

Op-amps with resistive components

Q1. [Problem 8.53]

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Solution:

- Known quantities:

For the circuit shown in Figure P8.53:

$$C = 1 \mu\text{F} \quad R_1 = 1.8 \text{ k}\Omega \quad R_2 = 8.2 \text{ k}\Omega \quad R_L = 333 \Omega.$$

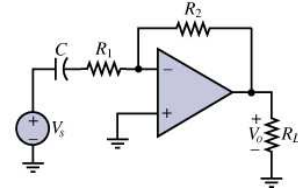


Figure P8.53

- Find:

- Whether the circuit is a low- or high-pass filter.
- The gain V_o/V_s in decibel in the passband.
- The cutoff frequency.

- Analysis:

- From Figure 8.25, it results that the amplifier in Figure P8.53 is a high-pass filter.

In fact, the output voltage is $V_o(j\omega) = -\frac{j\omega CR_2}{1 + j\omega CR_1} V_s(j\omega)$

$$\text{b) } \lim_{\omega \rightarrow \infty} \left| \frac{V_o(j\omega)}{V_s(j\omega)} \right|_{dB} = 20 \text{Log} \frac{R_2}{R_1} = 20 \text{Log} \frac{8.2}{1.8} = 13.17 \text{ dB}$$

$$\text{c) } \omega_0 = \frac{1}{CR_1} = \frac{1}{1 \cdot 10^{-6} \cdot 1.8 \cdot 10^3} = 5555 \text{ rad/s}$$

Q2. [Problem 8.57]

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Solution:

- Known quantities:

For the circuit shown in Figure P8.57:

$$C = 0.47 \mu\text{F} \quad R_1 = 9.1 \text{ k}\Omega \quad R_2 = 22 \text{ k}\Omega \quad R_L = 2.2 \text{ k}\Omega.$$

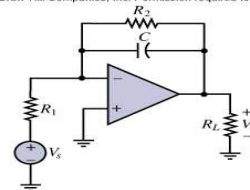


Figure P8.57

- Find:

- Whether the circuit is a low- or high-pass filter.
- An expression in standard form for the voltage transfer function.
- The gain in decibels in the passband, that is, at the frequencies being passed by the filter, and the cutoff frequency.

- Analysis:

- From Figure 8.22, it results that the amplifier in Figure P8.57 is a low-pass filter. In fact, the output voltage is

$$V_o(j\omega) = -\frac{R_2}{R_1} \frac{1}{1 + j\omega CR_2} V_s(j\omega)$$

$$\text{b) } H_v(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{R_2}{R_1} \frac{1}{1 + j\omega CR_2}$$

$$\text{c) } \text{The gain in decibel is obtained by evaluating } |H_v(j\omega)| \text{ at } \omega=0, |H_v(j0)|_{dB} = 20 \text{Log} \frac{R_2}{R_1} = 20 \text{Log} \frac{22}{9.1} = 7.66 \text{ dB}.$$

$$\text{The cutoff frequency is } \omega_0 = \frac{1}{CR_2} = \frac{1}{0.47 \cdot 10^{-6} \cdot 22 \cdot 10^3} = 96.71 \text{ rad/s}$$

Q3. [Problem 8.61]Solution:

- Known quantities:

For the circuit shown in Figure P8.61:

$$C_1 = 2.2 \mu\text{F} \quad C_2 = 1 \text{ nF} \quad R_1 = 2.2 \text{ k}\Omega \quad R_2 = 100 \text{ k}\Omega.$$

- Find:

Determine the passband gain.

- Analysis:

The voltage frequency response is

$$A_{BP}(j\omega) = -\frac{\mathbf{Z}_2}{\mathbf{Z}_1} = -\frac{j\omega C_1 R_2}{(1 + j\omega C_1 R_1)(1 + j\omega C_2 R_2)} \Rightarrow |A_{BP}(j\omega)| = \frac{\omega C_1 R_2}{\sqrt{1 + \omega^2 (C_1 R_1)^2} \sqrt{1 + \omega^2 (C_2 R_2)^2}}$$

The cutoff frequencies

$$\text{are } \omega_1 = \frac{1}{C_1 R_1} = \frac{1}{2.2 \cdot 10^{-6} \cdot 2.2 \cdot 10^3} = 206.6 \text{ rad/s} \quad \omega_2 = \frac{1}{C_2 R_2} = \frac{1}{1 \cdot 10^{-9} \cdot 100 \cdot 10^3} = 10000 \text{ rad/s}$$

The passband gain can be calculated approximately by evaluating the magnitude of the frequency response at frequencies greater than ω_1 and smaller than ω_2 , i.e.

$$\omega_1 \ll \omega \ll \omega_2 \Rightarrow 1 + \omega^2 (C_1 R_1)^2 \approx \omega^2 (C_1 R_1)^2, \quad 1 + \omega^2 (C_2 R_2)^2 \approx 1 \quad |A_{BP}| \cong \frac{\omega C_1 R_2}{\sqrt{\omega^2 (C_1 R_1)^2}} = \frac{R_2}{R_1} = \frac{100}{2.2} = 45.45$$

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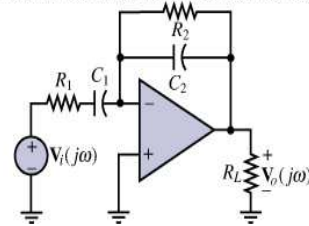


Figure P8.61

Q4. [Problem 8.59]Solution:

- Known quantities:

For the circuit shown in Figure P8.59:

$$C_1 = C_2 = 0.1 \mu\text{F} \quad R_1 = R_2 = 10 \text{ k}\Omega.$$

- Find:

- The passband gain.
- The resonant frequency.
- The cutoff frequencies.
- The circuit Q.
- The Bode plot.

- Analysis:

- From Figure 8.27, we have

$$A_{BP}(j\omega) = -\frac{\mathbf{Z}_2}{\mathbf{Z}_1} = -\frac{j\omega C_1 R_2}{(1 + j\omega C_1 R_1)(1 + j\omega C_2 R_2)} = -\frac{j\omega C_1 R_1}{(1 + j\omega C_1 R_1)^2} = -\frac{j\omega C_1 R_1}{(j\omega)^2 (C_1 R_1)^2 + 2j\omega C_1 R_1 + 1}$$

The magnitude of the frequency response is $|A_{BP}(j\omega)| = \frac{\omega C_1 R_1}{1 + \omega^2 (C_1 R_1)^2}$

The passband gain is the maximum over ω of the magnitude of the frequency response, i.e. $\left| A_{BP}\left(j\frac{1}{C_1 R_1}\right) \right| = \frac{1}{2}$

- The resonant frequency is $\omega_n = \frac{1}{R_1 C_1} = 1000 \text{ rad/s}$

- The cutoff frequencies are obtained by solving with respect to ω the equation

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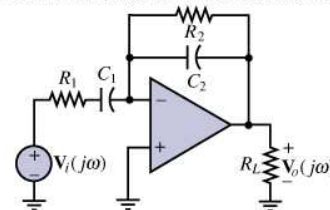


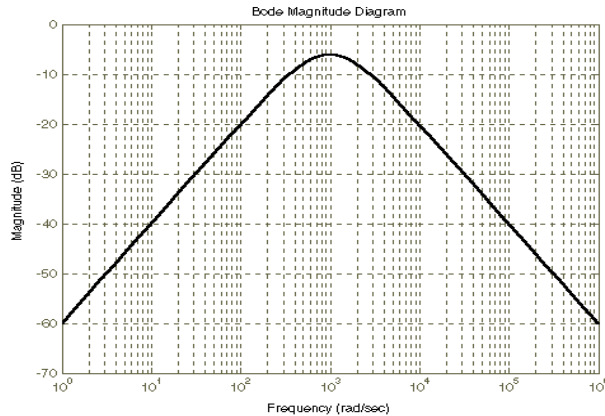
Figure P8.59

$$|A_{BP}(j\omega)| = \frac{\omega C_1 R_1}{1 + \omega^2 (C_1 R_1)^2} = \frac{1}{2\sqrt{2}} \Rightarrow \omega^2 - 2\sqrt{2}\omega_n\omega + \omega_n^2 = 0 \Rightarrow \omega_{1,2} = \omega_n(\sqrt{2} \pm 1)$$

$$\omega_1 = 2414 \text{ rad/s} \quad \omega_2 = 414 \text{ rad/s}$$

d) In this case $\zeta=1$, which implies $Q = \frac{1}{2\zeta} = \frac{1}{2}$

e) The Bode plot of the frequency response is as shown.



Q5. [Problem 8.55]

Solution:

• Known quantities:

For the circuit shown in Figure P8.55:

$$C = 100 \text{ pF} \quad R_1 = 4.7 \text{ k}\Omega \quad R_2 = 68 \text{ k}\Omega \quad R_L = 220 \text{ k}\Omega$$

• Find:

Determine the cutoff frequencies and the magnitude of the voltage frequency response at very low and at very high frequencies.

• Analysis:

The output voltage in the frequency domain is

$$\mathbf{V}_0(j\omega) = \left(1 + \frac{R_2}{\frac{1}{j\omega C} + R_1} \right) \mathbf{V}_i(j\omega) = \frac{1 + j\omega C(R_1 + R_2)}{1 + j\omega C R_1} \mathbf{V}_i(j\omega)$$

The circuit passes all frequencies, with a higher gain at higher than at lower frequencies. The cutoff frequencies are

$$\omega_1 = \frac{1}{C R_1} = \frac{1}{100 \cdot 10^{-12} \cdot 4.7 \cdot 10^3} = 2.127 \cdot 10^6 \text{ rad/s}$$

$$\omega_2 = \frac{1}{C(R_1 + R_2)} = \frac{1}{100 \cdot 10^{-12} \cdot 72.7 \cdot 10^3} = 1.375 \cdot 10^5 \text{ rad/s}$$

For high frequencies we have

$$A_\infty = \lim_{\omega \rightarrow \infty} \left| \frac{\mathbf{V}_0(j\omega)}{\mathbf{V}_i(j\omega)} \right| = 1 + \frac{R_2}{R_1} = 1 + \frac{68}{4.7} = 15.46$$

At low frequencies

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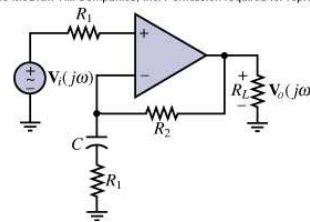


Figure P8.55

$$A_0 = \lim_{\omega \rightarrow 0} \left| \frac{V_0(j\omega)}{V_i(j\omega)} \right| = 1$$

Q6. [Problem 8.56]Solution:

- Known quantities:

For the circuit shown in Figure P8.56:

$$C = 20 \text{ nF} \quad R_1 = 1\text{k}\Omega \quad R_2 = 4.7\text{k}\Omega \quad R_3 = 80\text{k}\Omega.$$

- Find:

a) An expression for $H_v(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)}$.

- b) The cutoff frequencies.
c) The passband gain.
d) The Bode plot.

- Analysis:

- a) The frequency response is

$$H_v(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)} = 1 + \frac{R_3 \parallel \frac{1}{j\omega C}}{R_2} = \frac{R_2 + R_3 + j\omega C R_2 R_3}{R_2 + j\omega C R_2 R_3} = \left(1 + \frac{R_3}{R_2} \right) \frac{1 + j\omega C \frac{R_2 R_3}{R_2 + R_3}}{1 + j\omega C R_3}$$

- b) The cutoff frequencies are

$$\omega_1 = \frac{1}{C R_3} = \frac{1}{20 \cdot 10^{-9} \cdot 80 \cdot 10^3} = 625 \text{ rad/s}$$

$$\omega_2 = \frac{(R_2 + R_3)}{C R_2 R_3} = \frac{84.7 \cdot 10^3}{20 \cdot 10^{-9} \cdot 80 \cdot 10^3 \cdot 4.7 \cdot 10^3} = 11263 \text{ rad/s}$$

- c) The passband gain is obtained by evaluating the frequency response at low frequencies,

$$A_0 = \lim_{\omega \rightarrow 0} |H_v(j\omega)| = 1 + \frac{R_3}{R_2} = 1 + \frac{80}{4.7} = 18$$

- d) The magnitude Bode plot for the given amplifier is as shown.

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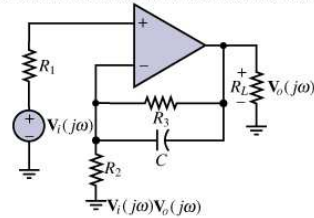
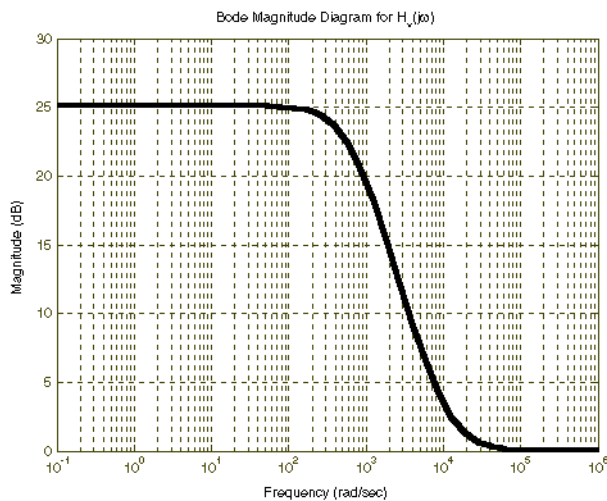


Figure P8.56



Q7. [Problem 8.11]Solution:

- Known quantities:

The circuit of Figure 8.11.

- Find:

The impedance of $Z_{in} = V_1/I_1$

- Assumptions

Ideal op-amp open-loop voltage gain A is very big.

- Analysis

Applying KVL to node 1,

$$V_o = V_1 - RI_1 \quad (1)$$

Applying KVL to node 2,

$$\frac{V_2}{Z_L} = \frac{V_o - V_2}{R} \quad (2)$$

$$\text{At the same time, } V^+ = V^- \Rightarrow V_2 = V_1 \quad (3)$$

From (2) and (3), we have:

$$\frac{V_1}{Z_L} = \frac{V_o - V_1}{R} \quad (4)$$

From (1) and (4), we have:

$$\frac{V_1}{Z_L} = -I_1 \Rightarrow Z_{in} = \frac{V_1}{I_1} = -Z_L \quad (5)$$

So,

$$\text{a. when } Z_L = R, \Rightarrow Z_{in} = -R$$

$$\text{b. when } Z_L = 1/j\omega C, \Rightarrow Z_{in} = -1/j\omega C$$

Q8. [Problem 8.12]Solution:

- Known quantities:

The circuit of Figure P8.12. $C_1=2Q$, $C_2=1/2Q$.

- Find:

Determine the gain function V_{out}/V_{in}

- Analysis:

Applying KVL at node 1:

$$\frac{V_{in} - V_1}{1} + \frac{V_{out} - V_1}{Z_1} = \frac{V_1 - V_2}{1} \quad (1)$$

Again, at node 2:

$$\frac{V_1 - V_2}{1} = \frac{V_2}{Z_2} \Rightarrow V_1 = (1 + 1/Z_2)V_2 \quad (2)$$

And easy to get:

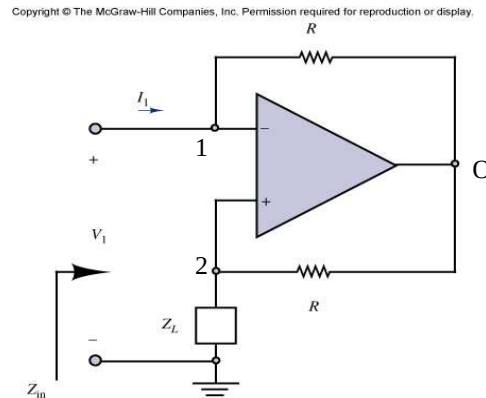


Figure P8.11

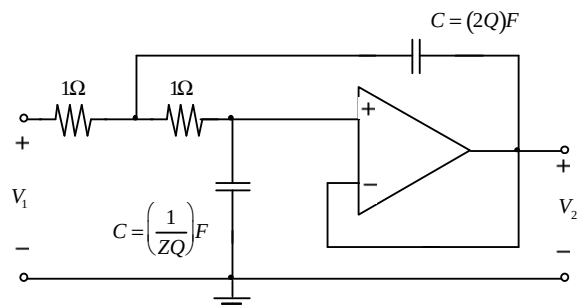


Figure P8.12

$$V_{out} = V_2 \quad (3)$$

Substituting (3) and (3) into (1):

$$\frac{V_{out}}{V_{in}} = \frac{Z_1 Z_2}{Z_1 Z_2 + 2Z_1 + 1}$$

Note that $Z_1 = 1/j\omega 2Q$, $Z_2 = 2Q/j\omega$

$$\text{We have } \frac{V_{out}}{V_{in}} = \frac{1}{(j\omega)^2 + j\omega/Q + 1}$$

Q9. [Problem 8.14]

Solution:

- Known quantities:
The circuit of P8.14.

- Find:

$$Z_{in} = \frac{V_1}{I_1}$$

- Analysis:

We know that the voltage at point 2 is equal to V_1 . Thus, the current through from point 2 to ground is V_1/R . Hence the current from O2 to point 2 is also V_1/R . Hence the voltage at O2 is $V_1 + V_1/(j\omega RC)$.

Since the voltage at point 3 is V_1 , the current from O2 to point 3 is $(V_1 + V_1/(j\omega RC) - V_1) = V_1/(j\omega R^2 C)$. But this must also be the current from point 3 to O1; hence the voltage at O1 is $V_1 + (1/j\omega C)V_1/(j\omega R^2 C) = V_1 + V_1/(-\omega^2 R^2 C^2)$. The voltage from O1 to point 1 is therefore $V_1/(-\omega^2 R^2 C^2)$, The current through the top resistor R is therefore $V_1/(-\omega^2 R^3 C^2)$. Thus, the current I_1 must be equal and opposite:

$$I_1 = V_1/(\omega^2 R^3 C^2).$$

And the impedance is

$$Z_{in} = \frac{V_1}{I_1} = \omega^2 C^2 R^3$$

Hence, this circuit behaves as a frequency-dependent resistor!

Q10. [Problem 8.22]

Solution:

- Known quantities:
The circuit of Figure P8.22.

- Find:

The peak amplitude and phase shift of $v_{out}(t)$.

- Analysis:

Replacing the components with impedances Z_f and Z_s yields the circuit shown below.

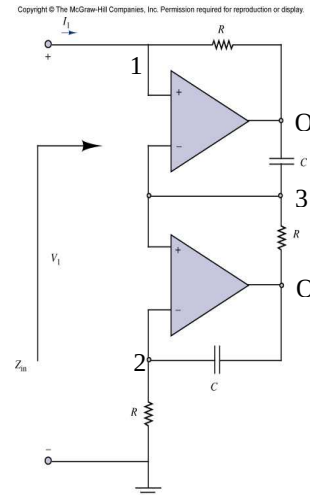


Figure P8.14

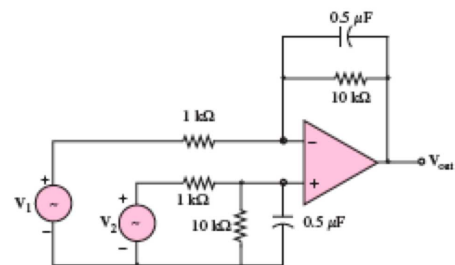


Figure P8.22

Recognizing that the circuit is a differential amplifier we can directly write:

$$v_{out}(t) = \frac{Z_f}{Z_s} (V_1 - V_2)$$

$$Z_s = 1000 \, \Omega \quad Z_f = \frac{10000}{1 + j\omega 5 \cdot 10^{-3}}$$

$$V_{out} = \frac{10}{1 + j\omega 5 \cdot 10^{-3}} (V_1 - V_2)$$

(a) Evaluating this voltage at $\omega=1,000$, we have

$$v_{out}(t) = 1.961 \cos(1000t - 78.7^\circ)$$

Thus, the peak value is 1.961 V.

(b) The phase shift of $v_{out}(t)$ is $\phi = -\tan^{-1}\left(\frac{1000(5 \cdot 10^{-3})}{1}\right) = -78.7$

