

Problem Set 2: Higher Order Linear ODEs with Constant Coefficients

- Find the general solutions of the following differential equations:
 (a) $y'' + 3y' = 0$, (b) $y'' - 3y' + 2y = 0$, (c) $y''' - 8y = 0$,
 (d) $y^{(4)} - 10y'' + 9y = 0$, (e) $y^{(4)} + 2y'' + y = 0$.
- Using the method of variation of parameters, find a particular integral of $y'' - 3y' + 2y = (1+x)e^x$.
- Find the general solution of the following differential equations:
 (a) $y'' + 4y' + 13y = e^{-2x} \cos x$.
 (b) $y'' + 6y' + 13y = xe^{-3x} \cos 2x$
- Using the method of undetermined coefficients to solve the following initial value problem
 $y''' + y'' = x^2 e^{-x}$ with $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$.
- Using the method of undetermined coefficients, find the general solution of
 $y'' - 9y = x^2 + \cosh 3x$.
- Find the general solution of the differential equation $y'' + 4y = \sec 2x$, using variation of parameters.
- Find a particular integral of $\frac{d^3x}{dt^3} - 7\frac{d^2x}{dt^2} + 15\frac{dx}{dt} - 9x = e^{3t}t^3$.
- If $y = y(x)$ and $x = e^t$, show that $\frac{dy}{dt} = x\frac{dy}{dx}$ and $\frac{d^2y}{dt^2} = x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$. Hence, or otherwise, find the particular solution of the differential equation $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{x}$ with $y(1) = 0$ and $y(2) = 0$.
- An inductor of 2 henries, resistor of 12 ohms and capacitor of 0.02 farads are connected in series with a battery of electromotive force $E = 100\sin 4t$. If the charge on the capacitor and current in the circuit are zero at time $t = 0$, find the charge and current when $t > 0$.

(Hint: Let $Q(t)$ and $I(t)$ be the instantaneous charge and current at time t .

By Kirchhoff's laws: $2\frac{dI}{dt} + 12I + \frac{Q}{0.02} = 100\sin 4t$.)

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