

GE1359 Algebra
Summer 2021
Test

Date: Tuesday, 13 July 2021
Time allowed: 90 minutes (14:15 – 15:45)
Submission time: 15 minutes (15:45 – 16:00)

Instructions to candidates:

This paper has FOUR pages (including this cover page).

The formula sheet of trigonometric identities is provided on page 4.

This paper has SEVEN questions.

The questions will begin on page 2.

Answer ALL questions. Show ALL steps to get full mark.

Please submit your work as a single pdf file (with the signed Academic Honesty Pledge as the front page) to Canvas before 16:00 HK time on 13 July 2021.

Question 1

For the following real-valued function, find its largest possible domain and largest possible range.

Give your answers in set/interval notations.

$$f(x) = 3 - \sqrt{4 - x^2}. \quad [12 \text{ marks}]$$

Question 2

Consider the real-valued function

$$g(x) = \frac{4}{\sqrt{1-x}} \quad \text{for } x \leq -3.$$

- (a) Determine whether $g(x)$ is a one-to-one function. [4 marks]
- (b) Find $g^{-1}(x)$, the inverse function of $g(x)$, if it exists. [4 marks]
- (c) Find the largest possible domain and largest possible range of $g^{-1}(x)$. [10 marks]

Question 3

- (a) Given that $\sin \alpha = \frac{3}{5}$, where $90^\circ < \alpha < 180^\circ$. Without using calculator, find the values of $\sin(2\alpha)$ and $\cos(2\alpha)$, and determine the quadrant containing 2α . Give reasons. [10 marks]
- (b) If $A + B = \frac{\pi}{4}$ (in radians), find the value of $(1 + \tan A)(1 + \tan B)$. [8 marks]

Question 4

- (a) Prove the identity: $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$. [6 marks]
- (b) Using (a), or otherwise, find the general solution (in radians) of the equation

$$8 \cos^3(2x) \sin x - 6 \cos(2x) \sin x = \sin x. \quad [10 \text{ marks}]$$

Question 5

Express $\frac{x^3 - 6x^2 + 6x + 2}{x^4 - x^3 - x + 1}$ into partial fractions. [14 marks]

Question 6

Solve the following inequality. Give your answers in interval/set notations.

$$|x| + 8 < 3|x - 2|. \quad [10 \text{ marks}]$$

Question 7

Using Gaussian elimination, determine the value(s) of p for which the system of linear equations

$$\begin{cases} x + 3y + 2z - w = 4 \\ -2x - 4y + 2z + 5w = -p \\ 3x + 5y + pz - 9w = -3 \end{cases}$$

is consistent and then solve the system in this case. [12 marks]

- End -

Trigonometric identities - Formula sheet

➤ Compound Angle Formulae

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

➤ Double Angle Formulae

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

➤ Half Angle Formulae

$$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$$

$$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$$

➤ Product-to-Sum Formulae

$$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

$$\sin A \sin B = -\frac{1}{2}[\cos(A + B) - \cos(A - B)]$$

➤ Sum-to-Product Formulae

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$