

EE2800 Assignment 2

Due: 9:00 AM, Mar. 5, 2024

1. In class, we have given the primitive cells of BCC and FCC lattices, which are also copied here as Fig. 1. The primitive basis vectors of the two primitive cells can be written as:

BCC lattice: $\mathbf{a} = \frac{a}{2}(-\hat{i} + \hat{j} - \hat{k})$, $\mathbf{b} = \frac{a}{2}(-\hat{i} + \hat{j} + \hat{k})$, and $\mathbf{c} = \frac{a}{2}(-\hat{i} - \hat{j} + \hat{k})$

FCC lattice: $\mathbf{a} = \frac{a}{2}(-\hat{i} + \hat{j})$, $\mathbf{b} = \frac{a}{2}(\hat{j} + \hat{k})$, and $\mathbf{c} = \frac{a}{2}(-\hat{i} + \hat{k})$

(30 marks)

- Using the equations provided in lecture notes, calculate the **reciprocal** lattice basis vectors, $\mathbf{a}^*$, $\mathbf{b}^*$, and $\mathbf{c}^*$, of this BCC lattice. (12 marks)
- Using the reciprocal lattice basis vectors obtained in part a), draw the reciprocal primitive cell in a similar style as that in Fig. 1(b), and show that the reciprocal lattice of a BCC direct lattice is an FCC lattice (10 marks).

Note: the reciprocal lattice you get may not look the same as in Fig. 1(b) since there are more than one way to choose a primitive cell.

- Calculate the volume of the primitive cell of this reciprocal lattice, V_C^* . Show that $V_C^* = (2\pi)^3 / V_C$, where V_C is the volume of the primitive cell of the direct lattice. (8 marks)

Note: in this part you are welcome to directly use the conclusions you obtained in Assignment 1, Q5.

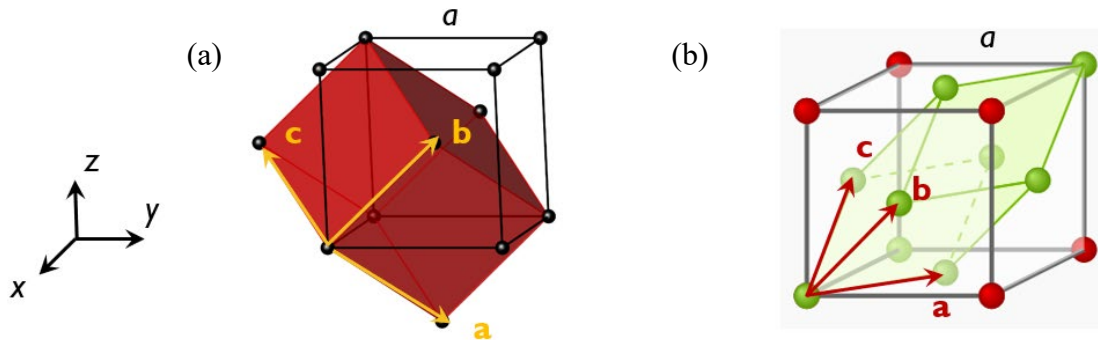


Figure 1. Primitive cells of a BCC and an FCC lattice.

2. For a Si sample at room temperature (300 K), assuming the effective densities of states are $N_C = 2.8 \times 10^{19} \text{ cm}^{-3}$ and $N_V = 2.65 \times 10^{19} \text{ cm}^{-3}$, the bandgap is 1.12 eV. In this question, we assume that all dopant atoms are fully ionized. (40 marks)
- What is the intrinsic free carrier density n_i ? (6 marks)

- b. If we dope this Si sample with $2 \times 10^{14} \text{ cm}^{-3}$ of phosphorous atoms (is this donor or acceptor?), calculate the free electron density n and hole density p . (8 marks)

Note: in this question the calculation can be simplified by taking proper approximation, but make sure you state clearly the reasoning before doing so.

- c. Following the assumption in part b), calculate the relative location of the Fermi level with respect to: (8 marks)
- the intrinsic Fermi level, *i.e.* $E_F - E_i$.
 - the conduction band edge, *i.e.* $E_F - E_C$.
 - the valence band edge, *i.e.* $E_F - E_V$.
 - the respective donor/acceptor impurity level, *i.e.* $E_F - E_{D/A}$. The relative location of various impurity levels in Si is given in Fig. 2.
- d. Using your result in part c), calculate the ionization ratio, *i.e.* $N_{D/A}^+ / N_{D/A}$, of this dopant. Is your result consistent with our initial assumption that all dopant atoms are (nearly) fully ionized? (8 marks)
- e. If instead of phosphorous, the Si sample is doped with $2 \times 10^{10} \text{ cm}^{-3}$ of boron atoms (is this donor or acceptor?), calculate the free electron density n and hole density p . (10 marks)

Note: check whether the same approximation as in part b) is still valid or not.

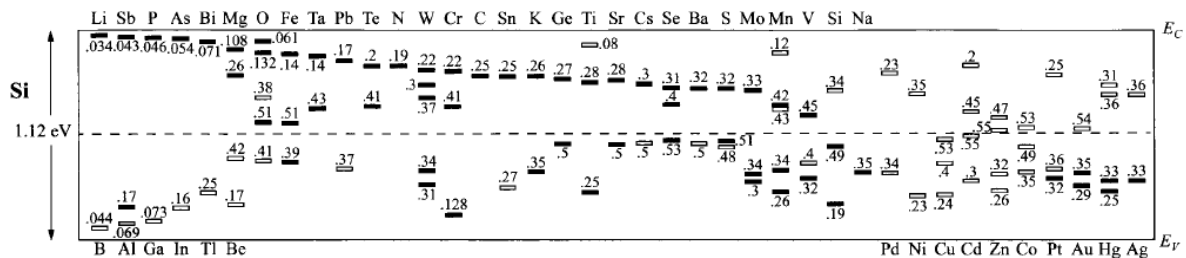


Figure 2. Energy levels of various impurities in Si. The numbers labeled near each impurity level refers to $E_C - E_D$ for donors and $E_A - E_V$ for acceptors.

3. A Si sample is doped with 10^{16} cm^{-3} of boron atoms, and a certain number of shallow donors at 300 K. The Fermi level is 0.36 eV above E_i at 300 K. Assuming the intrinsic carrier concentration n_i is the same as your result in Q2(a) and all dopants are completely ionized, calculate the donor concentration N_D . (10 marks)
4. Find the resistivity at 300 K for a Si sample that is **simultaneously** doped with $1.0 \times 10^{14} \text{ cm}^{-3}$ of phosphorous atoms, $8.5 \times 10^{12} \text{ cm}^{-3}$ of arsenic atoms, and $1.2 \times 10^{13} \text{ cm}^{-3}$ of boron

atoms. Assume that the dopants are completely ionized, and the mobilities are $\mu_n = 1450 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ and $\mu_p = 500 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$, independent of impurity concentrations. (10 marks)

5. How long does it take an average electron to drift $1 \text{ }\mu\text{m}$ in pure Si at an electric field of:
- a. 100 V/cm
 - b. 10^5 V/cm

In this question, we assume the saturation velocity is $v_s = 10^7 \text{ cm/s}$, and the mobilities at low fields are $\mu_n = 1450 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$ and $\mu_p = 500 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$, respectively (10 marks)