

Department of Electronic Engineering
EE 3109 Applied Electromagnetics

Test Solutions

Q1

(a) Linear +x-polarized

(b) The +z direction

(c)

$$u_p = \frac{c}{\sqrt{\epsilon_r}} = 1.6 \times 10^8$$
$$\Rightarrow \epsilon_r = 3.516$$

(d)

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{\eta_0}{\sqrt{\epsilon_r}} = 201 \, \Omega$$

(e)

$$u_p = \frac{\omega}{k}$$
$$\Rightarrow f = \omega / 2\pi = u_p k / 2\pi = 1.528 \text{GHz}$$

(f)

$$\vec{E} = -\eta \hat{z} \times \vec{H}$$
$$= -\eta (\hat{z} \times \hat{y}) 3 \times 10^{-4} \cos(2\pi ft - kz)$$
$$= \hat{x} 0.06 \cos(9.6 \times 10^9 t - 60z) \text{ V/m}$$

Q2

(a)

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0 / \sqrt{4} - \eta_0}{\eta_0 / \sqrt{4} + \eta_0} = -0.333$$

(b)

$$T = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2\eta_0 / \sqrt{4}}{\eta_0 / \sqrt{4} + \eta_0} = 0.667$$

(c)

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 2$$

(d)

$$\beta_0 = \omega / c = 20 \text{ rad/s}$$

$$\begin{aligned}\vec{E}_1 &= \vec{E}_i + \vec{E}_r \\ &= \hat{x}E_{i0}(e^{-j\beta_0 z} + \Gamma e^{j\beta_0 z}) \\ &= \hat{x}E_{i0}[Te^{-j\beta_0 z} + \Gamma(j2 \sin \beta_0 z)] \\ &= \hat{x}5(0.667e^{-j20z} - j0.666 \sin 20z) \text{ V/m}\end{aligned}$$

(e)

$$\beta = \frac{\omega}{u_p} = \frac{\omega \sqrt{\epsilon_r}}{c} = \sqrt{\epsilon_r} \beta_0 = 40 \text{ rad/s}$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}} = \frac{\eta_0}{\sqrt{\epsilon_r}} = 60\pi \text{ } \Omega$$

$$\vec{E}_2 = \hat{x}TE_{i0}e^{-j\beta z} = \hat{x}3.335e^{-j40z} \text{ V/m}$$

$$\begin{aligned}\vec{H}_2 &= \frac{1}{\eta} \hat{z} \times \vec{E}_2 = \hat{y} \frac{3.335}{60\pi} e^{-j40z} \\ &= \hat{y}0.0177e^{-j40z} \text{ A/m}\end{aligned}$$

(f)

$$\begin{aligned}P_{av,1} &= \frac{1}{2} \text{Re}(\vec{E}_1 \times \vec{H}_1^*) = \hat{z} \frac{E_{i0}^2}{2\eta_0} (1 - \Gamma^2) \\ &= \hat{z}0.0295 \text{ W}\end{aligned}$$

Q3

(a)

$$Z_L = 15 + j10\Omega \Rightarrow z_L = 0.3 + j0.2 \quad (\text{point A})$$

From Smith Chart, $\text{SWR} \cong 3.5$ (point B)

(b)

From Smith Chart,

$$|\Gamma| \cong 0.55, \theta_r = 155.3^\circ \quad (\text{point A})$$

$$\Gamma = -0.5 + j0.23$$

(c)

On Smith Chart,

$$y_L = 2.31 - j1.54 \quad (\text{point C})$$

$$Y_L = 0.046 - j0.031 \text{ S}$$

(d)

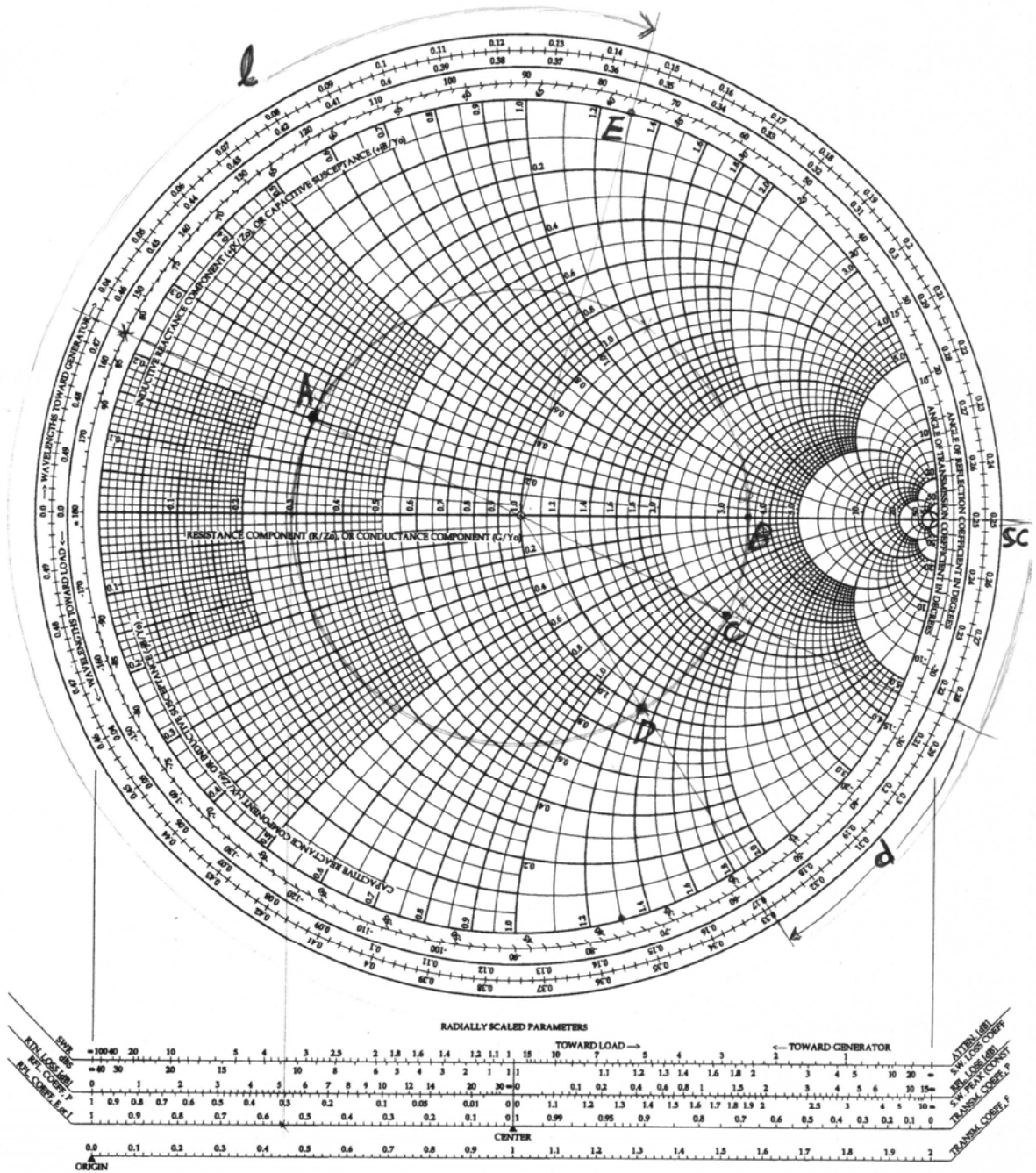
From Smith Chart

Position of stub (from the load): $d \cong 0.04 \lambda$ (point D)

Length of stub: $l \cong 0.40 \lambda$ (point E)

The Complete Smith Chart

Black Magic Design



Q4

(a)

TM₁₁ mode

(b)

$$f_c = \frac{v}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = 4.86 \text{ GHz}$$

(c)

$$\beta = k \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \frac{\omega}{c} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 73.72 \text{ rad/s}$$

(d)

$$u_g = u \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = c \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 1.76 \times 10^8 \text{ m/s}$$

(e)

$$\eta_{TM} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = \eta_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 221 \text{ } \Omega$$