

EE1001 Fundamentals of Digital Techniques

Assignment 4 (Solution)

Marking Policy:

- Q.1 (15 marks), Q.3 (15 marks), and Q.6 (20 marks) will not be marked. All 50 marks will be given to students who have provided answers to these questions.
- For Q.2 and Q.5, each carries 15 marks.
- Q.4 carries 20 marks.

1. The statement is true, which can be proved as follows:

Proof: Suppose k is any [particular but arbitrarily chosen] odd integer and m is any even integer. [We must show that $k^2 + m^2$ is odd.] By definition of odd and even, $k = 2a + 1$ and $m = 2b$ for some integers a and b . Then

$$\begin{aligned}k^2 + m^2 &= (2a + 1)^2 + (2b)^2 && \text{by substitution} \\&= 4a^2 + 4a + 1 + 4b^2 \\&= 4(a^2 + a + b^2) + 1 \\&= 2(2a^2 + 2a + 2b^2) + 1 && \text{by algebra.}\end{aligned}$$

But $2a^2 + 2a + 2b^2$ is an integer because it is a sum of products of integers. Thus $k^2 + m^2$ is twice an integer plus 1, and so $k^2 + m^2$ is odd [as was to be shown].

2. The statement can be disproved by giving a counter-example. For example, if $n = 5$, then $(n-1)/2 = 2$, which is *not* odd.

3. The proof is shown below:

Proof by contradiction: Suppose not. That is, suppose there is an irrational number x such that the square root of x is rational. *[We must derive a contradiction.]* By definition of rational, $\sqrt{x} = \frac{a}{b}$ for some integers a and b with $b \neq 0$. By substitution,

$$(\sqrt{x})^2 = \left(\frac{a}{b}\right)^2,$$

and so, by algebra,

$$x = \frac{a^2}{b^2}.$$

But a^2 and b^2 are both products of integers and thus are integers, and b^2 is nonzero by the zero product property. Thus $\frac{a^2}{b^2}$ is rational. It follows that x is both irrational and rational, which is a contradiction. *[This is what was to be shown.]*

4. Suppose r and s are positive real numbers such that $r \leq 10$ and $s \leq 10$.

(Multiplying 10 to both sides of the first inequality, we obtain $10r \leq 100$. Multiplying r to both sides of the second inequality, we obtain $rs \leq 10r$. *This step is quite obvious, and may be skipped.*)

Therefore, $rs \leq 100$, which means that their product *cannot* be greater than 100. Hence, the statement is true.

5. Yes, it is valid by universal modus tollens.

6. Inference steps:

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|---|----------------------------|
| 1) $\forall x, K(x) \rightarrow E(x)$ | |
| 2) $\sim(\exists x, E(x) \wedge K(x))$ | |
| 3) $\forall x, \sim K(x) \vee E(x)$ | (contrapositive 1) |
| 4) $\forall x, \sim E(x) \vee \sim K(x)$ | (negation of quantifier 2) |
| 5) $\forall x, ((\sim K(x) \vee E(x)) \wedge (\sim K(x) \vee \sim E(x)))$ | (equiv. 3, 4) |
| 6) $\forall x, (\sim K(x) \vee (E(x) \wedge \sim E(x)))$ | (distributive 5) |
| 7) $\forall x, (\sim K(x) \vee c)$ | (negation) |
| 8) $\forall x, \sim K(x)$ | (identity) |

The last statement means "Nothing is known".