

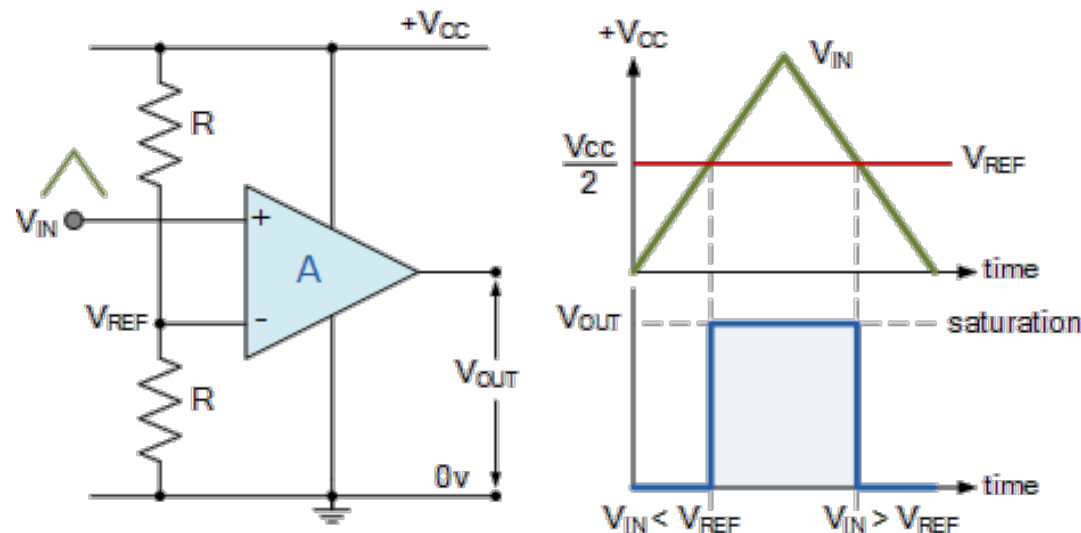
Outline of Contents

- > Review of Op Amps with Examples
- > Passive filters:
 - Review of 1st order passive filters
 - Passive 2nd order filters
- > Active filters
 - 1st order active filters
 - Active band pass filter
 - 2nd order active filters
 - Multiple feedback narrow band filters
 - Sallen-Key filter topologies

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Open-Loop Op Amp Circuit : Non-inverting Comparator

A comparator is an example of an op amp operated in open loop
Open loop means that no feedback is applied



$$V_{out} = A(V_{+} - V_{-})$$

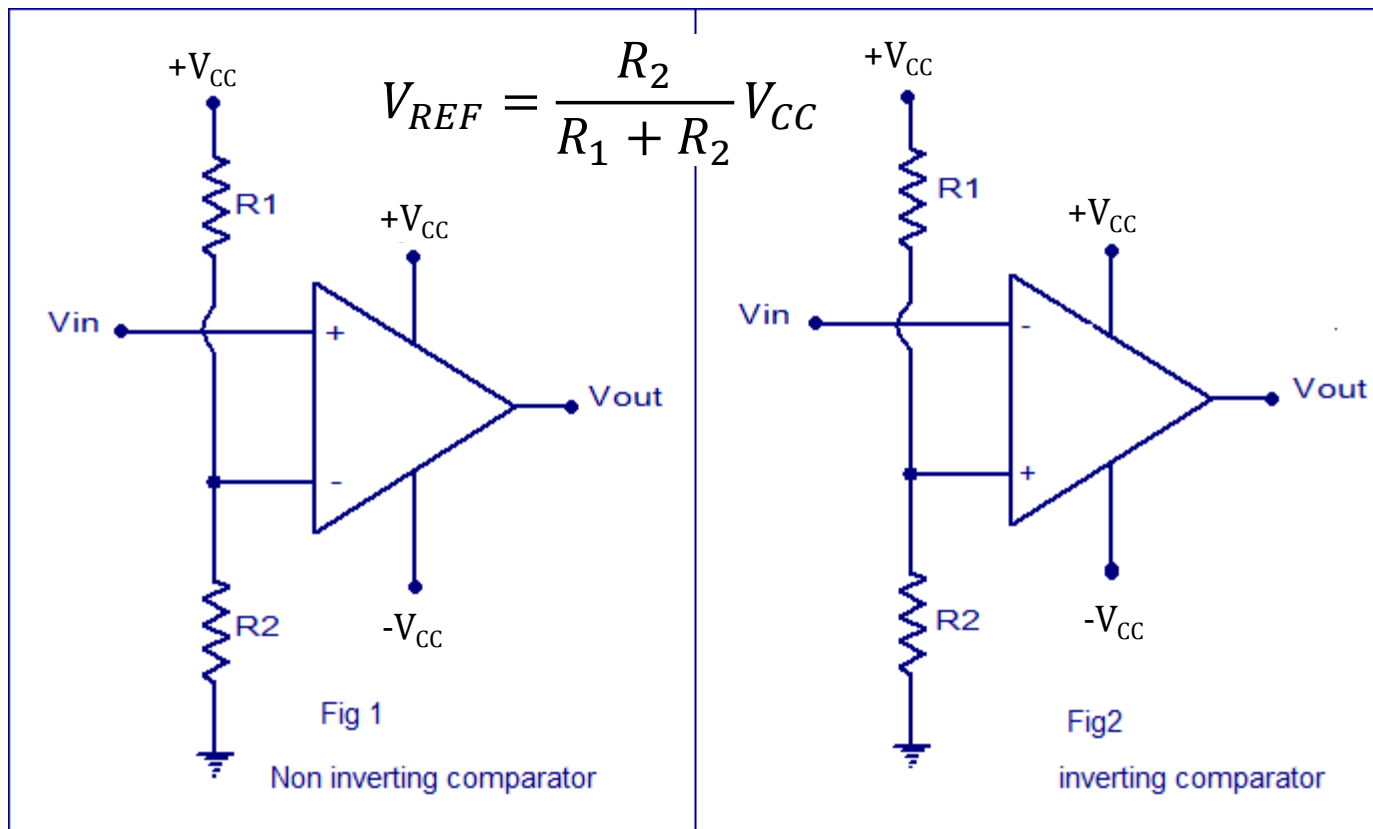
For an ideal op amp ($A \rightarrow \infty$),
given there is no feedback
between inputs and output:

When $V_{IN} > V_{REF}$: $V_{out} = +V_{CC}$;

When $V_{IN} < V_{REF}$: $V_{out} = 0$

In the above example, $V_{REF} = V_{CC}/2$ (no input current into op amp) Note
that V_{REF} can be set to any desired value by the choice of resistors

Open-Loop Op Amp Circuit : Inverting Comparator



When $V_{IN} > V_{REF}$: $V_{out} = +V_{CC}$;

When $V_{IN} < V_{REF}$: $V_{out} = -V_{CC}$

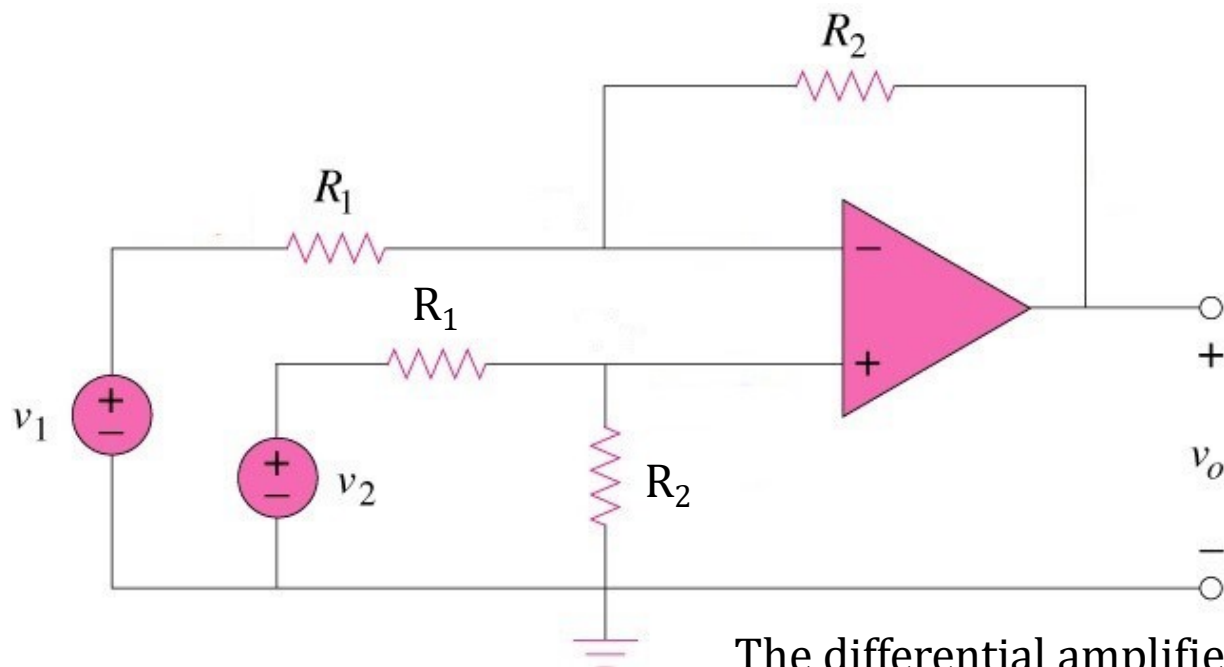
When $V_{IN} > V_{REF}$: $V_{out} = -V_{CC}$;

When $V_{IN} < V_{REF}$: $V_{out} = +V_{CC}$

Closed-Loop Application: Ideal differential op amp

We look at several op amp circuits that use negative feedback, starting with the differential amplifier from the previous lecture.

Negative feedback: Taking part or all of the output and returning it back to the input out of phase. The aim is to stabilize the output.



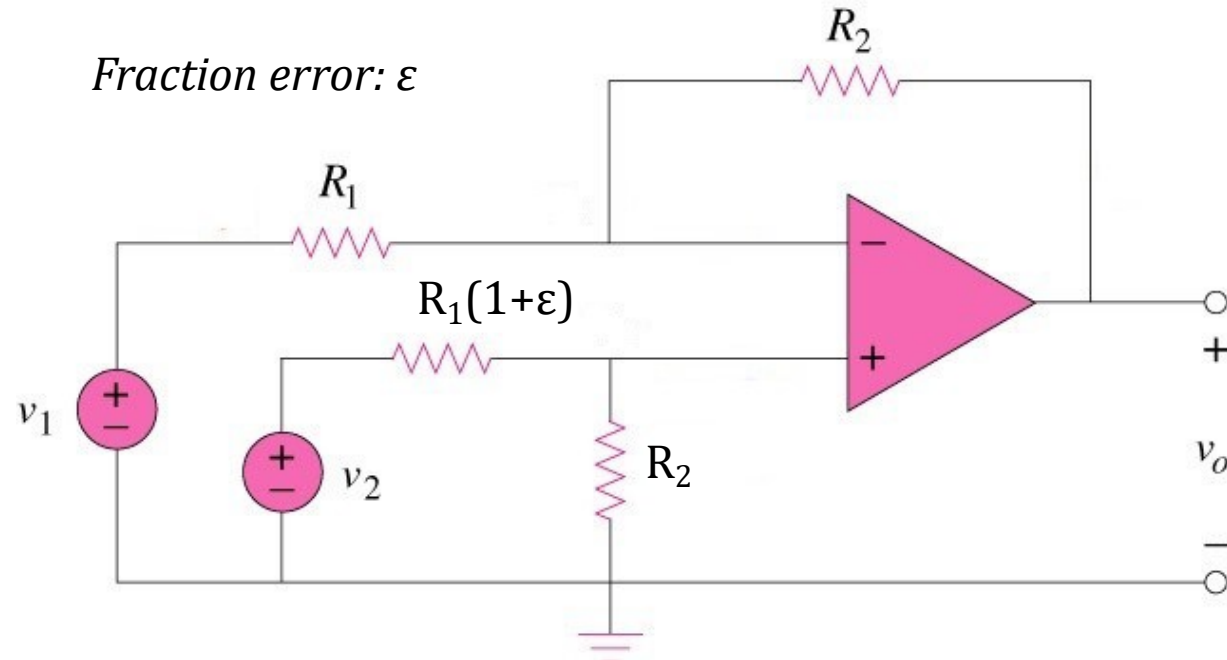
$$V_o = \frac{R_2}{R_1} (V_2 - V_1)$$

The differential amplifier is supposed to amplify only differences between the inputs.

Differential op amp in practice

In practice, it is not possible for two resistors to be exactly equal. So let us consider what happens when there is some error between just one of the resistors.

Fraction error: ε

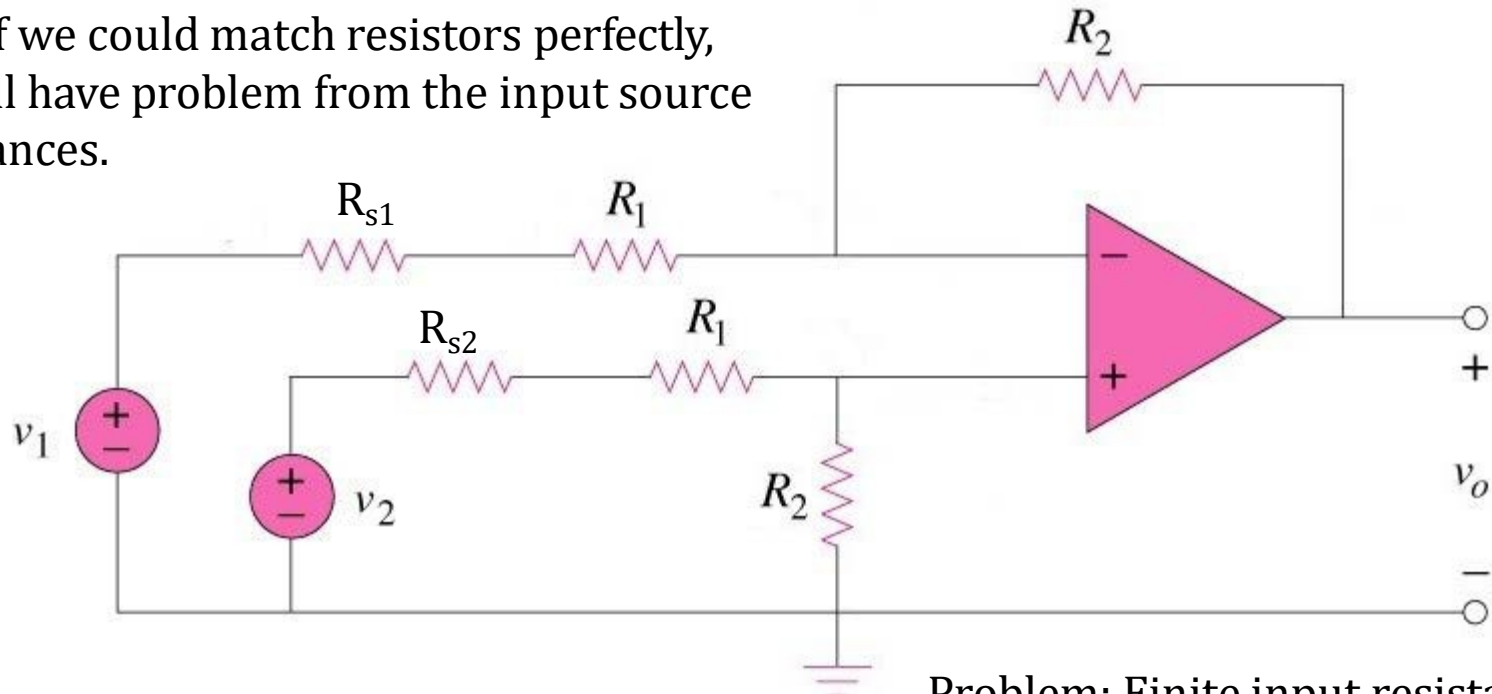


$$V_o = \frac{R_2}{R_1} (V_2 - V_1) - \frac{\varepsilon R_2}{R_1(1 + \varepsilon) + R_2} V_2$$

We can see that V_o no longer depends on only the difference between the inputs; the value of the inputs also matter now.

Effect of source resistance

Even if we could match resistors perfectly, we will have problem from the input source resistances.



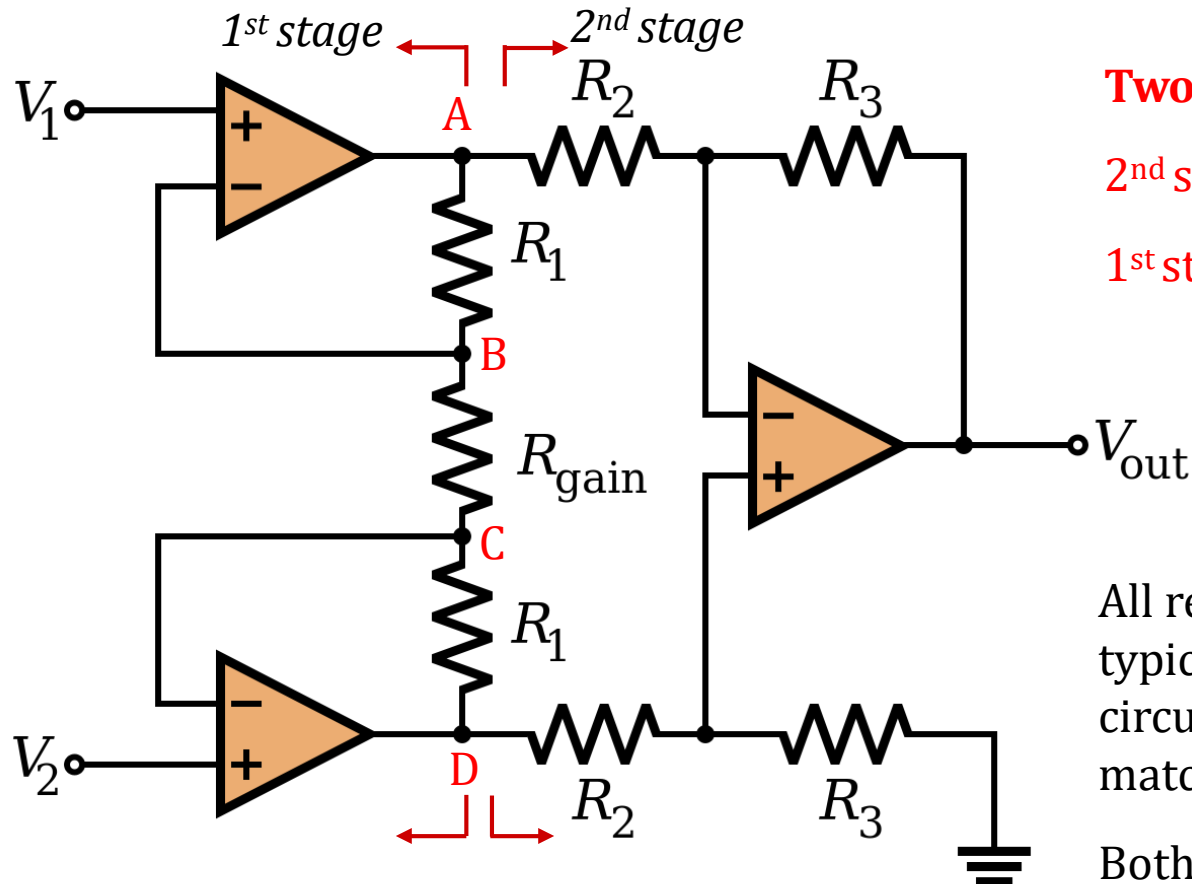
Problem: Finite input resistance

$$V_o = \left(\frac{R_2}{R_1 + R_{s1}} \right) \left[\left(\frac{R_2 + R_1 + R_{s1}}{R_1 + R_2 + R_{s2}} \right) V_2 - V_1 \right]$$

Clearly V_o does not depend just on the difference between the inputs.

If $V_2 = V_1$, V_o would not be zero.

Instrumentation amplifier



Two stage cascaded amplifier

2nd stage: Differential amplifier

1st stage: Non-inverting amplifier

All resistors (except R_{gain}) are typically built into the integrated circuit and thus can be precisely matched.

Both inputs see the large input resistance of the op amp

⇒ Source resistance has little effect

$$V_{out} = \left(1 + \frac{2R_1}{R_{gain}}\right) \left(\frac{R_3}{R_2}\right) (V_2 - V_1)$$

Instrumentation amplifier: Analysis

Current from A → B = Current from B → C = Current from C → D

Node A: V_{o1} ; Node B: V_1 ; Node C: V_2 ; Node D: V_{o2}

1st stage

$$\frac{V_{o1} - V_1}{R_1} = \frac{V_1 - V_2}{R_{gain}} \Rightarrow V_{o1} = \left(1 + \frac{R_1}{R_{gain}}\right) V_1 - \left(\frac{R_1}{R_{gain}}\right) V_2$$
$$\frac{V_2 - V_{o2}}{R_1} = \frac{V_1 - V_2}{R_{gain}} \Rightarrow V_{o2} = \left(1 + \frac{R_1}{R_{gain}}\right) V_2 - \left(\frac{R_1}{R_{gain}}\right) V_1$$

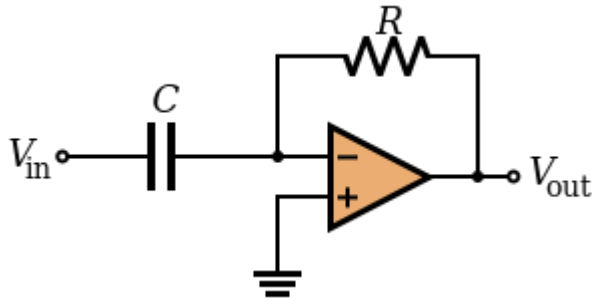
None of the above intermediate output voltages are affected by the source resistance of the inputs

2nd stage

$$V_{out} = \frac{R_3}{R_2} (V_{o2} - V_{o1}) \quad V_{o2} - V_{o1} = \left(1 + \frac{R_1}{R_{gain}}\right) (V_2 - V_1) - \left(\frac{R_1}{R_{gain}}\right) (V_1 - V_2)$$

$$\Rightarrow V_{out} = \frac{R_3}{R_2} \left(1 + \frac{2R_1}{R_{gain}}\right) (V_2 - V_1)$$

Differentiating amplifier



KCL at inverting input:

$$I_S = I_F = -V_{out}/R \quad \dots (1)$$

Voltage across C = V_{in} :

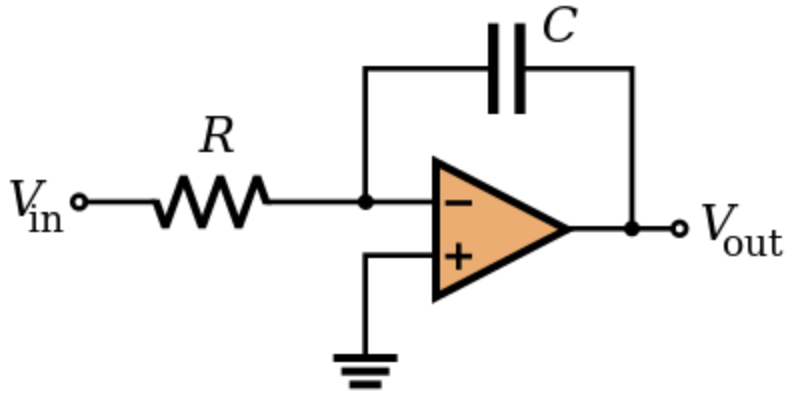
$$I_S = C \left(\frac{dV_{in}}{dt} \right) \quad \dots (2)$$

Substitute (1) into (2):

$$V_{out} = -RC \left(\frac{dV_{in}}{dt} \right)$$

Output is the time-differential of the input.

Integrating amplifier



KCL at inverting input:

$$I_S = I_F = V_{in}/R \quad \dots (1)$$

Voltage across C = $-V_{out}$:

$$I_F = -C \left(\frac{dV_{out}}{dt} \right) \quad \dots (2)$$

Substitute (1) into (2):

$$\frac{V_{in}}{R} = -C \left(\frac{dV_{out}}{dt} \right)$$

$$V_{out} = -\frac{1}{RC} \int V_{in} dt$$

Output is the time-integral of the input

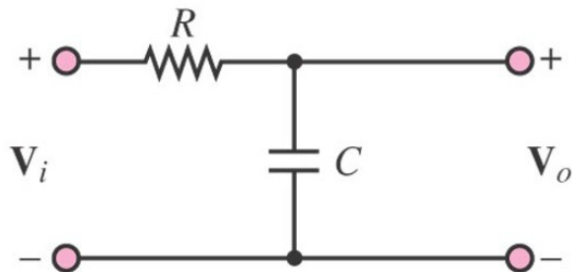
Review: Passive Filters

- Made up of passive components only, e.g. R, L, C
- No amplifying elements, e.g. transistors, op amps
- Load dependent
- No power supply needed
- Scale better to large signals
- Can work at very high frequencies

Passive 1st order low pass filter

$$\frac{V_o}{V_i}(j\omega) = \frac{1}{1 + j(\omega/\omega_c)}$$

$$\omega_c = \frac{1}{RC}$$

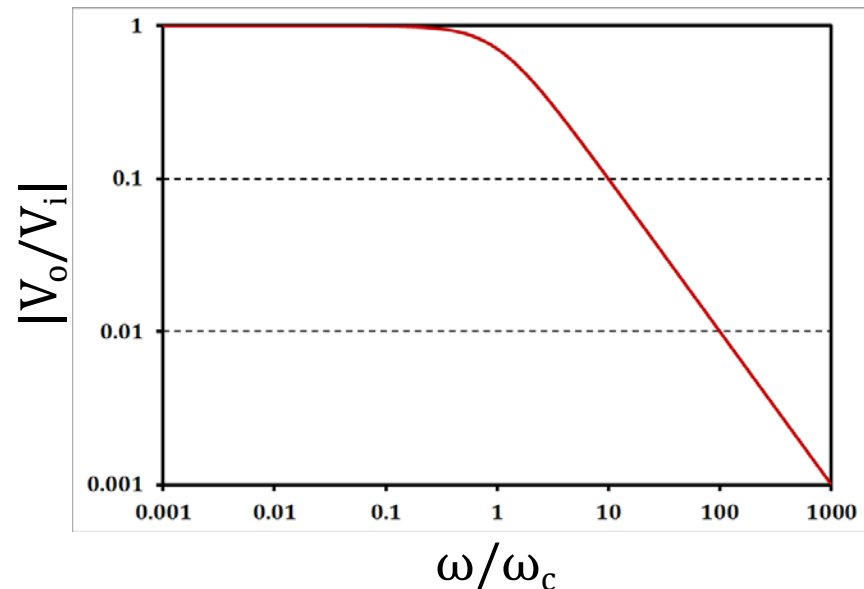


Recall that ω_c is a constant that is determined by the circuit components, referred to as the corner frequency.

For an input at a frequency of ω_c , $V_o = V_i/\sqrt{2}$

For frequencies $< \omega_c$, $|V_o| \approx |V_i|$ (i.e. nearly constant with frequency)

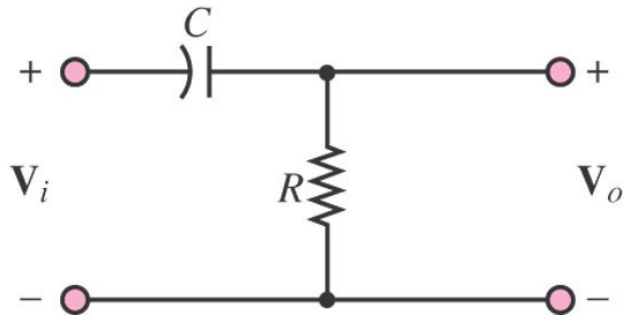
For frequencies $> \omega_c$, $|V_o| \approx |V_i| \omega_c/\omega$ (i.e. inversely proportional to the frequency)



Passive 1st order high pass filter

$$\frac{V_o}{V_i}(j\omega) = \frac{j(\frac{\omega}{\omega_c})}{1 + j(\omega/\omega_c)}$$

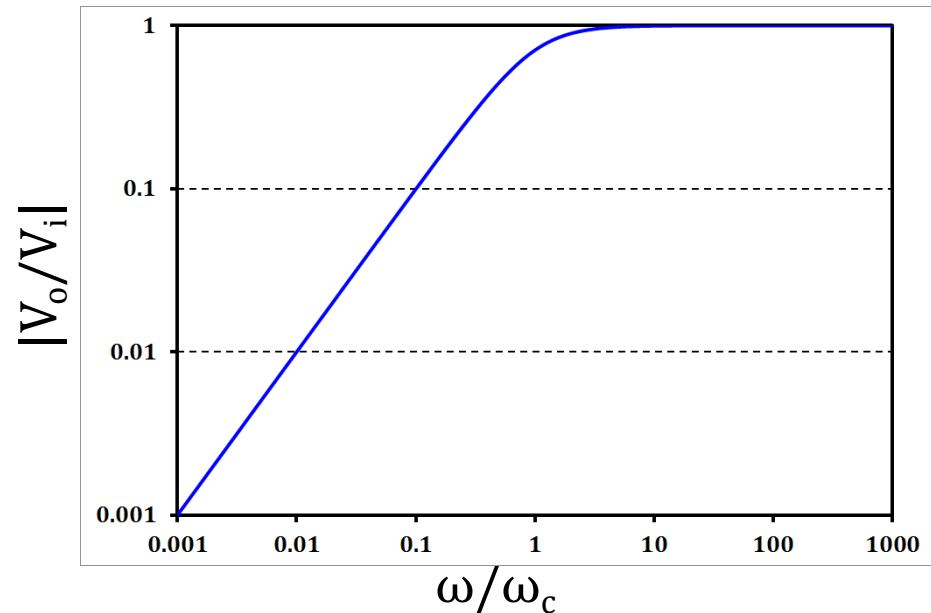
$$\omega_c = \frac{1}{RC}$$



For an input at a frequency of ω_c , $V_o = V_i/\sqrt{2}$

For frequencies $> \omega_c$ $|V_o| \approx |V_i|$ (i.e. nearly constant with frequency)

For frequencies $< \omega_c$ $|V_o| \approx |V_i| \omega/\omega_c$ (i.e. proportional to the frequency)



Decibels (dB)

$$\text{Gain in decibels (dB)} = 10 \log \frac{P_{out}}{P_{in}}$$

$$\text{Gain in decibels (dB)} = 20 \log \frac{V_{out}}{V_{in}}$$

$$\text{Gain} = 1 \Rightarrow 0\text{dB}$$

$$\text{Gain} = 1/\sqrt{2} \Rightarrow -3\text{dB}$$

$$\text{Gain} = 0.1 \Rightarrow -20\text{dB}$$

$$\text{Gain} = 0.01 \Rightarrow -40\text{dB}$$

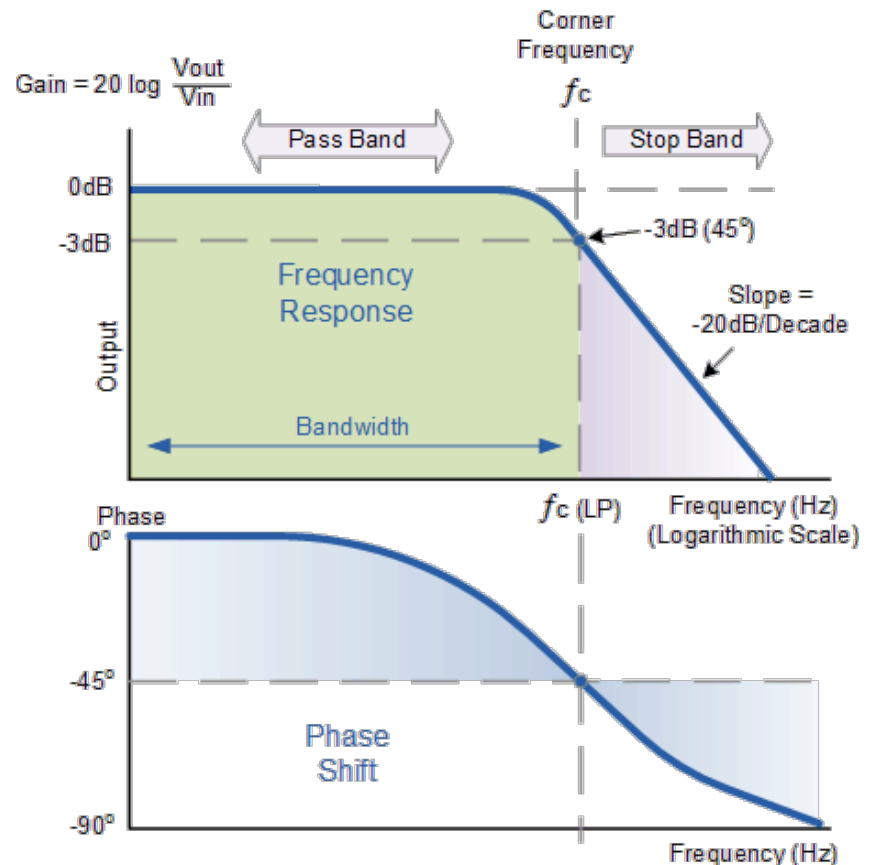
Every factor of 10 corresponds to a change of 20dB

$$\text{Times 10} \Rightarrow +20\text{dB}$$

$$\text{Divide by 10} \Rightarrow -20\text{dB}$$

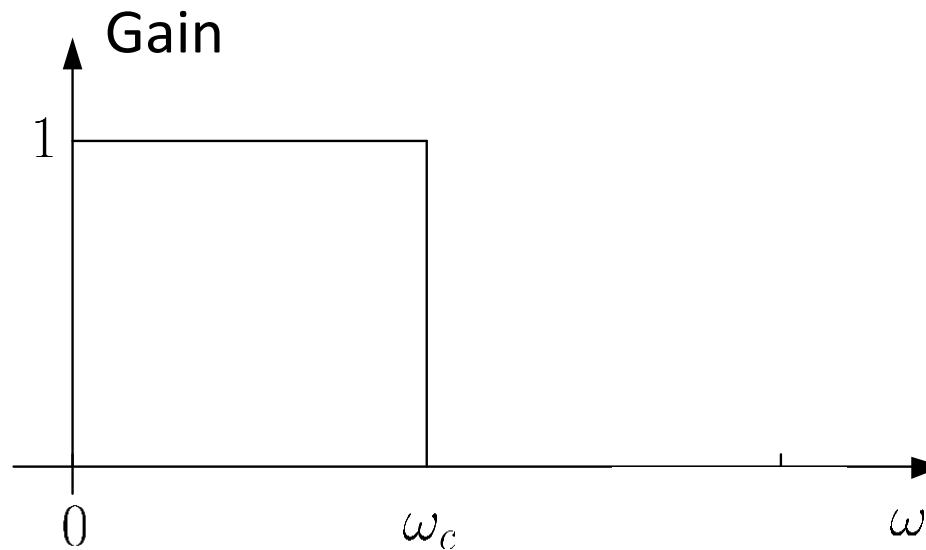
Recall: $P = V^2/R$

At the corner frequency (f_c), gain drops by 3dB relative to the passband



Ideal Low-Pass Filter Specification

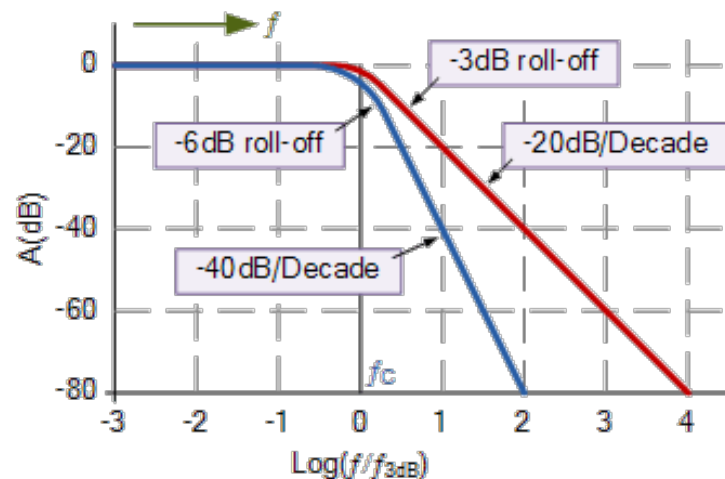
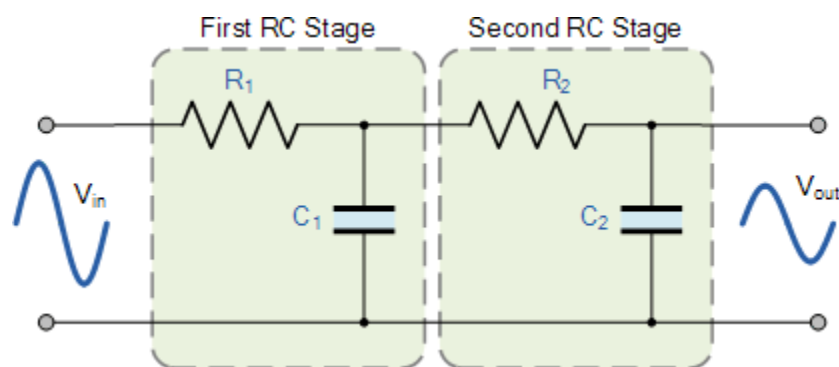
- Unity gain for the whole range of $\omega \in (0, \omega_c)$
- Complete suppression for $\omega > \omega_c$
- Step change in frequency response at $\omega = \omega_c$



Passive 2nd order low pass filter

We can increase the slope of the roll off beyond the cut-off frequency by cascading several stages of low pass filters.

The example here shows a two stage passive low pass filter.



By choice of values, if $(R_2 + 1/j\omega C_2) \gg 1/j\omega C_1$:

$$\frac{V_o}{V_i}(j\omega) \approx \left(\frac{1}{1 + j(\omega/\omega_{c1})} \right) \left(\frac{1}{1 + j(\omega/\omega_{c2})} \right)$$

$$\omega_{c1} = \frac{1}{R_1 C_1} \quad \omega_{c2} = \frac{1}{R_2 C_2}$$

If we chose the values of components so that $\omega_{c1} = \omega_{c2} = \omega_c$:

$$\frac{V_o}{V_i}(j\omega) \approx \frac{1}{\left[1 + j \left(\frac{\omega}{\omega_c} \right) \right]^2}$$

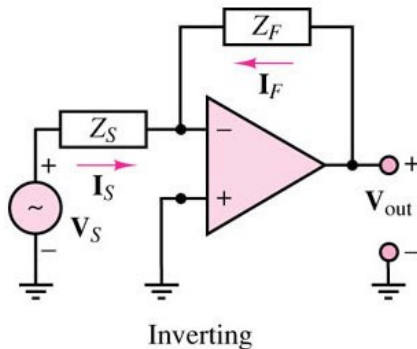
Passive 2nd order low pass filter

- > Slope:
 - 2nd order filter: -40dB/decade ($|V_o/V_i|$ drops by 100 times for 10 times increase in frequency)
 - 1st order filter: -20dB/decade ($|V_o/V_i|$ drops by 10 times for 10 times increase in frequency)
- > At cut-off frequency (f_c),
 - 2nd order filter: gain drops by -6dB
 - 1st order filter: gain drops by -3dB
- > Coupling issues: Impedance of stage 2 chosen to be much larger than load impedance of stage 1
 - What happens as we increase the number of stages in the cascade for higher orders?
- > Passband gain: Passband gain is still less than 1

Active filters

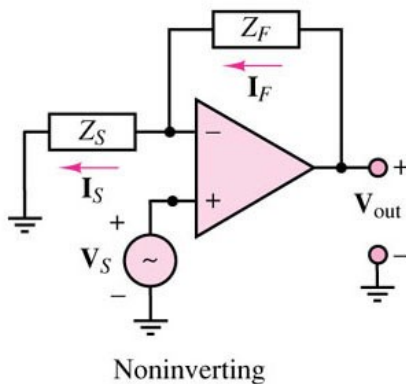
- Range of applications is greatly expanded if reactive components are used
- Addition of reactive components allows us to shape the frequency response
- **Active filters:** Op-amp provides amplification (gain) in addition to filtering effects

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$$I_S = \frac{V_S}{Z_S}; I_F = \frac{V_{out}}{Z_F}; I_S + I_F = 0 \Rightarrow \frac{V_S}{Z_S} + \frac{V_{out}}{Z_F} = 0 \Rightarrow$$

$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$



$$I_S = \frac{V_S}{Z_S}; I_F = \frac{V_{out} - V_S}{Z_F}; I_F = I_S \Rightarrow \frac{V_S}{Z_S} = \frac{V_{out} - V_S}{Z_F} \Rightarrow$$

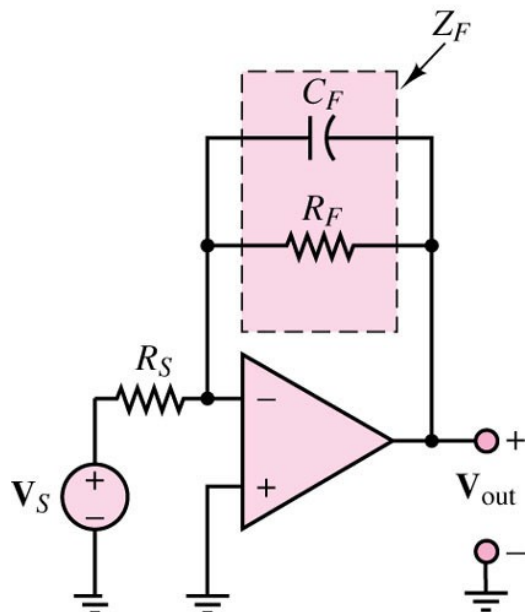
$$Z_F V_S = Z_S V_{out} - Z_S V_S \Rightarrow V_S (Z_F + Z_S) = Z_S V_{out} \Rightarrow$$

$$\frac{V_{out}}{V_S}(j\omega) = \frac{Z_F + Z_S}{Z_S} = 1 + \frac{Z_F}{Z_S}$$

Z_F and Z_S can be arbitrary (i.e. any) complex impedance

Active 1st order low pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F \parallel C_F = \frac{R_F}{1 + j\omega C_F R_F}$$

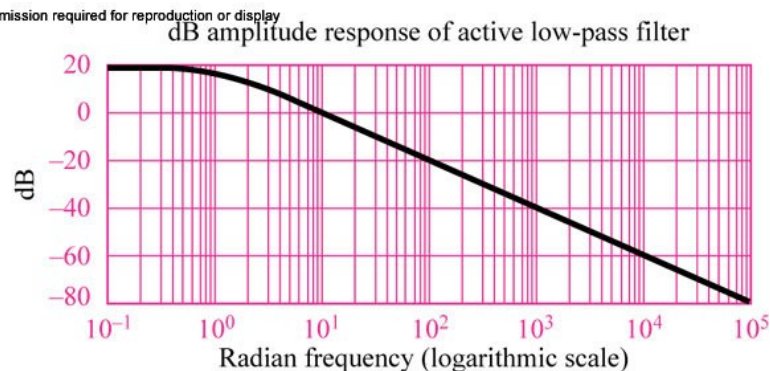
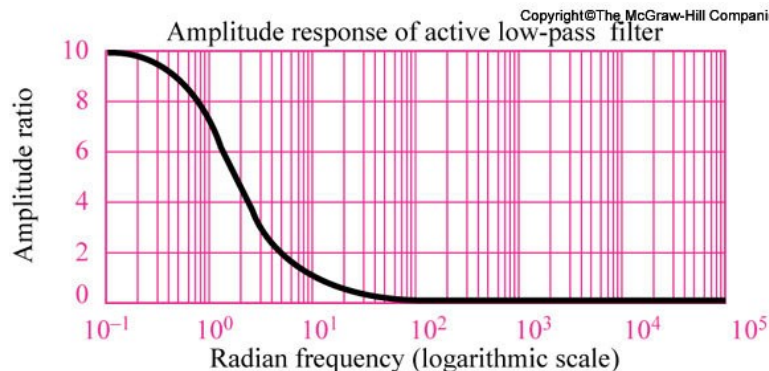
$$Z_S = R_S$$

$$\frac{V_{out}}{V_S}(j\omega) = -\frac{R_F / R_S}{1 + j\omega C_F R_F}$$

Passband gain is R_F/R_S

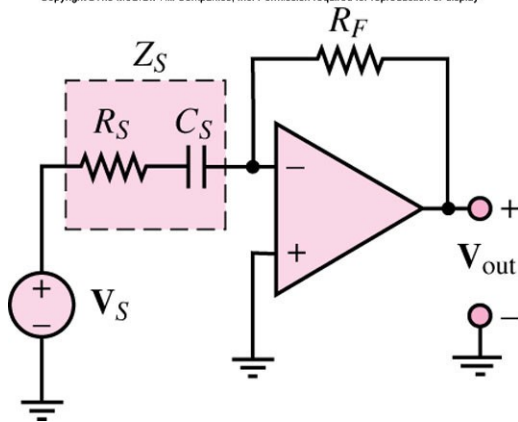
$$\omega_c = \frac{1}{R_F C_F}$$

If we choose $R_F/R_S = 10$, frequency response becomes:



Active 1st order high pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_F$$

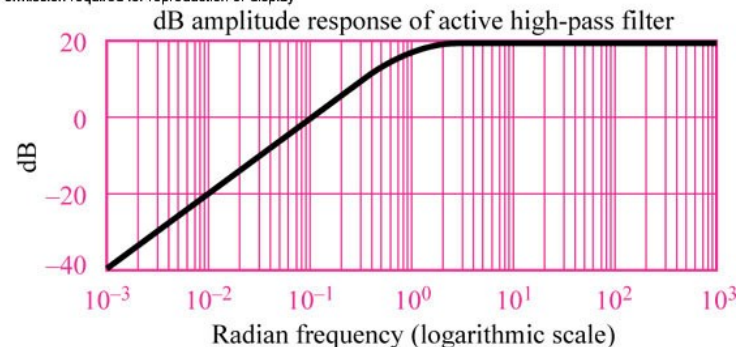
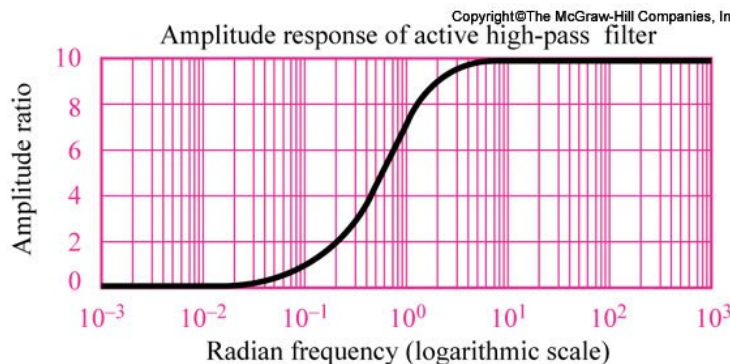
$$Z_S = R_S + 1/j\omega C_S$$

$$\frac{V_{out}}{V_S}(j\omega) = -\frac{R_F/R_S}{1 + (1/j\omega C_S R_S)}$$

Passband gain is R_F/R_S

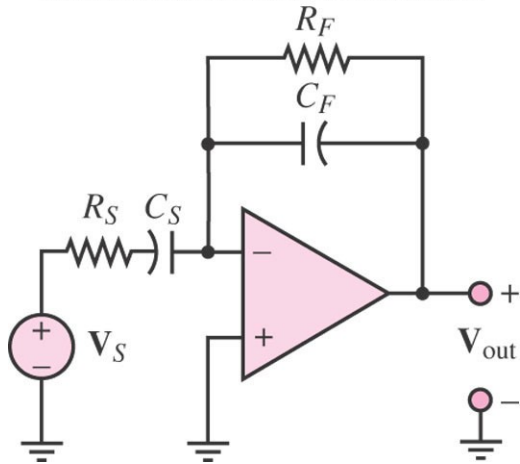
$$\omega_c = \frac{1}{R_S C_S}$$

If we choose $R_F/R_S = 10$, frequency response becomes:



Active band pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_S = R_S + \frac{1}{j\omega C_S}$$

$$Z_F = R_F \parallel C_F = \frac{R_F}{1 + j\omega C_F R_F}$$

$$\frac{V_{out}}{V_S}(j\omega) = \frac{-R_F / R_S}{(1 + 1/j\omega C_S R_S)(1 + j\omega C_F R_F)}$$

Z_S : C_S blocks low frequency inputs but lets high frequency inputs through

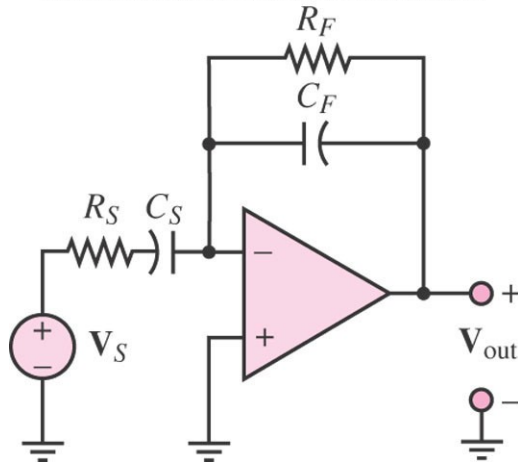
⇒ High pass filter

Z_F : C_F shorts R_F at high frequency (reducing the gain), but otherwise looks just like a low pass filter at lower frequencies

⇒ Low pass filter

Active band pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = \frac{-R_F/R_S}{(1 + \omega_{HP}/j\omega)(1 + j\omega/\omega_{LP})}$$

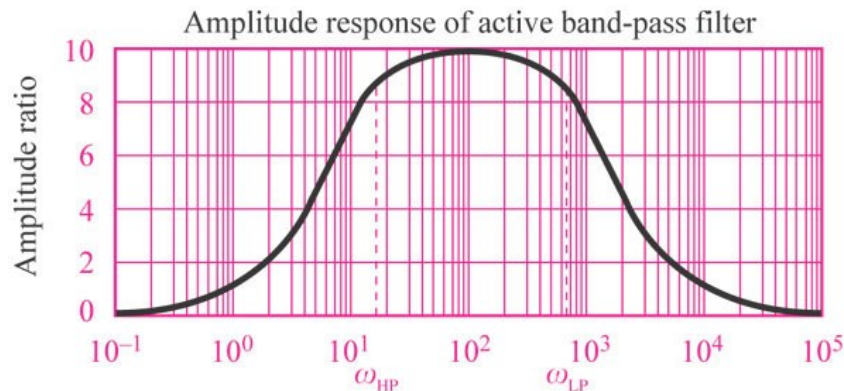
$\omega_{HP} = 1/(C_S R_S)$ – Lower cut-off frequency

$\omega_{LP} = 1/(C_F R_F)$ – Upper cut-off frequency

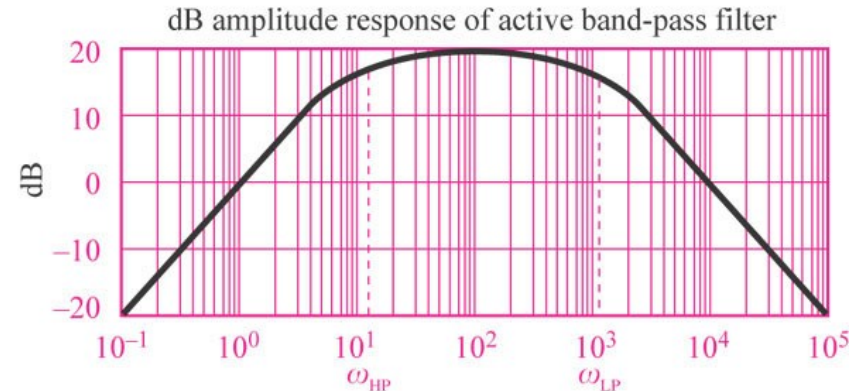
Passband gain is R_F/R_S

For $\omega_{HP} < \omega_{LP}$ and $R_F/R_S = 10$: Frequency response curve is shown below

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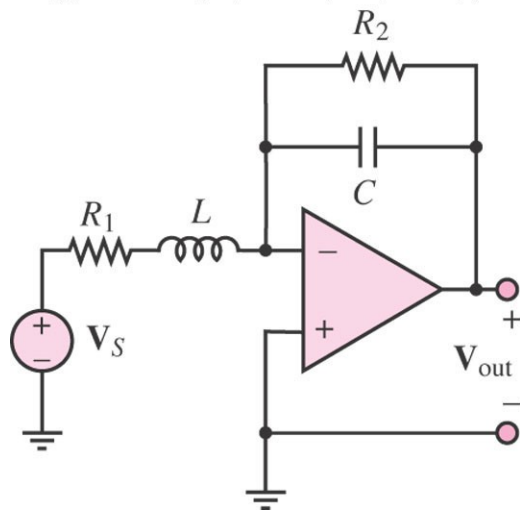
Radian frequency (logarithmic scale)



Radian frequency (logarithmic scale)

Active 2nd order low pass filter

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$$\frac{V_{out}}{V_S}(j\omega) = -\frac{Z_F}{Z_S}$$

$$Z_F = R_2 \parallel 1/j\omega C = \frac{R_2}{1 + j\omega CR_2} \quad Z_S = R_1 + j\omega L$$

$$\frac{V_{out}}{V_S}(j\omega) = -\frac{R_2/R_1}{(1 + j\omega CR_2)(1 + j\omega L/R_1)}$$

If R_1 , R_2 , C and L are chosen so that: $\omega_c = 1/(CR_2) = R_1/L$.

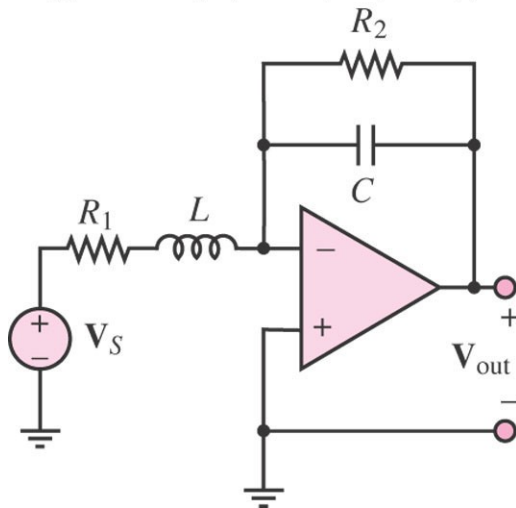
The frequency response function then simplifies to:

$$\frac{V_{out}}{V_S}(j\omega) = -\frac{R_2/R_1}{(1 + j\omega/\omega_c)^2}$$

Passband gain is R_F/R_S

Active 2nd order low pass filter

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$$H_v(j\omega) = -\frac{R_2/R_1}{(1+j\omega/\omega_c)^2}$$

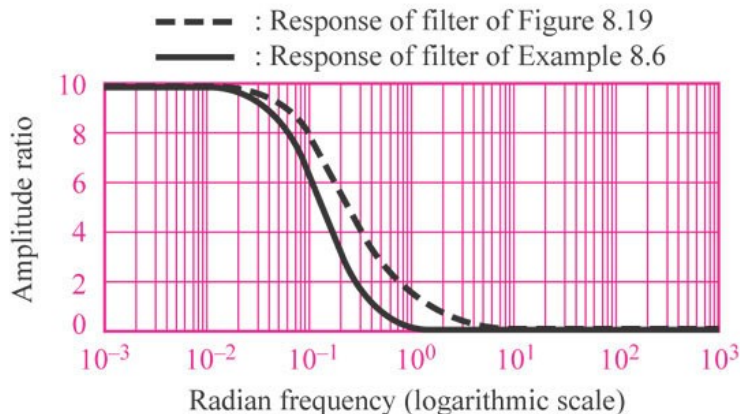
Above ω_c , H_v is reduced by a factor of **100** for a ten fold increase in ω (40dB drop per decade)

Recall for 1st order filter: H_v is reduced by a factor of **10** for a ten fold increase in ω (20dB drop per decade)

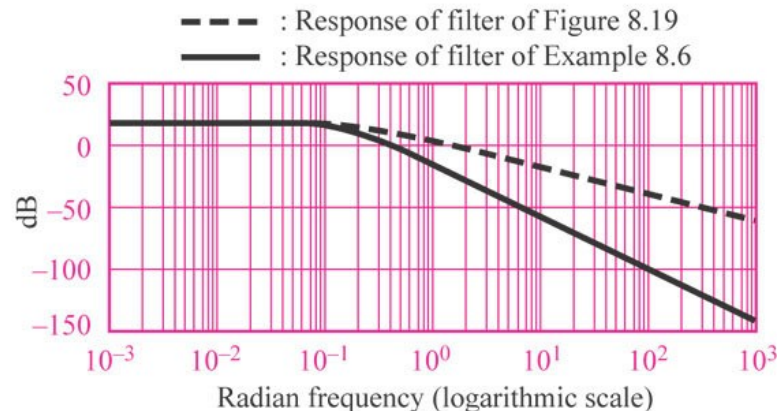
If $R_2/R_1 = 10$: Frequency response curve is shown below

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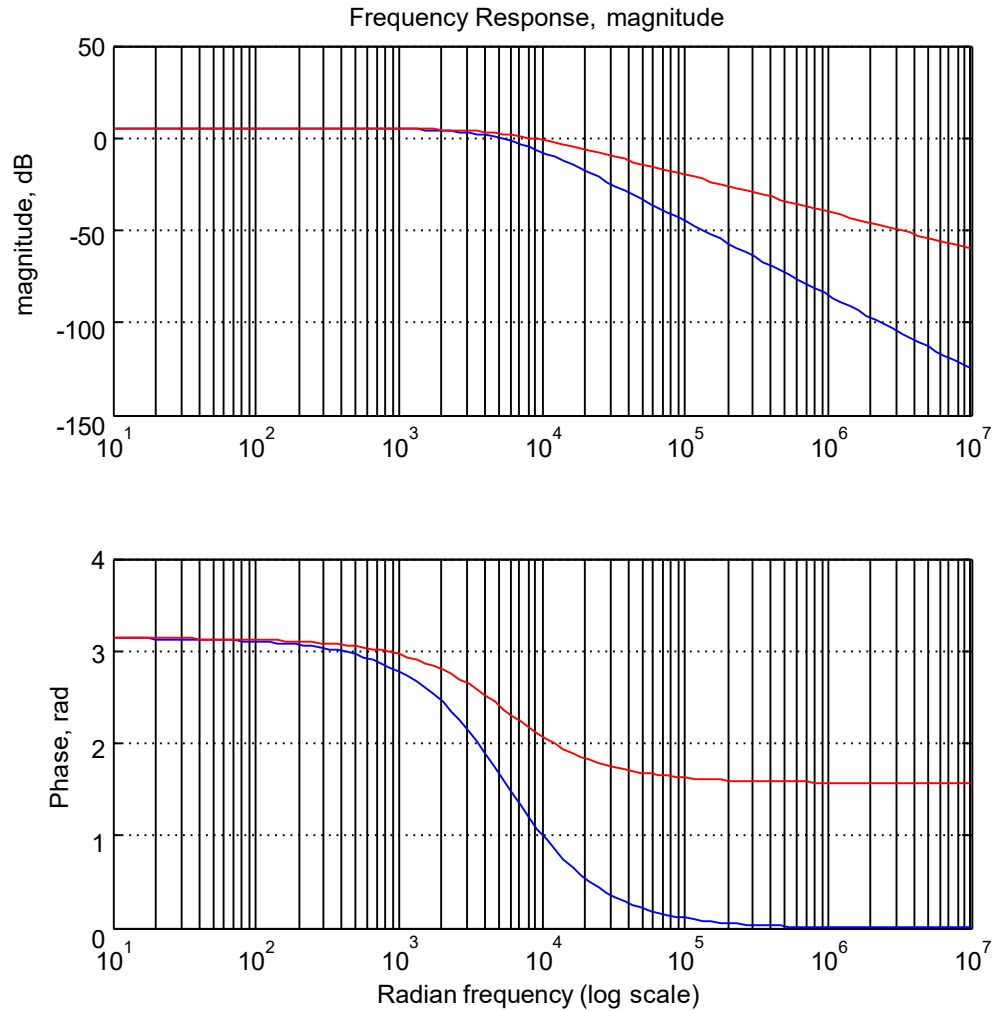
Comparison of active low-pass filters



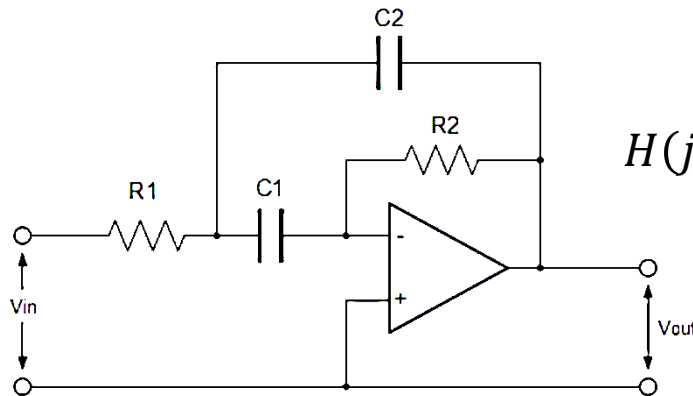
Comparison of active low-pass filters (dB plot)



Bode plot of 2nd order low pass filter



Multiple Feedback Narrow Band Filter



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{-\frac{1}{R_1 C_2} (j\omega)}{(j\omega)^2 + \left(\frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} \right) (j\omega) + \frac{1}{R_1 R_2 C_1 C_2}}$$

1. Assuming $C_1 = C_2 = C$, and convert the above equation to the following form

$$H(j\omega) = \frac{\frac{\omega_o}{Q} K \times (j\omega)}{(j\omega)^2 + \left(\frac{\omega_o}{Q} \right) (j\omega) + \omega_o^2}$$

2. The center frequency ω_o is given by

$$\omega_o = \frac{1}{C \sqrt{R_1 R_2}}$$

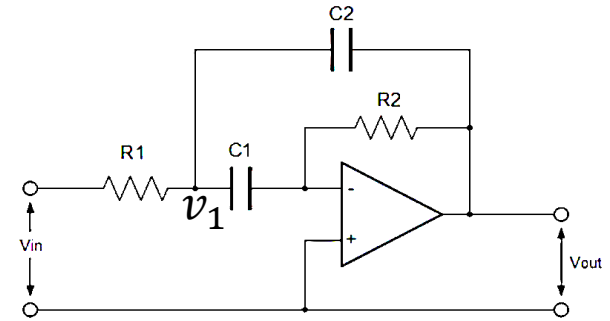
3. The Q and K factor are respectively given by

$$Q = \frac{1}{2} \sqrt{\frac{R_2}{R_1}} \quad K = -2Q^2 = -\frac{R_2}{2R_1}$$

Derivation of $H(j\omega)$ for Multiple Feedback Narrow Band Filter

$$v_- = 0 \text{ V}$$

$$\frac{v_1}{\frac{1}{j\omega C_1}} = -\frac{v_{out}}{R_2} \Rightarrow v_1 = \frac{-v_{out}}{j\omega C_1 R_2}$$



$$\frac{v_{in} - v_1}{R_1} = \frac{v_1 - v_{out}}{\frac{1}{j\omega C_2}} + \frac{v_1}{\frac{1}{j\omega C_1}} \Rightarrow \frac{v_{in} - v_1}{j\omega R_1 C_1 C_2} = \frac{v_1 - v_{out}}{C_1} + \frac{v_1}{C_2}$$

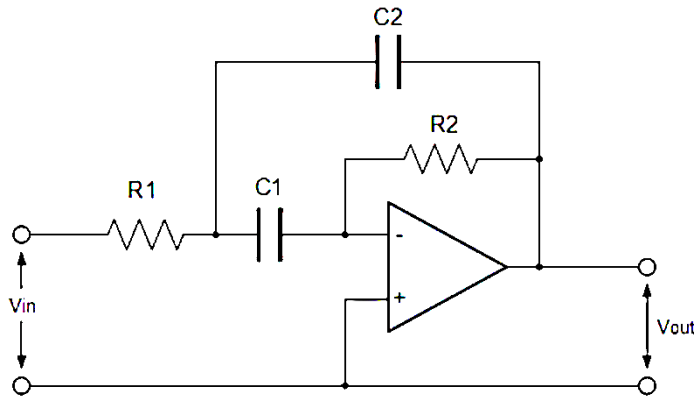
$$\Rightarrow \frac{v_{in}}{j\omega R_1 C_1 C_2} = \left(\frac{-1}{j\omega C_1^2 R_2} - \frac{1}{C_1} - \frac{1}{j\omega C_1 C_2 R_2} - \frac{1}{(j\omega)^2 R_1 R_2 C_1^2 C_2} \right) v_{out}$$

$$\Rightarrow v_{in} = -v_{out} \left(\frac{R_1 C_2}{R_2 C_1} + j\omega R_1 C_2 + \frac{R_1}{R_2} + \frac{1}{j\omega R_2 C_1} \right)$$

$$= -v_{out} \left(\frac{j\omega R_1 (C_1 + C_2) + (j\omega)^2 R_1 R_2 C_1 C_2 + 1}{j\omega R_2 C_1} \right)$$

$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{-\frac{1}{R_1 C_2} (j\omega)}{(j\omega)^2 + \left(\frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} \right) (j\omega) + \frac{1}{R_1 R_2 C_1 C_2}}$$

Multiple Feedback Narrow Band Filter



$$H(j\omega) = \frac{\frac{\omega_o}{Q} K \times (j\omega)}{(j\omega)^2 + \left(\frac{\omega_o}{Q}\right) (j\omega) + \omega_o^2}$$

$$\omega_o = \frac{1}{C\sqrt{R_1 R_2}} \quad Q = \frac{1}{2} \sqrt{\frac{R_2}{R_1}} \quad K = -\frac{R_2}{2R_1}$$

$$f_0 = 1.125\text{kHz}$$

Given

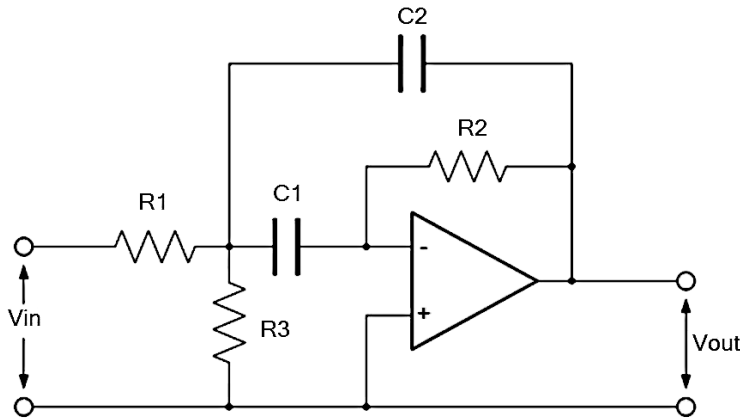
1. $R_1 = 100\Omega$; $R_2 = 20\text{k}\Omega$; $C_1 = C_2 = 100\text{nF}$, calculate ω_0 and Q . $K = -100$, $Q = 7.07$
2. $R_1 = 1\text{k}\Omega$; $R_2 = 2\text{k}\Omega$; $C_1 = C_2 = 100\text{nF}$, calculate ω_0 and Q . $K = -1$, $Q = 0.707$

Plot the frequency response of your above two cases using

NI MultiSim: <http://www.ni.com/multisim>; <http://www.ni.com/multisim/mobile>

Multiple Feedback Narrow Band Filter

(A more General Form)



$$H(j\omega) = \frac{v_{out}}{v_{in}}$$

$$= \frac{-\frac{1}{R_1 C_2} (j\omega)}{(j\omega)^2 + \left(\frac{1}{R_2 C_1} + \frac{1}{R_2 C_2}\right) (j\omega) + \frac{1}{R_2 C_1 C_2} \left(\frac{1}{R_1} + \frac{1}{R_3}\right)}$$

1. Assuming $C_1 = C_2 = C$, and convert the above equation to the following form

$$H(j\omega) = \frac{\frac{\omega_o}{Q} K \times (j\omega)}{(j\omega)^2 + \left(\frac{\omega_o}{Q}\right) (j\omega) + \omega_o^2}$$

2. The center frequency ω_o is given by

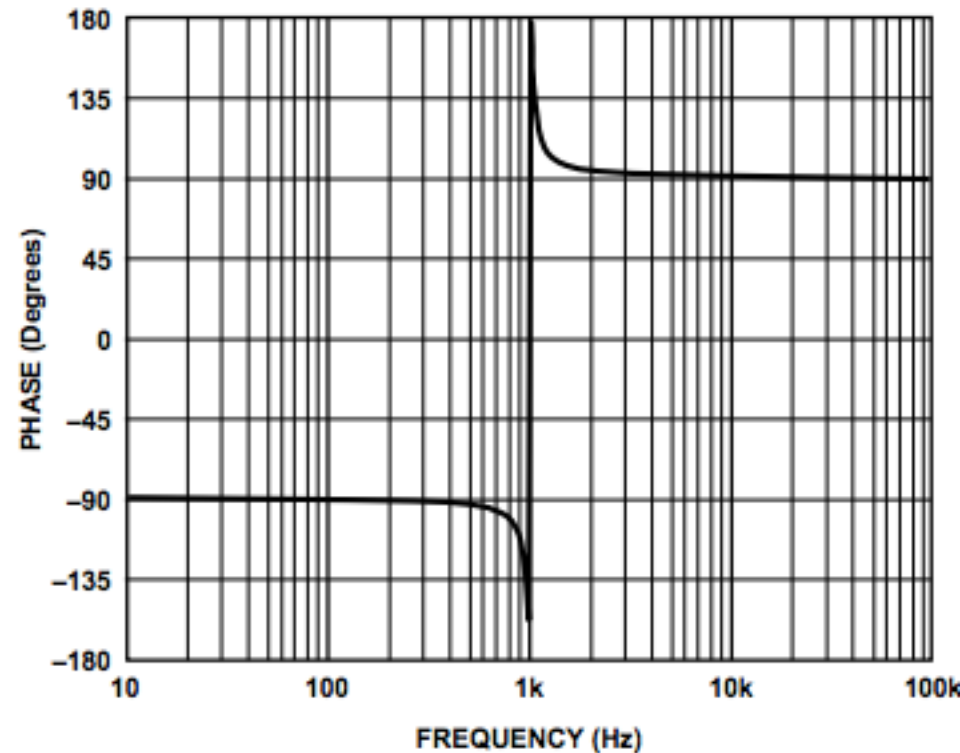
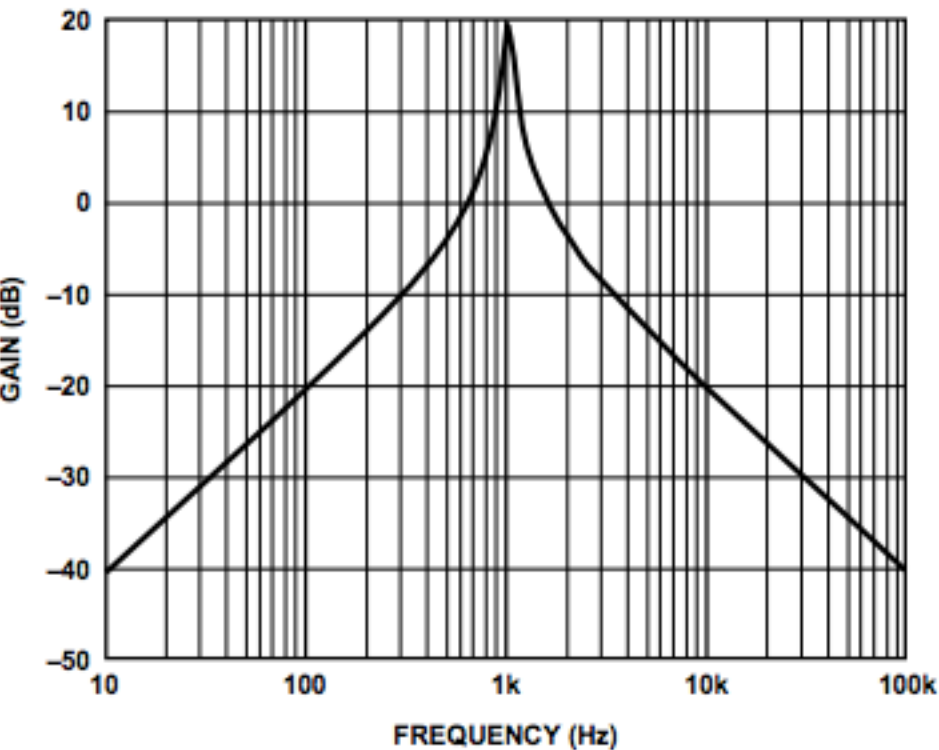
$$\omega_o = \frac{1}{C} \sqrt{\frac{1}{R_2} \left(\frac{1}{R_1} + \frac{1}{R_3}\right)}$$

3. The Q and K factor are respectively given by

$$Q = \frac{1}{2} \sqrt{R_2 \left(\frac{1}{R_1} + \frac{1}{R_3}\right)} \quad K = -\frac{R_2}{2R_1}$$

Frequency Response of a Multiple Feedback Band-Pass Filter

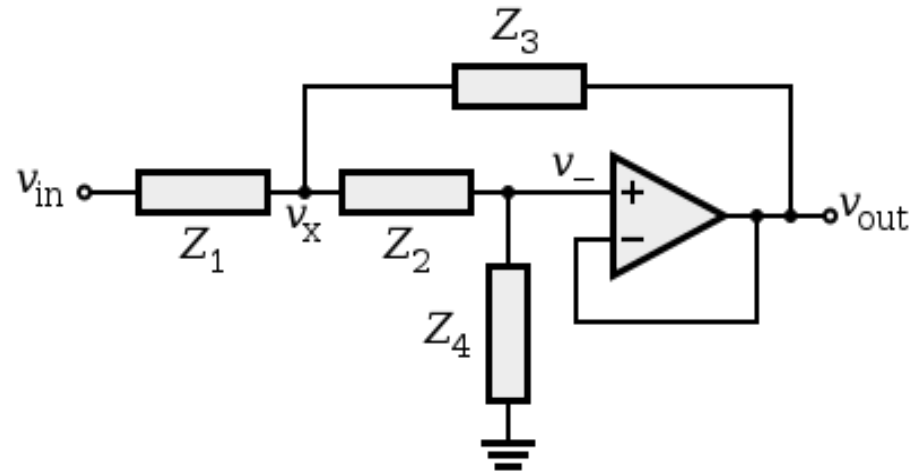
Simple and reliable band-pass implementation



<https://www.analog.com/media/en/training-seminars/tutorials/MT-218.pdf>

Sallen-Key Filter Topology

Assuming that the op-amp is ideal, transfer function of this filter is given by

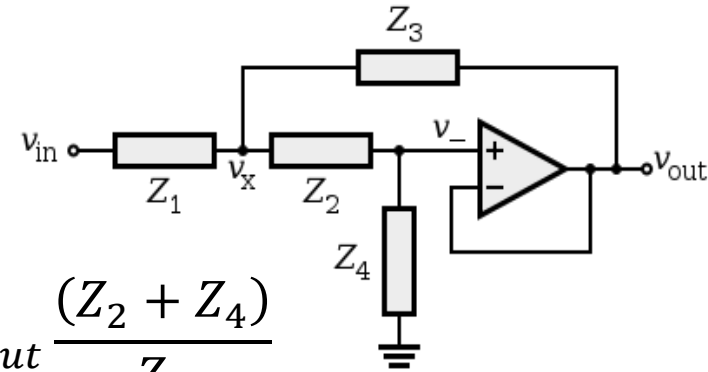


$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

different types of R, L and C for Z's components
→ different kinds of filter responses.

Derivation of $H(j\omega)$ in Sallen-Key Filter Topology

$$v_- = v_{out}$$



$$v_x \times \frac{Z_4}{Z_2 + Z_4} = v_- = v_{out} \Rightarrow v_x = v_{out} \frac{(Z_2 + Z_4)}{Z_4}$$

$$\frac{v_{in} - v_x}{Z_1} + \frac{v_{out} - v_x}{Z_3} = \frac{v_x}{Z_2 + Z_4}$$

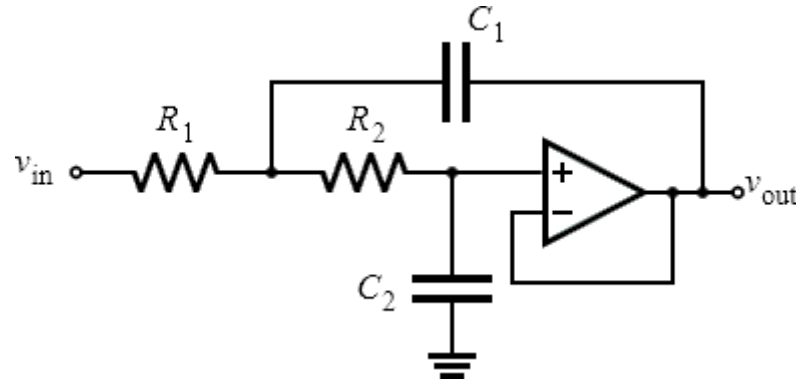
$$\Rightarrow \frac{Z_3 v_{in} + Z_1 v_{out} - (Z_1 + Z_3) v_x}{Z_1 Z_3} = \frac{v_{out}}{Z_4}$$

$$\Rightarrow Z_3 v_{in} + Z_1 v_{out} - \frac{(Z_1 + Z_3)(Z_2 + Z_4)}{Z_4} v_{out} = \frac{Z_1 Z_3}{Z_4} v_{out}$$

$$\Rightarrow Z_3 Z_4 v_{in} = (Z_1 Z_3 + Z_1 Z_2 + Z_1 Z_4 + Z_2 Z_3 + Z_3 Z_4 - Z_1 Z_4) v_{out}$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3(Z_1 + Z_2) + Z_3 Z_4}$$

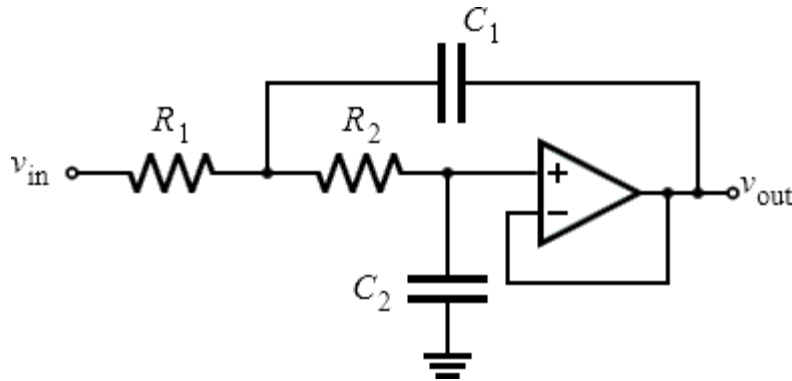
Operations of Sallen-Key Low-Pass Filter



- At low frequencies, C_1 and C_2 appear as open circuits $\Rightarrow v_{out} \approx v_{in}$
- At high frequencies, C_1 and C_2 appear as short circuits $\Rightarrow v_{out} \approx 0\text{ V}$
- Near cut-off frequencies, impedance of C_1 and C_2 is on the same order as R_1 and R_2 $\Rightarrow$ positive feedback through C_1 provides Q enhancement of the signal

<https://www.ti.com/lit/an/sloa024b/sloa024b.pdf>

Sallen-Key Filter Topology: Low Pass Filter



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

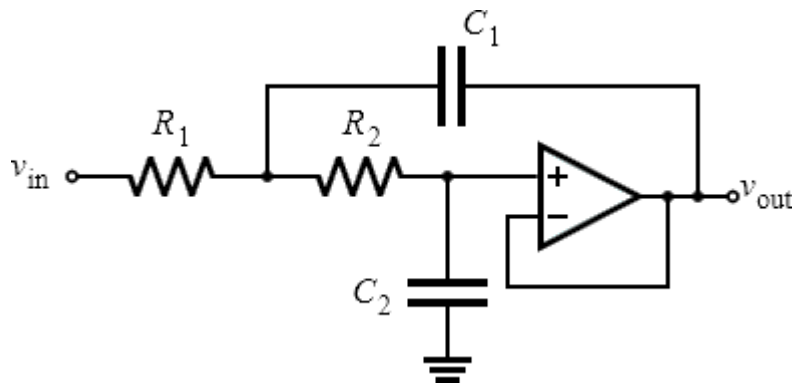
1. It can be formed by choosing $Z_1=R_1$, $Z_2=R_2$, $Z_3=1/j\omega C_1$, $Z_4=1/j\omega C_2$;

2. Convert the above equation to the following form $H(j\omega) = \frac{\omega_o^2}{(j\omega)^2 + \left(\frac{\omega_o}{Q}\right)(j\omega) + \omega_o^2}$

3. The undamped natural frequency ω_o is given by $\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$

4. The Q factor is given by $Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)}$

Sallen-Key Filter Topology: Low Pass Filter



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

$$\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)}$$

$$f_0 = 999\text{Hz}$$

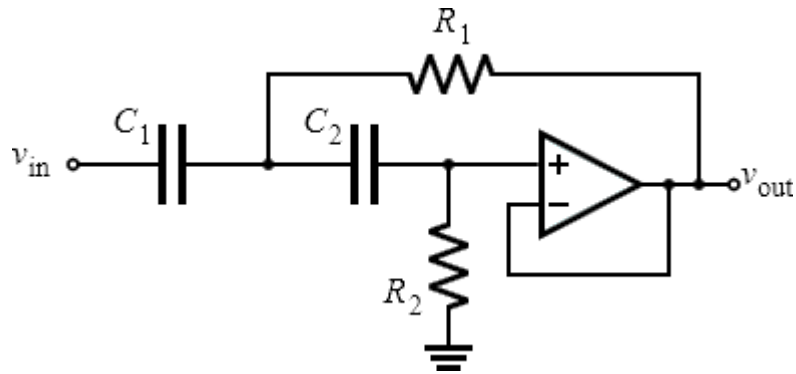
Given:

1. $R_1 = 30\text{k}\Omega$; $R_2 = 18\text{k}\Omega$; $C_1 = 10\text{nF}$; $C_2 = 4.7\text{nF}$, calculate ω_o and Q . ($Q = 0.7$)
2. $R_1 = 30\text{k}\Omega$; $R_2 = 18\text{k}\Omega$; $C_1 = 20\text{nF}$; $C_2 = 2.35\text{nF}$, calculate ω_o and Q . ($Q = 1.4$)

Plot the frequency response of your above two cases using

NI MultiSim: <http://www.ni.com/multisim>; <http://www.ni.com/multisim/mobile>

Sallen-Key Filter Topology: High Pass Filter



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

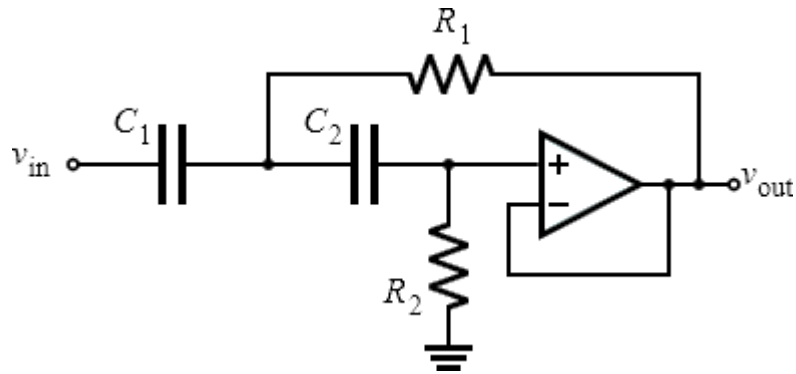
1. It can be formed by choosing $Z_1 = 1/j\omega C_1$, $Z_2 = 1/j\omega C_2$, $Z_3 = R_1$, $Z_4 = R_2$;

2. Convert the above equation to the following form $H(j\omega) = \frac{(j\omega)^2}{(j\omega)^2 + \left(\frac{\omega_o}{Q}\right)(j\omega) + \omega_o^2}$

3. The undamped natural frequency ω_o is given by $\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$

4. The Q factor is given by $Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2)}$

Sallen-Key Filter Topology: High Pass Filter



$$H(j\omega) = \frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3 (Z_1 + Z_2) + Z_3 Z_4}$$

$$\omega_o = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \quad Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2)}$$

$$f_0 = 999\text{Hz}$$

Given

1. $R_1 = 7.5\text{k}\Omega$; $R_2 = 72\text{k}\Omega$; $C_1 = 10\text{nF}$; $C_2 = 4.7\text{nF}$, calculate ω_o and Q . ($Q = 1.44$)
2. $R_1 = 15\text{k}\Omega$; $R_2 = 36\text{k}\Omega$; $C_1 = 10\text{nF}$; $C_2 = 4.7\text{nF}$, calculate ω_o and Q . ($Q = 0.72$)

Plot the frequency response of your above two cases using

NI MultiSim: <http://www.ni.com/multisim>; <http://www.ni.com/multisim/mobile>