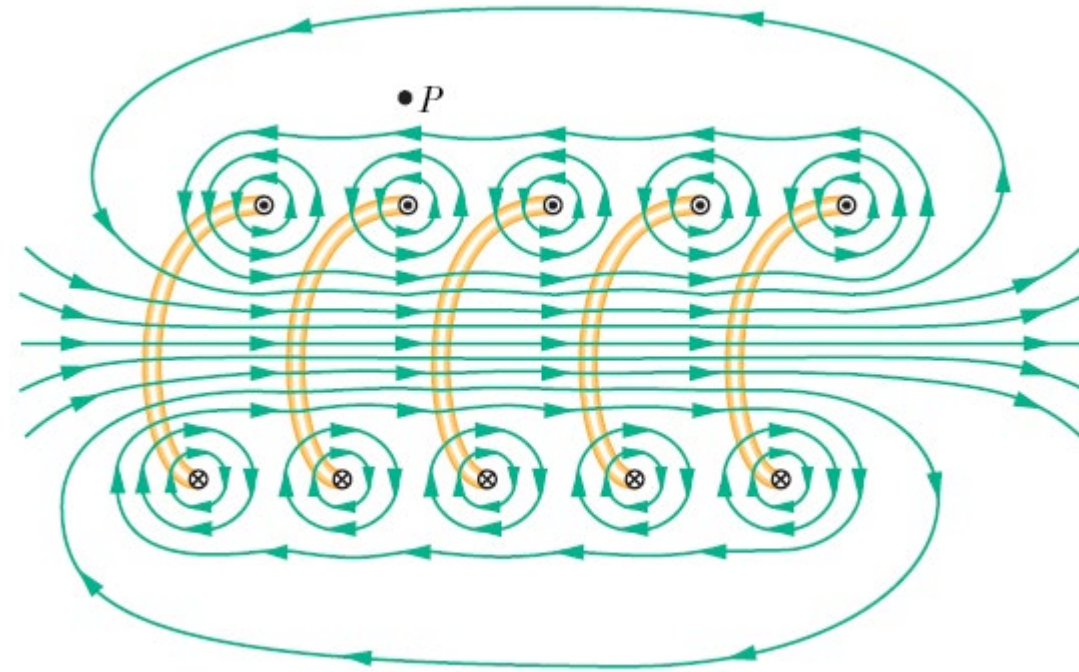
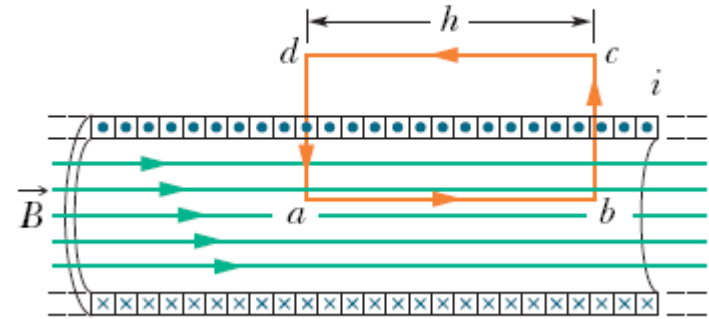


Fig. 29-17 A vertical cross section through the central axis of a “stretched-out” solenoid. The back portions of five turns are shown, as are the magnetic field lines due to a current through the solenoid. Each turn produces circular magnetic field lines near itself. Near the solenoid’s axis, the field lines combine into a net magnetic field that is directed along the axis. The closely spaced field lines there indicate a strong magnetic field. Outside the solenoid the field lines are widely spaced; the field there is very weak.

29.5: Solenoids:

Fig. 29-19 Application of Ampere's law to a section of a long ideal solenoid carrying a current i . The Amperian loop is the rectangle $abcd$.



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}},$$

$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}.$$

$i_{\text{enc}} = i(nh)$. Here n be the number of turns per unit length of the solenoid

$$Bh = \mu_0 in h$$

$$B = \mu_0 in \quad (\text{ideal solenoid}).$$

29.6: A Current Carrying Coil as a Magnetic Dipole:

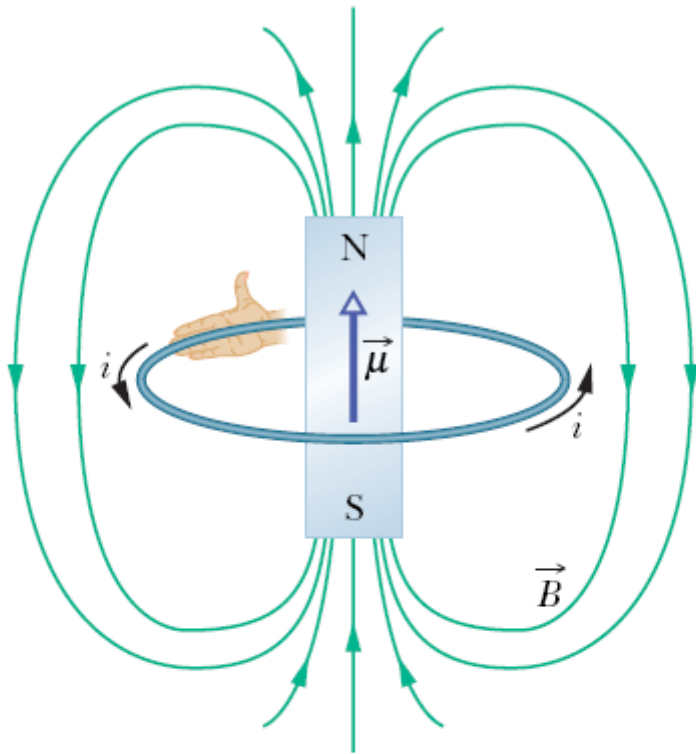


Fig. 29-21 A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles. The magnetic dipole moment $\vec{\mu}$ of the loop, its direction given by a curled–straight right-hand rule, points from the south pole to the north pole, in the direction of the field $\vec{B}$ within the loop.

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}},$$

$$B(z) \approx \frac{\mu_0 i R^2}{2z^3}. \quad z \gg R$$

$$B(z) = \frac{\mu_0}{2\pi} \frac{NiA}{z^3}.$$

*Magnetic Dipole
Moment of N
current loops:*

$$\mu = NiA = Ni\pi R^2$$

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$$

**$\vec{B}$ due to the
magnetic dipole**

$$\vec{\mu} = Ni\vec{A}$$



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29.7: Summary:

- Biot-Savart Law:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \hat{r}}{r^2}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$

- B** field of a long straight wire

$$B = \frac{\mu_0 i}{2\pi R}$$

- B** field of a circular arc

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

B due to the circular current arc of angle ϕ

$$B = \frac{\mu_0 i (2\pi)}{4\pi R} = \frac{\mu_0 i}{2R}$$

B at center of a full circle of current.

- Force between parallel currents

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}$$

- Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

- B** field of solenoid & Toroid

$$B = n\mu_0 i$$

solenoid

$$B = \frac{N\mu_0 i}{2\pi} \frac{1}{r}$$

toroid

- Magnetic Dipole

$$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$$



Faraday's Law of Induction:



The magnitude of the emf $\mathcal{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

The *magnetic flux* is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \quad (\text{magnetic flux through area } A).$$

If the loop lies in a plane and the magnetic field is perpendicular to the plane of the loop, and if the magnetic field is constant, then

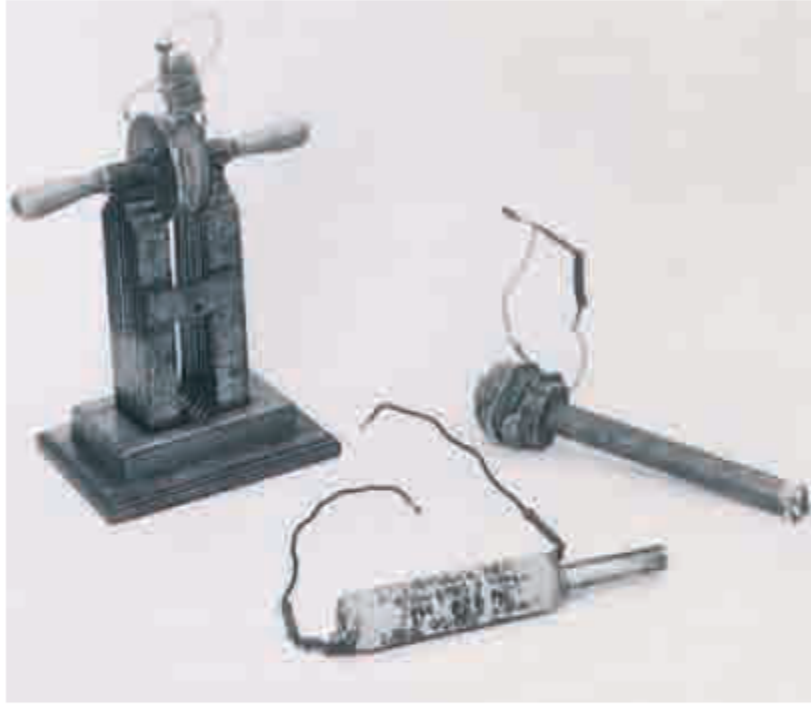
$$\Phi_B = BA \quad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform}).$$

The SI unit for magnetic flux is the tesla–square meter, which is called the *weber* (abbreviated *Wb*):

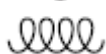
$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2.$$

$$\mathcal{E} = - \frac{d\Phi_B}{dt} \quad (\text{Faraday's law}),$$

Inductors and Inductance:



The crude inductors with which Michael Faraday discovered the law of induction. In those days amenities such as insulated wire were not commercially available. It is said that Faraday insulated his wires by wrapping them with strips cut from one of his wife's petticoats. (*The Royal Institution/Bridgeman Art Library/NY*)

An **inductor** (symbol ) can be used to produce a desired magnetic field.

If we establish a current i in the windings (turns) of the solenoid which can be treated as our inductor, the current produces a magnetic flux Φ_B through the central region of the inductor.

The inductance of the inductor is then

$$L = \frac{N\Phi_B}{i} \quad (\text{inductance defined})$$

The SI unit of inductance is the tesla-square meter per ampere ($\text{T m}^2/\text{A}$). We call this the **henry (H)**, after American physicist Joseph Henry.

Inductance of a Solenoid:

Consider a long solenoid of cross-sectional area A , with number of turns N , and of length l . The flux is $N\Phi_B = (nl)(BA)$,

Here n is the number of turns per unit length.

The magnitude of B is given by: $B = \mu_0 in$,

Therefore,
$$L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 in)(A)}{i}$$
$$= \mu_0 n^2 l A.$$

The *inductance per unit length near the center* is therefore:

$$\frac{L}{l} = \mu_0 n^2 A \quad (\text{solenoid}).$$

$$\begin{aligned}\mu_0 &= 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A} \\ &= 4\pi \times 10^{-7} \text{ H/m}.\end{aligned}$$

Self-Induction:



An induced emf $\mathcal{E}_L$ appears in any coil in which the current is changing.

This process (see Fig. 30-13) is called **self-induction**, and the emf that appears is called a **self-induced emf**. It obeys Faraday's law of induction just as other induced emfs do.

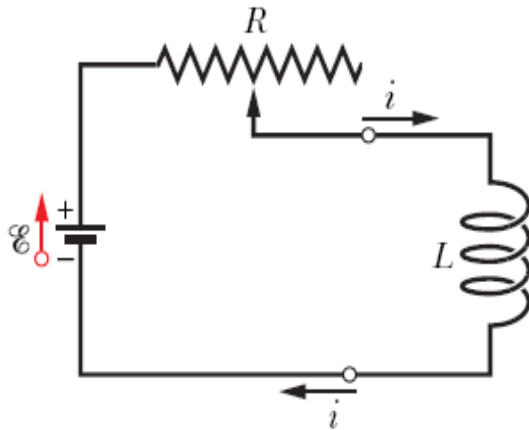


Fig. 30-13 If the current in a coil is changed by varying the contact position on a variable resistor, a self-induced emf $\mathcal{E}_L$ will appear in the coil *while the current is changing*.

$$N\Phi_B = Li.$$

$$\mathcal{E}_L = - \frac{d(N\Phi_B)}{dt}.$$

$$\mathcal{E}_L = -L \frac{di}{dt} \quad (\text{self-induced emf}).$$

This is defined in the direction of the current.