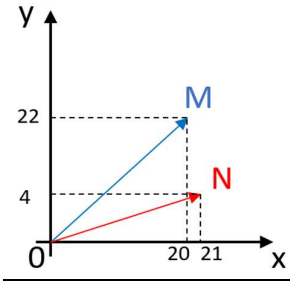
City University of Hong Kong
Electrical Engineering
Semester A 2022-2023

EE2104 Introduction to Electromagnetics
Assignment 1 Solution

Deadline (for uploading): 11:59 pm, 16 Sep 2022
Please put down your name and university ID.

1. Given the vectors $\boldsymbol{M} = 20\hat{\boldsymbol{x}} + 22\hat{\boldsymbol{y}}$ and $\boldsymbol{N} = 21\hat{\boldsymbol{x}} + 4\hat{\boldsymbol{y}}$. Draw the two vectors and then find:
   (a) the unit vector pointing from $\boldsymbol{M}$ to $\boldsymbol{N}$;
   (b) $\boldsymbol{M} \cdot \boldsymbol{N}$;
   (c) the angle between $\boldsymbol{M}$ and $\boldsymbol{N}$;

   Solution:



   (a) The vector pointing from $\boldsymbol{M}$ to $\boldsymbol{N}$ can be expressed as

$$\boldsymbol{R} = \boldsymbol{N} - \boldsymbol{M} = \hat{\boldsymbol{x}} - 18\hat{\boldsymbol{y}}$$

   We have

$$|\boldsymbol{R}| = \sqrt{1^2 + 18^2} = 18$$

   Therefore, the unit vector pointing from $\boldsymbol{M}$ to $\boldsymbol{N}$ is

$$\frac{\boldsymbol{R}}{|\boldsymbol{R}|} = \frac{\hat{\boldsymbol{x}} - 18\hat{\boldsymbol{y}}}{18} = 0.056\hat{\boldsymbol{x}} - \hat{\boldsymbol{y}}$$

   (b)

$$\boldsymbol{M} \cdot \boldsymbol{N} = (20\hat{x} + 22\hat{y}) \cdot (21\hat{x} + 4\hat{y}) = 420 + 88 = 508$$

   (c) The angle between $\boldsymbol{M}$ and $\boldsymbol{N}$ is

$$\theta = \cos^{-1}\left(\frac{\boldsymbol{M} \cdot \boldsymbol{N}}{|\boldsymbol{M}||\boldsymbol{N}|}\right) = \cos^{-1}\left(\frac{508}{\sqrt{20^2 + 22^2} \times \sqrt{21^2 + 4^2}}\right) = 0.64 \text{ (rad)} = 37°$$

2. A field is given as

$$\boldsymbol{G}(x,y) = \frac{y\hat{\boldsymbol{x}} - x\hat{\boldsymbol{y}}}{x^2 + y^2}$$

Find:
(a) the unit vector in the direction of **G** at $P(0, 3)$ and
(b) the angle between $\boldsymbol{G}(x, y)$ and $\hat{\boldsymbol{x}}$ at P.

Solution:

(a) At point $P(0, 3)$, we have

$$\boldsymbol{G} = \frac{3\hat{\boldsymbol{x}} - 0\hat{\boldsymbol{y}}}{0^2 + 3^2} = 0.33\hat{\boldsymbol{x}}, \qquad |\boldsymbol{G}| = \sqrt{0.33^2} = 0.33$$

Therefore, the unit vector in the direction of **G** at $P(0, 3)$ is

$$\frac{\boldsymbol{G}}{|\boldsymbol{G}|} = \hat{\boldsymbol{x}}$$

(b) At point $P(0, 3)$, the angle between $\boldsymbol{G}$ and $\hat{\boldsymbol{x}}$ is

$$\theta = \cos^{-1}\left(\frac{\boldsymbol{G}\cdot\hat{\boldsymbol{x}}}{|\boldsymbol{G}||\hat{\boldsymbol{x}}|}\right) = \cos^{-1}\left[\frac{0.33\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{x}}}{0.33}\right] = 0$$

3. For two vector fields

$$\boldsymbol{A}(\rho,\phi) = \rho^{\frac{3}{2}}\cos\phi\hat{\boldsymbol{\rho}} + \rho^{\frac{3}{2}}\sin\phi\hat{\boldsymbol{\phi}},$$
$$\boldsymbol{B}(\rho,\phi) = \rho^{\frac{1}{2}}\cos\phi\hat{\boldsymbol{\rho}} + \rho^{\frac{1}{2}}\sin\phi\hat{\boldsymbol{\phi}},$$

is there any point in space with $\boldsymbol{A}$ and $\boldsymbol{B}$ being perpendicular? Please provide clear reasoning using mathematics.

Solution:

We have

$$\boldsymbol{A}\cdot\boldsymbol{B} = (\,\rho^2\cos^2\phi + \rho^2\sin^2\phi) = \rho^2\,(\cos^2\phi + \sin^2\phi) = \rho^2$$

For any point (other than the origin), ρ is non-zero. So $\boldsymbol{A}$ and $\boldsymbol{B}$ are not perpendicular
For the origin, both $\boldsymbol{A}$ and $\boldsymbol{B}$ are zero.

Thus, there is no point in space for the two vectors being perpendicular physically.

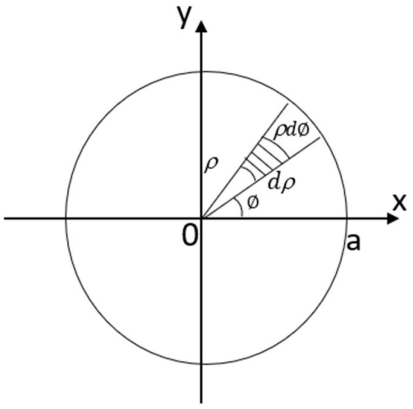
4. Consider a line at $y = L$ to $2L$. It contains electric charge of density $\rho_L = \rho_{L0}(y/L)^{-22}$ for some constant ρ_{L0} in C/m. Find the total charge of this line.

Solution:
The total charge is

$$\begin{aligned} Q &= \int_L^{2L} \rho_L dy \\ &= \int_L^{2L} \rho_{L0}(y/L)^{-22} dy \\ &= \rho_{L0} \cdot L^{22} \int_L^{2L} y^{-22} dy \\ &= \rho_{L0} \cdot L^{22} \left[\frac{L^{-21}(1 - 2^{-19})}{21} \right] \\ &= \frac{L \cdot \rho_{L0}}{21} \text{ (C)} \end{aligned}$$

5. In the polar coordinate system (ρ, ϕ), a disc of radius a centered at the origin holds charges of density $\rho_{s0}(\rho/a)\sin(\phi/2)$, where ρ_{s0} is a constant in the unit of C/m^2. Find the total charge.

Solution:
Consider the charged disc:



The differential radial length = $d\rho$
The differential radial length = $\rho d\phi$
The differential area = $\rho d\rho d\phi$

Total charge = sum of (charge density × differential area)

$$= \int_{Disc} \rho_{s0}(\rho/a)\sin(\phi/2))\rho d\rho d\phi$$
$$= (\rho_{s0}/a)\int_{\rho=0}^{a} \rho^2 d\rho \int_{\phi=0}^{2\pi} \sin(\phi/2) d\phi$$
$$= \frac{4}{3}\rho_{s0}a^2 \quad \text{(C)}$$