

Test2A

ⓘ This is a preview of the draft version of the quiz

Started: Nov 18 at 6:13pm

Quiz Instructions

Test 2 will be conducted during lecture on Nov 19 starting at 2:00 pm. The deadline for completion and uploading the answers is 2:40 pm. Please make sure you follow all instructions from the University, ARRO, and EE. In addition, please note the following points.

1. The OPEN-BOOK test covers Chapters 1 to 10. Students can read the lecture notes and/or other materials available online.

2. There will be two parts to be conducted via Canvas\Assignments\Tests

Test2A: Short questions requiring selecting/typing the answers to Canvas. On Canvas, special notations are adopted. Should you encounter any issues reading the equations/files, raise your question to the Invigilator immediately. For example: (a) micrometer is written as μm (instead of " μm "), (b) The unit vectors are represented by $\mathbf{a}$ (instead of adding the "hat"); and (c) $\mathbf{a}_x$ is the unit vector along the x-direction, which is the same as " $\mathbf{x}$ hat" in the lectures.

Test2B: Long questions requiring handwriting on papers for uploading to Canvas.

Different from the previous test, the submissions of the two parts should be done separately. You are responsible to receive the questions on Canvas, hand-write all answers, compile the answers into a single PDF file, and upload the file before the deadline of the test/exam. (If possible, try to keep the file size to be less than 20 MB while maintaining a good resolution.)

3. Students can choose to complete the test at home or in the classroom.

(a) Students taking the test at home are required to be connected via Zoom with video.
-You are responsible for joining the Zoom meeting 10 minutes before the start time of the exam. You are encouraged to use a second device on Zoom and also record a video with sound as evidences.
-Stay on Zoom until the examiner allows you to leave. The examiner may request you via Canvas Inbox for individual Breakout Room Zoom meeting during the test and/or within 15 min. after the test.

(b) Students taking the test at the classroom do not need Zoom.

-You need to bring your own computer, blank papers, and scanning device.
-You have to sign-up via the [Survey for Week 12 \(including Test 2\)](#) at least **24 hours** before the test. Approval from the Course is automatically granted for entering the classroom (unless announced otherwise).

4. Canvas Inbox will be the main channel of communications in case students need to contact the Course Examiner.

5. On the first page of your answer sheets, you will be asked to sign, date, and show your student ID card: "I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties."

Students are NOT allowed to communicate with ANYONE during the test/exam. Answers, steps, and results that are highly similar amongst students would be SERIOUSLY scrutinized. Students caught committing any of the offenses could be heavily penalized. For example, if you allow your classmates to copy from you, you would receive heavy penalty as well. As always, you have to acknowledge the Pledge for following the Rules on Academic Honesty: *"I pledge that the answers in this exam are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically, (a) I will not plagiarize (copy without citation) from any source; (b) I will not communicate or attempt to communicate with any other person during the exam/quiz; neither will I give or attempt, directly or indirectly, to give or get assistance to or from another student taking the exam/quiz; and (c) I will use only approved devices (e.g., calculators) and/or approved device models. I understand that any act of academic dishonesty can lead to disciplinary action."*

Question 1

0 pts

When you have finished answering Test2A, press submit and continue for Test2B. Note that once the submission button is pressed, you cannot change the answers.

"I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties."

☐ True

☐ False

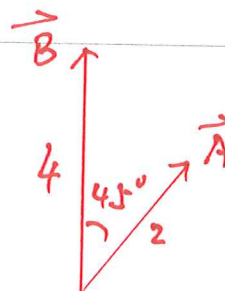
Question 2

5 pts

Consider vectors $\mathbf{A} = 2^{0.5}\mathbf{a}_x + 2^{0.5}\mathbf{a}_y$ and $\mathbf{B} = 4\mathbf{a}_y$

Find the cross product: $\mathbf{A} \times \mathbf{B}$

$$(4)(2)\sin 45^\circ \hat{z}$$



- ☐ $5.66\mathbf{a}_z$
- ☐ $-5.66\mathbf{a}_y$
- ☐ $5.66\mathbf{a}_x$
- ☐ $-5.66\mathbf{a}_z$

Question 3

5 pts

Consider an electric potential $V(x, y) = x^2/2$. Find the electric field $\mathbf{E}$ at $(x, y) = (1, 1)$.

- ☐ $+(1/2)\mathbf{a}_x$
- ☐ $-\mathbf{a}_x$
- ☐ $-(1/2)\mathbf{a}_y$
- ☐ $-1/2$

$$\begin{aligned}\vec{E} &= -\nabla V \\ &= -x \hat{x} \\ &= -(1) \hat{x}\end{aligned}$$

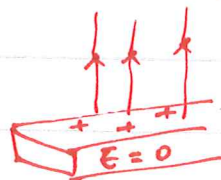
Question 4

5 pts

A rectangular metallic rod is placed inside an insulator. The surface charge density on the rod is $+8.9 \text{ pC/m}^2$. The insulator has a relative permittivity of 10. Find the electric field inside the metal.

- ☐ 0
- ☐ 0.1 V/m
- ☐ 8.9 V/m
- ☐ 1 V/m

①

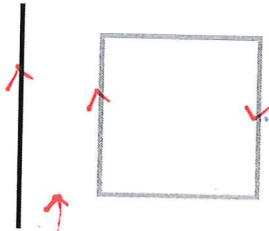


$$\begin{aligned}D &= 8.9 \text{ pC/m}^2 \quad (\text{B.C}) \\ E &= D / \epsilon_0 \epsilon_r \\ &= 10^{-12} / 10^{-12} \cdot 10 \\ \epsilon_r &= 10 \quad \text{② } E = 0\end{aligned}$$

Question 5

5 pts

Consider an infinitely long vertical wire carrying a current towards to the upward direction. A small rectangular coil is placed on the right of the wire. The coil carries a clockwise current. The force between the two objects is:


☒ Attractive

☐ Repulsive

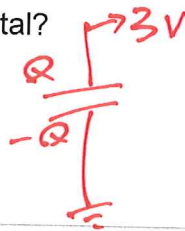
☐ Unknown

☐ Zero

Question 6

5 pts

Consider a coaxial cable with a capacitance 5 mF at a voltage of 3 V, how much charge is stored in the positive metal?



$$Q = CV$$

$$= 3 \times 10^{-3} \cdot 3 \text{ (C)}$$

unit!

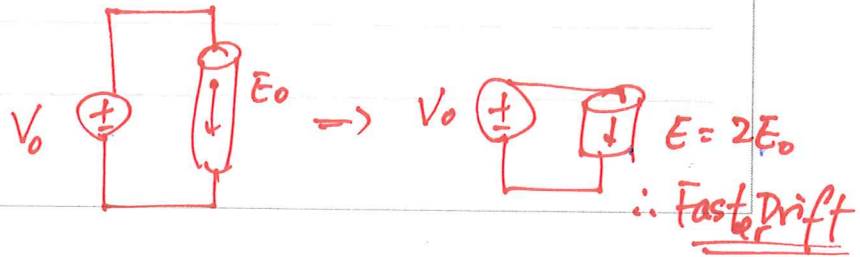
Question 7

5 pts

A voltage V_0 is applied across a piece of copper wire. The radius of the wire is kept constant. The drift velocity of the electrons in the wire can be increased by:

☐ None of the above

- ☐ Decreasing V_0
☐ Increasing the wire length
☒ Decreasing the wire length



Question 8

10 pts

Consider an electric field $\mathbf{E} = 5\mathbf{a}_y$ (V/m) and a magnetic field $\mathbf{B} = 2\mathbf{a}_z$ (T). A small charge of 4 nC is moving towards +z direction at speed 3 m/s. Find the magnitude of the electromagnetic force acting on the charge.

$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = q\vec{E}$

 $\therefore F = 4 \text{ nC} \cdot 5 \text{ V/m} = 20 \times 10^{-9} \text{ N}$

 $= 20 \text{ nN}$

$\vec{v} \parallel \vec{B} \Rightarrow \vec{v} \times \vec{B} = 0$

Question 9

10 pts

In rectangular coordinates (x, y, z), consider a field: $\mathbf{F} = (1+z)\mathbf{a}_x + z\mathbf{a}_y$. Find the path integral P along a straight line S from A(0, 0, 2) to B(0, 7, 2).

$P = \int_S \mathbf{F} \cdot d\mathbf{L} = 7(2)$

(If you cannot see the above equation of P, please inform the Invigilator immediately).

Question 10

10 pts

Placed in air are a pair of parallel metallic wires each of:

- 1 meters in length
- 2 mm in radius
- 10 mm in separation between the centers of the wires.

From Notes:

$$C = \frac{\pi \epsilon_0 \epsilon_r l}{\ln\left(\frac{2a}{b} - 1\right)} = 0.02 \text{ nF}$$

There is a voltage of 30 V between the two metals. Find the energy stored.

$$\begin{aligned}\text{Energy} &= \frac{1}{2} QV \\ &= \frac{1}{2} CV^2 \\ &= 9 \text{ nJ} \# \end{aligned}$$

Quiz saved at 6:13pm

Submit Quiz

City University of Hong Kong
Electrical Engineering
Semester A 2020-2021
EE2104 Introduction to Electromagnetics
Test 2B

On the first page of your answer sheets, copy the following sentence and sign it: *I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties.* (Signature)_____ (Date)_____

For Test 2, Part A on short questions should be submitted to Canvas directly, while Part B on the long questions below should be submitted by handwriting, scanning, and uploading to Canvas. The two submissions are independent.

Question B1 (20 points)

An infinitely long medium with a circular cross-section of radius a is placed along the z -axis. The region for radial coordinate ρ between a and $2a$ is filled with air, where the electric potential is known to be

$$V(\rho, \phi, z) = V_0 \left(\frac{\rho}{a}\right)^7 \cos(m\phi)$$

for some constant voltage V_0 and positive constant m . Note that the region other than $a < \rho < 2a$ is unknown so that the usual symmetry argument may not apply.

- (a) Find the magnitude of the electric field at $(x, y) = (a, 0)$.
- (b) Find m by providing a clear reasoning.

Solution:

- (a) Since $\mathbf{E} = -\nabla V$.

$$\begin{aligned}\mathbf{E} &= \frac{\partial V}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \hat{\mathbf{a}}_{\phi} \\ &= \frac{7V_0\rho^6}{a^7} \cos(m\phi) \hat{\rho} - \frac{V_0\rho^6 m}{a^7} \sin(m\phi) \hat{\mathbf{a}}_{\phi}\end{aligned}$$

At $(x, y) = (a, 0)$, $\rho = a$ and $\phi = 0$, so the electric field magnitude is

$$E = \frac{7V_0}{a}$$

- (b) Laplace equation: $\nabla^2 V = \nabla \cdot (\nabla V) = 0$

So we have

$$\nabla \cdot \left(\frac{7V_0\rho^6}{a^7} \cos(m\phi) \hat{\rho} - \frac{V_0\rho^6 m}{a^7} \sin(m\phi) \hat{\mathbf{a}}_{\phi} \right) = 0$$

Then we have

$$\frac{7^2 V_0 \rho^6}{a^7} \cos(m\phi) = \frac{m^2 V_0 \rho^6}{a^7} \cos(m\phi)$$

Therefore, $m = 7$.

Question B2 (20 points)

An infinitely long wire with a circular cross-section of radius b is placed along the z -axis. It carries a current density towards the $+z$ -direction with the magnitude being dependent on the radial coordinate ρ

$$J(\rho) = J_0 \rho^8 / b^8$$

for some constant J_0 (A/m²). The current generates a magnetic field $\mathbf{B}$. Also, there is a measurement point $P(x, y, z) = (b, b, 0)$ as expressed in the Cartesian coordinates.

- What is the direction of the magnetic field at point P ? Express the answer in terms of Cartesian unit vectors $\mathbf{a}_x$ and $\mathbf{a}_y$. Please provide the reason in one sentence.
- Does the field magnitude $B(\rho, \phi, z)$ depend on coordinate ρ ? Please provide the reason in one sentence.
- Find the magnitude B at point P .

Solution:

- The direction of the magnetic field at point P is towards $-\mathbf{a}_x + \mathbf{a}_y$.
This is because the current density is towards the $+z$ -direction and independent on (ϕ, z) .
- The field magnitude $B(\rho, \phi, z)$ is actually dependent on coordinate ρ , although it is independent of ϕ .
This is again because of symmetry Point P is outside the wire.
- Using Ampere's law, we have

$$\oint_L \mathbf{H} \cdot d\mathbf{L} = I$$

Thus,

$$2\sqrt{2}b\pi H = \int_{\phi=0}^{2\pi} \int_{\rho=0}^b J(\rho) \rho d\rho d\phi = \int_{\phi=0}^{2\pi} \int_{\rho=0}^b J_0 \rho^8 / b^8 \rho d\rho d\phi$$

Then we have

$$2\sqrt{2}b\pi H = \frac{\pi b^2 J_0}{5}$$

$$\text{So } H = \frac{\sqrt{2}J_0 b}{20}$$

$$\text{The magnitude } B \text{ at point } P \text{ is } \mu_0 \frac{\sqrt{2}J_0 b}{20}.$$