

EE2302 Foundations of Information and Data Engineering

Assignment 1 (Solution)

1.

- a. $C \subseteq D$ Proof: [We will show that every element of C is in D .] Suppose n is any element of C . Then $n = 6r - 5$ for some integer r . Let $s = 2r - 2$. Then s is an integer (because products and differences of integers are integers), and

$$3s + 1 = 3(2r - 2) + 1 = 6r - 6 + 1 = 6r - 5,$$

which equals n . Thus n satisfies the condition for being in D . Hence, every element in C is in D .

- b. $D \not\subseteq C$ because there are elements of D that are not in C . For example, 4 is in D because $4 = 3 \cdot 1 + 1$. But 4 is not in C because if it were, then $4 = 6r - 5$ for some integer r , which would imply that $9 = 6r$, or, equivalently, that $r = 3/2$, and this contradicts the fact that r is an integer.

2.

a)

False. Counterexample: Any sets A , B , and C where A and C have elements in common that are not in B will serve as a counterexample. For instance, let $A = \{1, 2, 3\}$, $B = \{2, 3\}$, and $C = \{3\}$. Then $B - C =$

$\{2\}$, and so $A - (B - C) = \{1, 2, 3\} - \{2\} = \{1, 3\}$. On the other hand $A - B = \{1, 2, 3\} - \{2, 3\} = \{1\}$, and so $(A - B) - C = \{1\} - \{3\} = \{1\}$. Since $\{1, 3\} \neq \{1\}$, $A - (B - C) \neq (A - B) - C$.

b)

True. Proof: Let A and B be any sets.

$A \cap (A \cup B) \subseteq A$: Suppose $x \in A \cap (A \cup B)$. By definition of intersection, $x \in A$ and $x \in A \cup B$. In particular $x \in A$. Thus, by definition of subset, $A \cap (A \cup B) \subseteq A$.

$A \subseteq A \cap (A \cup B)$: Suppose $x \in A$. Then by definition of union, $x \in A \cup B$. Hence $x \in A$ and $x \in A \cup B$, and so, by definition of intersection $x \in A \cap (A \cup B)$. Thus, by definition of subset, $A \subseteq A \cap (A \cup B)$.

Because both $A \cap (A \cup B) \subseteq A$ and $A \subseteq A \cap (A \cup B)$ have been proved, we conclude that $A \cap (A \cup B) = A$.

c)

True. Proof: Let A , B and C be any sets.

$A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$:

Suppose $x \in A \cap (B - C)$.

By the definition of intersection, $x \in A$ and $x \in (B - C)$, which implies $x \in A$, $x \in B$ and $x \notin C$.

Since $x \notin C$, by the definition of intersection, $x \notin A \cap C$.

Therefore, $x \in A \cap B$ and $x \notin A \cap C$.

This is equivalent to $x \in (A \cap B) - (A \cap C)$.

Hence, $A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$.

$(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$:

Suppose $x \in (A \cap B) - (A \cap C)$. By the definitions of intersection and set difference, $x \in A$, $x \in B$ and $x \notin (A \cap C)$.

Since $x \in A$ and $x \notin (A \cap C)$, we must have $x \notin C$. [You may draw a Venn diagram to see that.]

Since $x \in B$ and $x \notin C$, we have $x \in B - C$.

Since $x \in A$ and $x \in B - C$, we have $x \in A \cap (B - C)$.

Hence, $(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$.

Combining the two parts, we conclude that $A \cap (B - C) = (A \cap B) - (A \cap C)$.

3. No. Suppose there were a computer program P that had as output a list of all computer programs that do not list themselves in their output. Consider the following two cases:
- a) If P lists itself as output, then it would be on the output list of P , which means that P would not list itself in its output. A contradiction.
 - b) If P does not list itself as output, then it would be a member of the list of all computer programs that do not list themselves in their output, and this list is exactly the output of P . Hence, P would list itself as output. Again, a contradiction.

In both cases, the assumption of the existence of such a program P is contradictory, and so no such program exists.