

Test 1

7 Oct 2022: 6:00pm~6:40pm (YEUNG LT-7)

- As a closed-book test, you are not allowed to use any written materials online/offline.
- Write your answers on the question papers directly.
- Non-graphical calculators are allowed. Laptops/phones must be switched off.
- Unless otherwise specified, SI units are assumed in the questions and 3 significant figures should be used in numerical answers.
- The total score is 100. There are a total of **5 pages**.

The following formulas may or may not be useful.

$$\begin{aligned}\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \mathbf{E}(x, y, z) &= \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \hat{\mathbf{a}}_R & d\mathbf{B} &= \frac{\mu_0}{4\pi} \frac{Id\mathbf{L} \times \hat{\mathbf{a}}_R}{R^2} \\ \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} & \oint_S \mathbf{D} \cdot d\mathbf{S} &= Q & \oint_L \mathbf{H} \cdot d\mathbf{L} &= I \\ \nabla \cdot \mathbf{D} &= \rho_v & \mathbf{D} &= \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon_0 \epsilon_r \mathbf{E} & \mathbf{B} &= \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 \mu_r \mathbf{H} \\ \nabla \cdot \mathbf{B} &= 0 & \epsilon_0 &= 8.854 \times 10^{-12} \text{ (F/m)} & \mu_0 &= 4\pi \times 10^{-7} \text{ (H/m)}\end{aligned}$$

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\rho} \hat{\mathbf{a}}_\rho \quad \mathbf{E} = \frac{p}{4\pi\epsilon_0 r^3} (2\cos\theta \hat{\mathbf{a}}_r + \sin\theta \hat{\mathbf{a}}_\theta) \quad \mathbf{B} = \frac{\mu_0 I}{2\pi\rho} \hat{\mathbf{a}}_\phi \quad \mathbf{B} = \mu_0 n I \hat{\mathbf{a}}_z$$

$$\begin{aligned}V_{AB} &= -\int_B^A \mathbf{E} \cdot d\mathbf{L} & \nabla^2 V &= -\frac{\rho_v}{\epsilon} & V &= -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{S} \\ \nabla^2 &= \nabla \cdot \nabla\end{aligned}$$

$$\begin{aligned}\nabla &= \frac{\partial}{\partial x} \hat{\mathbf{a}}_x + \frac{\partial}{\partial y} \hat{\mathbf{a}}_y + \frac{\partial}{\partial z} \hat{\mathbf{a}}_z & \nabla \times \mathbf{F} &= \frac{1}{\rho} \begin{vmatrix} \hat{\mathbf{a}}_\rho & \rho \hat{\mathbf{a}}_\phi & \hat{\mathbf{a}}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_\rho & \rho F_\phi & F_z \end{vmatrix} \\ \nabla f &= \frac{\partial f}{\partial \rho} \hat{\mathbf{a}}_\rho + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{\mathbf{a}}_\phi + \frac{\partial f}{\partial z} \hat{\mathbf{a}}_z & \nabla \times \mathbf{F} &= \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{\mathbf{a}}_r & r \hat{\mathbf{a}}_\theta & r \sin\theta \hat{\mathbf{a}}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_r & r F_\theta & r \sin\theta F_\phi \end{vmatrix} \\ \nabla f &= \frac{\partial f}{\partial r} \hat{\mathbf{a}}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\mathbf{a}}_\theta + \frac{1}{r \sin\theta} \frac{\partial f}{\partial \phi} \hat{\mathbf{a}}_\phi & \nabla \times \mathbf{F} &= \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{\mathbf{a}}_r & r \hat{\mathbf{a}}_\theta & r \sin\theta \hat{\mathbf{a}}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_r & r F_\theta & r \sin\theta F_\phi \end{vmatrix}\end{aligned}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho F_\rho) + \frac{\partial}{\partial \phi} F_\phi + \frac{\partial}{\partial z} (\rho F_z) \right]$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r^2 \sin\theta} \left[\frac{\partial}{\partial r} (r^2 \sin\theta F_r) + \frac{\partial}{\partial \theta} (r \sin\theta F_\theta) + \frac{\partial}{\partial \phi} (r F_\phi) \right]$$

Question 1 (20 points)

Fill in the blanks.

- (a) Electromagnetics are governed by a set of four equations that are called “_____ Equations”.
- (b) An atom contains a positively charged _____ and negatively charged _____.
- (c) A scalar field is mathematically a _____ of position and time.
- (d) A unit vector must have a length of _____.

Question 2 (20 points)

In rectangular coordinates, consider a point $P: (x, y, z) = (0, 3, 4)$. The unit vectors are denoted as $\hat{x}$, $\hat{y}$, and $\hat{z}$.

- (a) Express P in the spherical coordinates (r, θ, ϕ) .
(Hint: Remember that θ and ϕ are the polar angle and the azimuthal angle, respectively.)
- (b) At P , express $\hat{r}$ of the spherical coordinate system in terms of $\hat{x}$, $\hat{y}$, and $\hat{z}$.
- (c) At P , express $\hat{\theta}$ of the spherical coordinate system in terms of $\hat{x}$, $\hat{y}$, and $\hat{z}$.

Question 3 (20 points)

In the cylindrical coordinate system, a field is given as:

$$\mathbf{F}(\rho, \phi, z) = 2 \exp\left(-\frac{\rho}{a}\right) \hat{\boldsymbol{\rho}} + \frac{\rho}{a} \hat{\boldsymbol{\phi}}$$

for some positive constant a (in meters). At point $(\rho, \phi, z) = (a, 0, 0)$:

(a) Find $\text{div } \mathbf{F}$.

(b) Find $\text{curl } \mathbf{F}$.

Question 4 (20 points)

Consider a ball of charge with a uniform charge density of $+4 \text{ (C/m}^3\text{)}$. The ball has a radius b of 3 mm. It is centered at the origin.

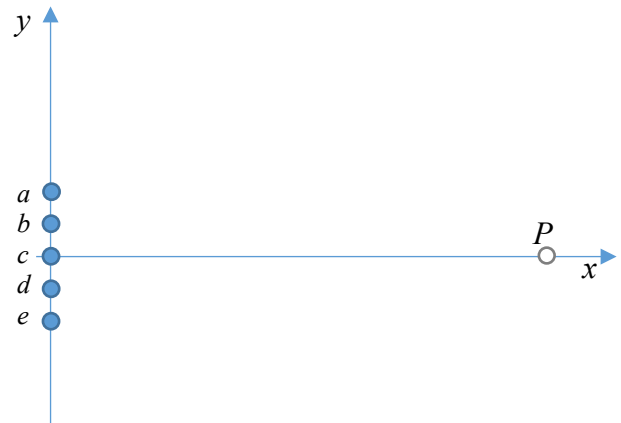
(a) Find the total charge.

(b) Find, or estimate, the electric field measured at $x = 22 \text{ m}$ and $y = z = 0$.

Question 5 (20 points)

Consider 5 point charges positioned on the y -axis as follows:

Position	Charge (pC)	Coordinate y (μm)
a	+2	+200
b	-1	+100
c	+4	0
d	+1	-100
e	-2	-200



The electric field $\mathbf{E}$ is measured at position P : $(x, y) = (3 \text{ m}, 0)$.

(a) Find E (i.e. magnitude of $\mathbf{E}$). Express the answer in 2 significant figures. State the reasoning.

(b) Find E_y (i.e. the y -component of $\mathbf{E}$). Express the answer in 2 significant figures. State the reasoning.

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