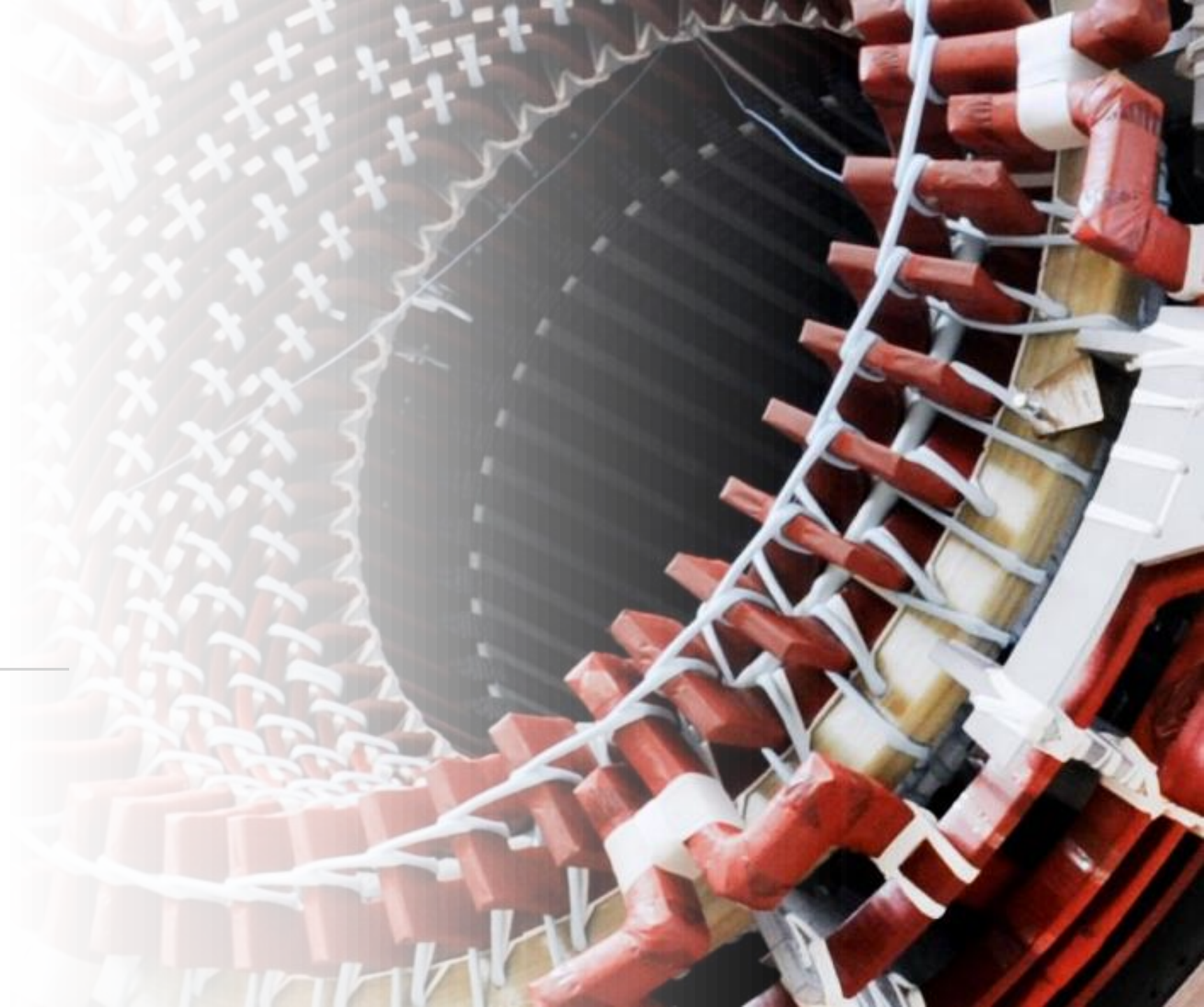




EE3124 Introduction to Electric Machines and Drives

2-Basics of Magnetic Circuit

Dr. CQ Jiang

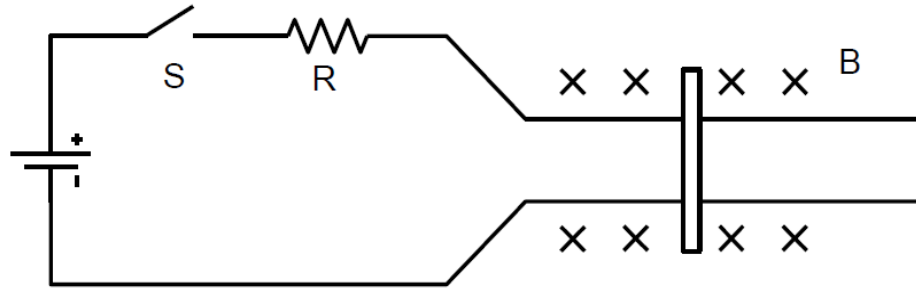


Outline

- Ampere's Law
- Faraday's Law of Induction
- Magnetic Flux Density and Permeability
- Relative Permeability - Magnetic Core Characteristics
- Magnetic Circuit – Reluctance
- Flux Linkage and Inductance

Revision of Fleming's Left- or Right-hand Rule

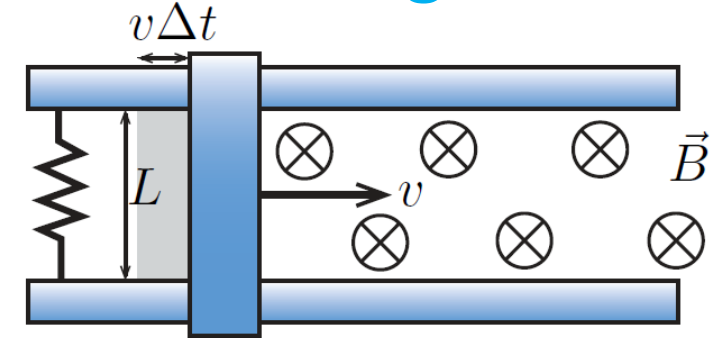
1) Where to go of the bar?



Along the bed of this railroad track is a constant, uniform-density magnetic field directed into the page.

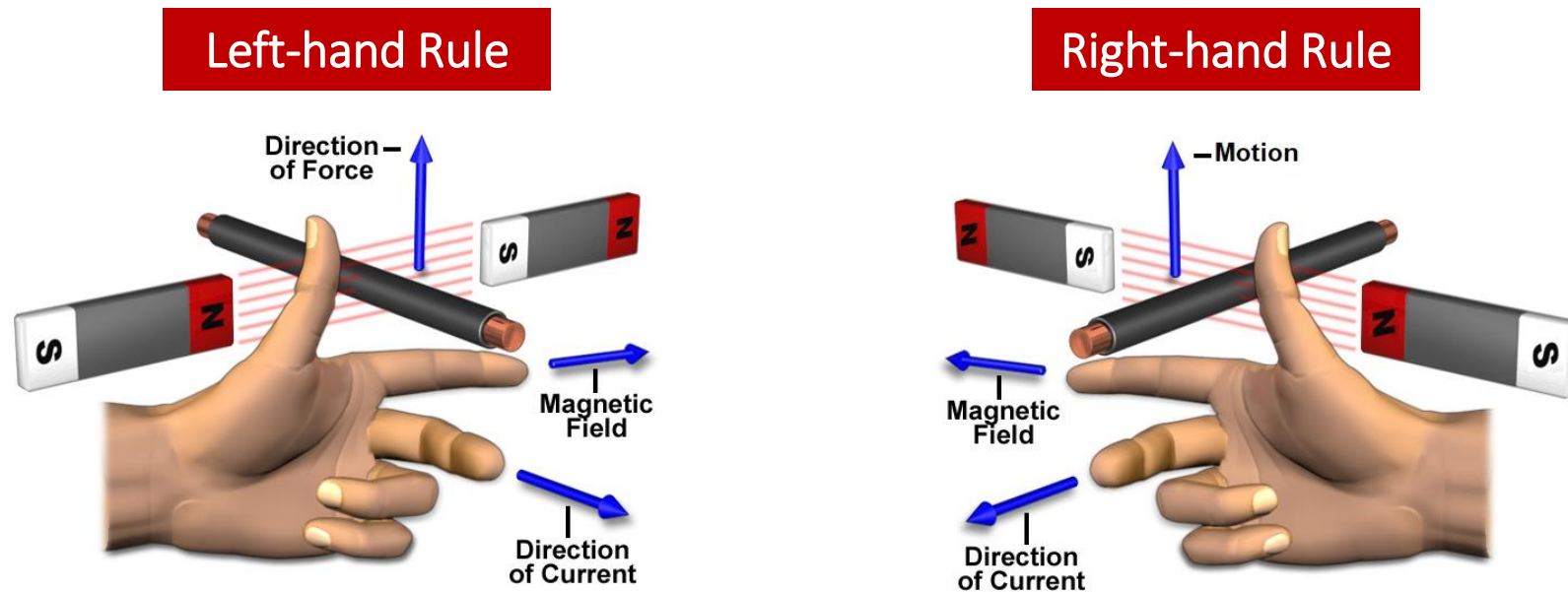
- It consists of a battery and a resistance connected through a switch to a pair of smooth, frictionless rails. A bar of conducting metal is lying across the tracks.
- Close the switch, the bar will accelerate to which direction?

2) What is the voltage across the bar?



- A bar of conducting metal is moving along the tracks connected to a resistance.
- Along the motion, what is the voltage across the bar?

Revision of Fleming's Left- or Right-hand Rule



- It is a bit confusing to use either Fleming's left- or right-hand rule.
- Easy to remember

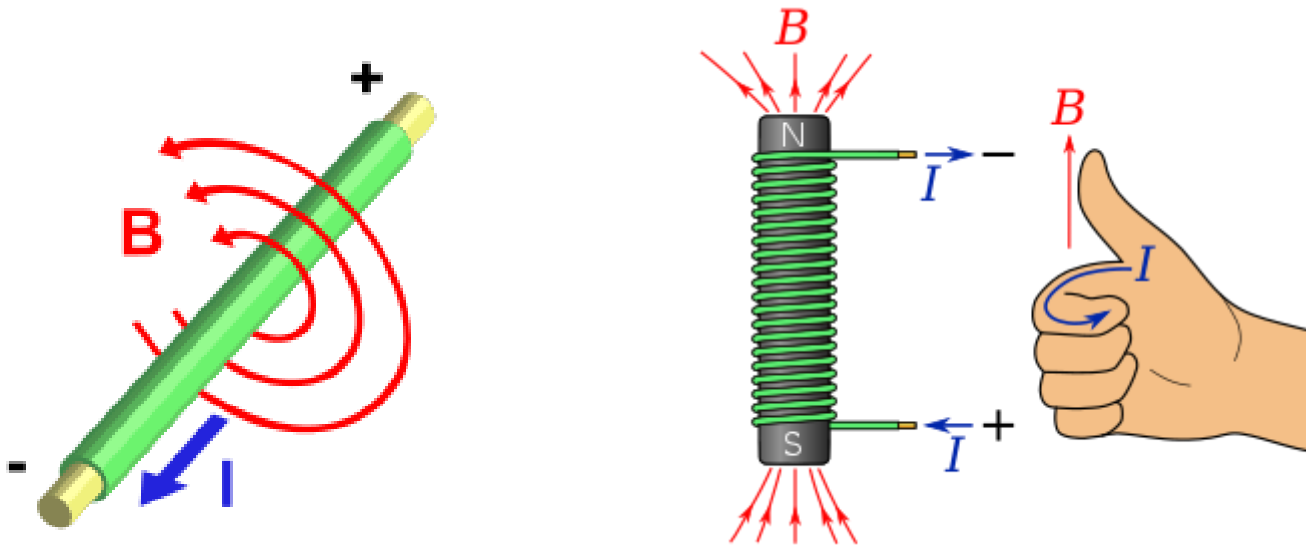
Left hand is more powerful (force) than your right hand!!!

- Left hand rule for creation of **force**.
- Right hand rule for creation of current or field.

Production of a Magnetic Field

- 1820:

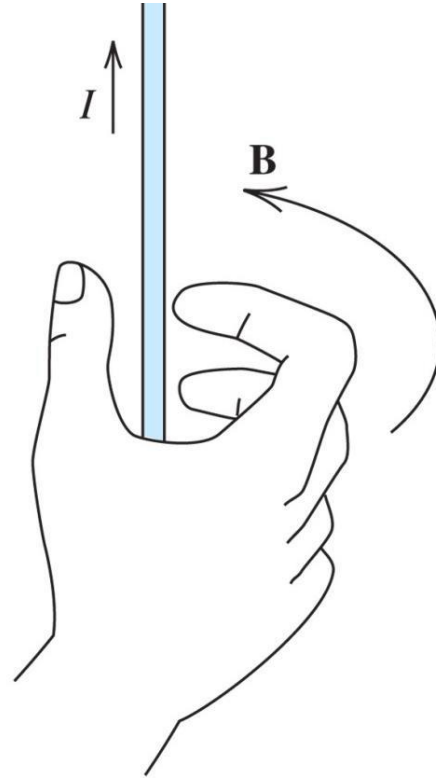
André-Marie Ampère developed Ampere's law showing that electric current produces a magnetic field. 電流產生磁力場



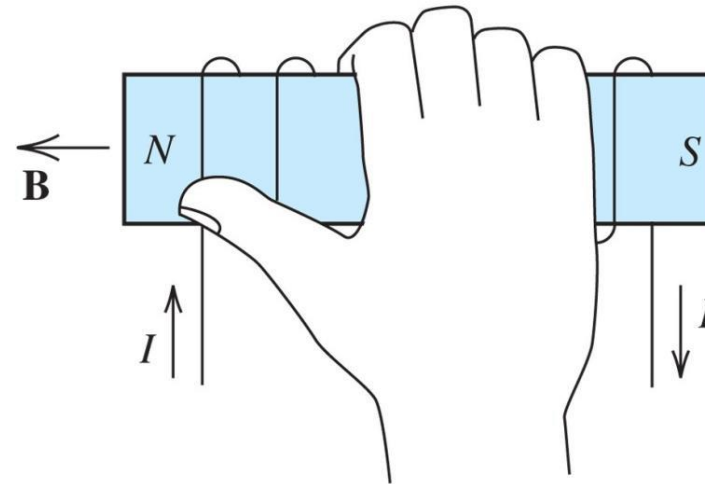
André-Marie Ampère
French Scientist

Production of a Magnetic Field

Right Hand Rule



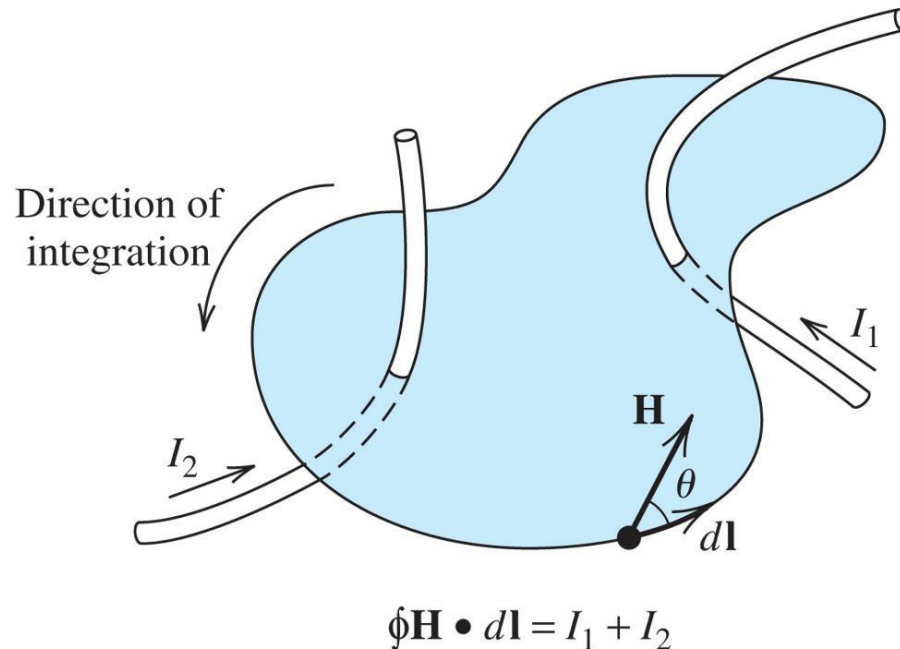
(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field



(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Ampere's Law

- The line integral of the magnetic field around a **closed path** L is equal to the algebraic sum of the currents flowing through the area enclosed by the path.

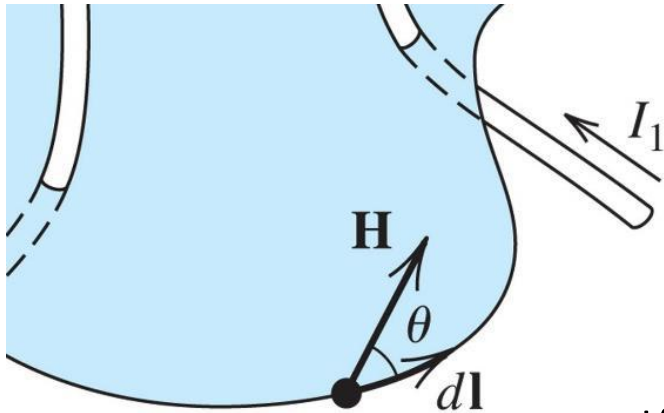


$$\oint \mathbf{H} \cdot d\mathbf{l} = \sum i$$

where $\mathbf{H}$ is the magnetic field intensity produced by the current i , i is measured in amperes and $\mathbf{H}$ is measured in ampere-turns per meter.

Ampere's Law

- The line integral of the magnetic field around a **closed path** L is equal to the algebraic sum of the currents flowing through the area enclosed by the path.



$$\oint \mathbf{H} \cdot d\mathbf{l} = I_1 + I_2$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = \sum i$$

The vector dot product is:

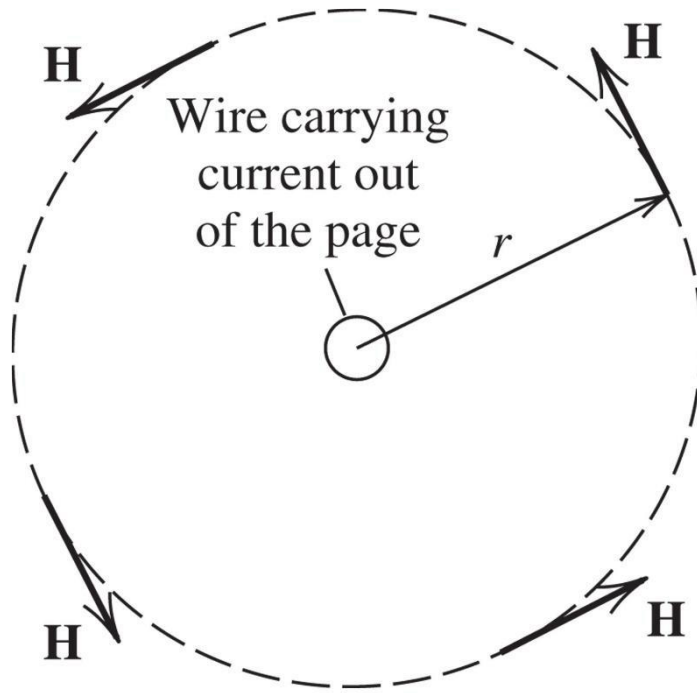
$$\mathbf{H} \cdot d\mathbf{l} = H dl \cos(\theta)$$

If the magnetic field is constant and points in the same direction as $d\mathbf{l}$

$$Hl = \sum i$$

Ampere's Law

- **Example:** A Conductor Carrying a Current

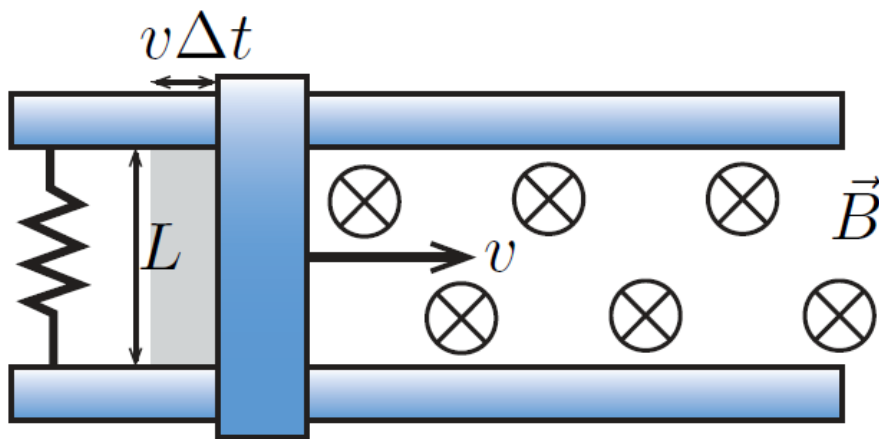


$$Hl = H(2\pi r) = i$$

$$H = \frac{i}{2\pi r}$$

Faraday's Law of Induction

- Basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an **electromotive force (EMF)**—a phenomenon known as electromagnetic induction.
- The electromotive force around a closed path is equal to the **negative** of the **time rate of change** of the magnetic flux enclosed by the path.



$$emf = -\frac{d\Phi_{mag}}{dt}$$

In a short time Δt the bar moves a distance $\Delta x = v\Delta t$, and the flux increases by $\Delta\Phi_{mag} = B(Lv\Delta t)$

$$emf = \frac{\Delta\Phi_{mag}}{\Delta t} = BLv$$

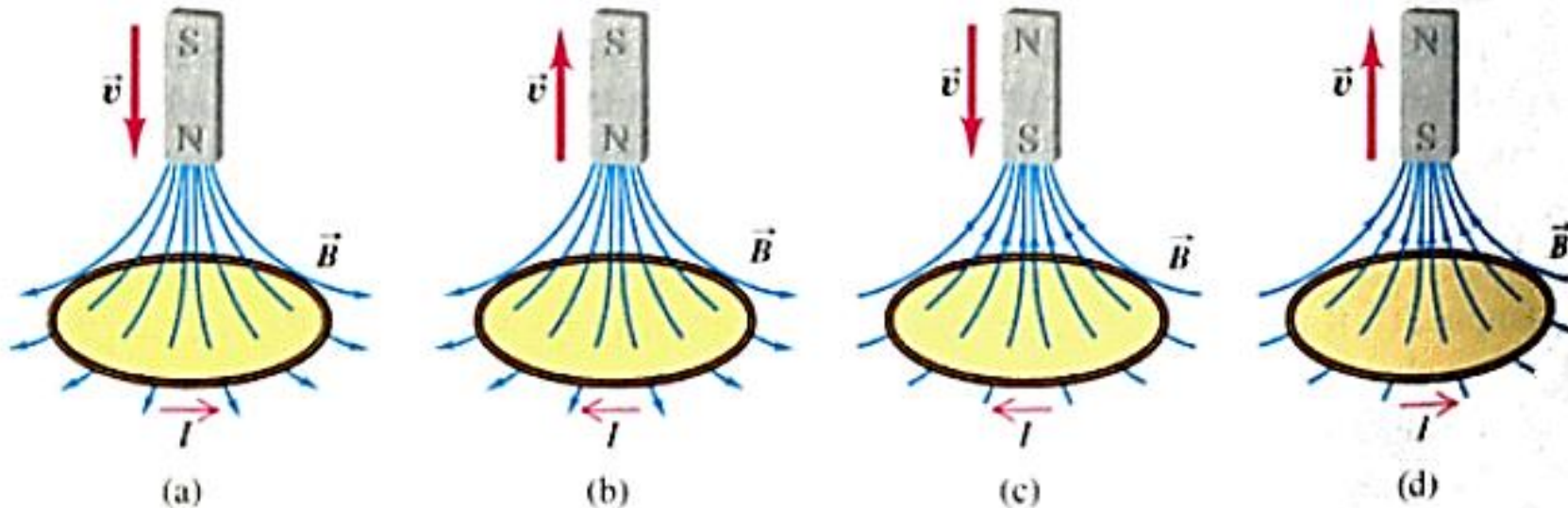
There is an increase in flux through the circuit as the bar of length L moves to the right (orthogonal to magnetic field H) at velocity, v .

Faraday's Law of Induction

- It indicates that when a magnet moves towards a closing coil, the induced current will be generated (Lenz Law). This simple law governs the basic operation principle of all electric machines.
- No motion, No current.

The induced emf in a coil of N turns is equal to N times the rate of change of the magnetic flux on one loop of the coil.

$$emf = -N \frac{d\Phi_{mag}}{dt}$$

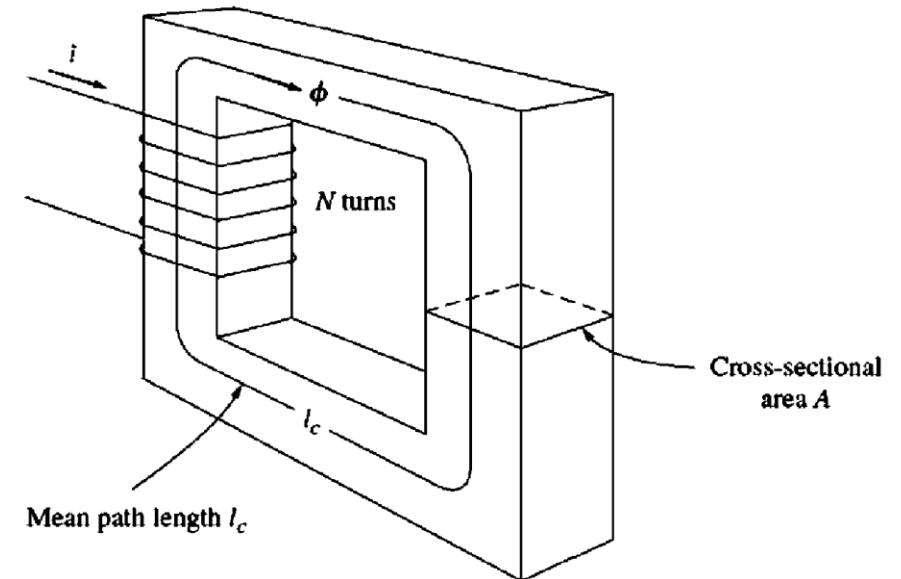


Simple Magnetic Circuit

- A rectangular core with a winding of N turns of wire wrapped about one leg of the core. If the core is composed of iron or certain other similar metals, essentially all the magnetic field produced by the current will remain inside the core, so the path of integration in Ampere's law is the mean path length of the core l_c .

Magnetomotive force (*mmf*) is a quantity appearing in the equation for the magnetic flux in a magnetic circuit.

$$\text{mmf: } \mathcal{F} = Hl_c = Ni$$

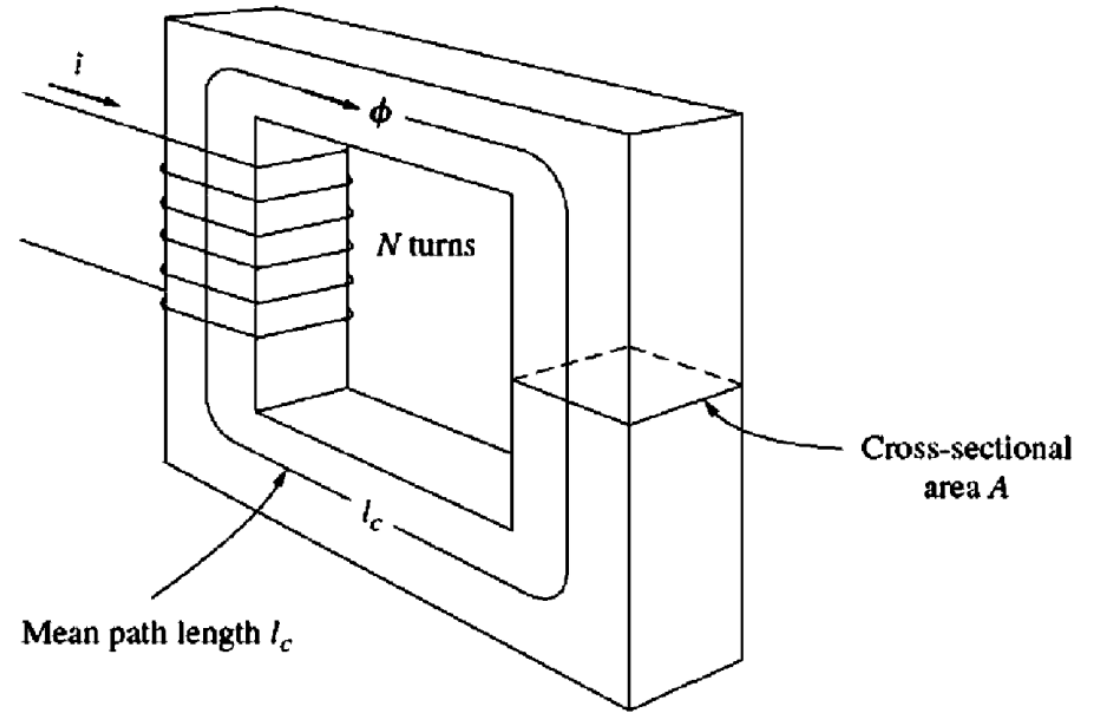


Simple Magnetic Circuit

- The current passing within the path of integration i is then Ni , since the coil of wire cuts the path of integration N times while carrying current i . Ampere's law thus becomes

$$Hl_c = Ni$$

$$H = \frac{Ni}{l_c}$$



Magnetic Flux Density and Permeability

- The magnetic field intensity H is in a sense a measure of the effort that a current is putting into the establishment of a magnetic field. The strength of the magnetic field flux produced in a core also depends on the material of the core.
- The relationship between the magnetic field intensity H and the resulting magnetic flux density B produced within a material is given by

$$B = \mu H$$

where

H = magnetic field **intensity**

μ = magnetic permeability of material

B = *resulting magnetic flux **density** produced*

Magnetic Flux

Magnetic Flux Density

Magnetic Field Intensity

Φ [Wb] (Webers)

B [Wb/m²] = T (Tesla)

H [Amp-turn/m]

Relative Permeability - Magnetic Core Characteristics

- The units of magnetic field intensity are ampere-turns per meter, the units of permeability are henrys per meter, and the units of the resulting flux density are webers per square meter, known as teslas (T)
- The permeability of free space is called μ_0 , and its value is

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

- The permeability of any other material compared to the permeability of free space is called its relative permeability

$$\mu_r = \frac{\mu}{\mu_0}$$

Relative Permeability - Magnetic Core Characteristics

- **Permeability** is the measure of magnetization that a material obtains in response to an applied magnetic field

	<i>Electrical Conductivity $\times 10^6$ S/m</i>	<i>Relative Magnetic Permeability @ B=20 Gauss</i>
Cold rolled steel	10.0	100
Iron	10.0	200
Purified iron	10.0	5,000
4% Silicon iron	1.7	500
45 Permalloy	2.2	2,500
78 Permalloy	6.3	8,000
4-79 Permalloy	1.8	20,000
2-81 Permalloy	0.0001	125
Supermalloy	1.7	100,000
Mu Metal	1.6	20,000
Hiperco	4.0	650
Hypernik	2.0	4,500
Monimax	1.3	2,000
Sinimax	1.1	3,000
Permendur	14.3	800
2V Permendur	3.8	800

Magnetic Materials

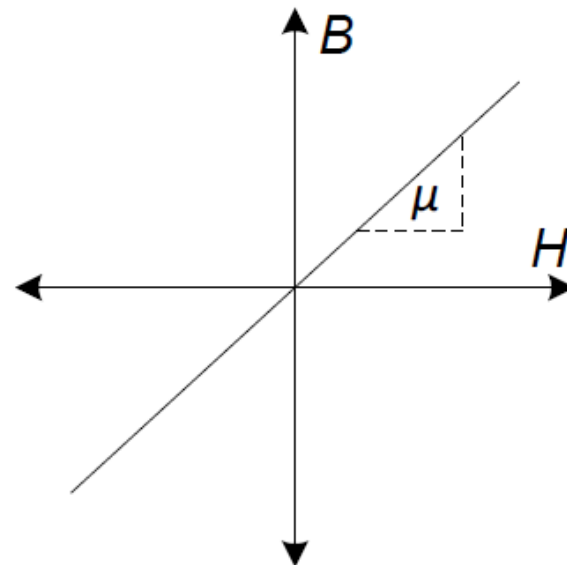
Material	Relative Permeability μ_r
Vacuum	1
Air	1.0000004
Water	0.999992
Copper	0.999994
Aluminum	1.00002
Silver	0.99998
Nickel	600
Iron	5000
Carbon Steel	100
Transformer Steel	2000
Mumetal	50,000
Supermalloy	1,000,000

Note: Values can often vary depending on purity and processing.

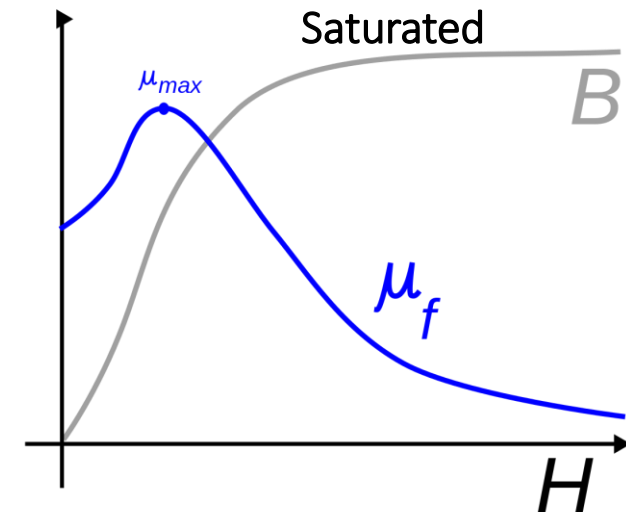
Relative Permeability - Magnetic Core Characteristics

- Relative permeability is a convenient way to compare the magnetizability of materials. For example, the steels used in modern machines have relative permeabilities of 2000 to 6000 or even more. This means that, for a given amount of current, 2000 to 6000 times more flux is established in a piece of steel than in a corresponding area of air.

$$B = \mu H$$
$$= \frac{\mu N i}{l_c}$$



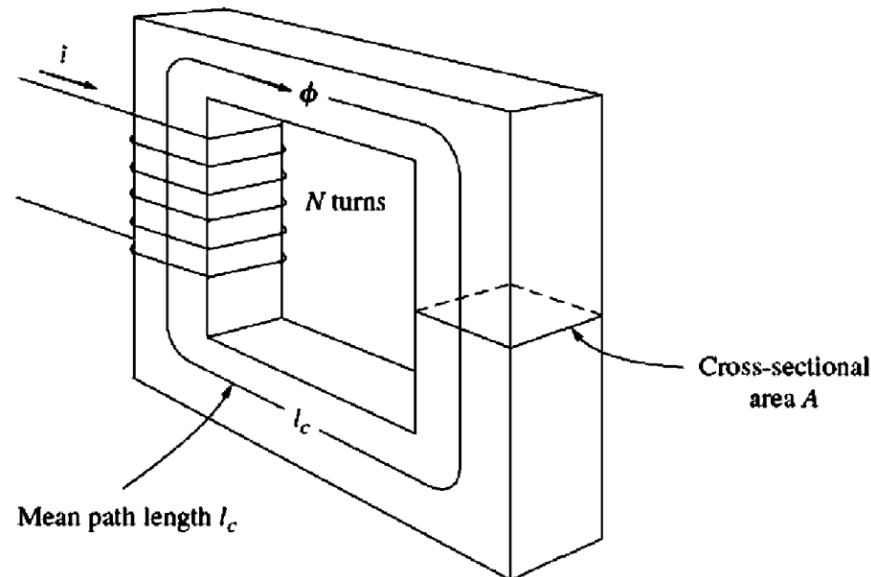
Ideal situation



Actual situation

Inside a Ferromagnetic Material

- Because the permeability of iron is so much higher than that of air, the great majority of the flux in an iron core remains inside the core instead of traveling through the surrounding air, which has much lower permeability.
- The small leakage flux that does leave the iron core is very important in determining the flux linkages between coils and the self-inductances of coils in transformers and motors.



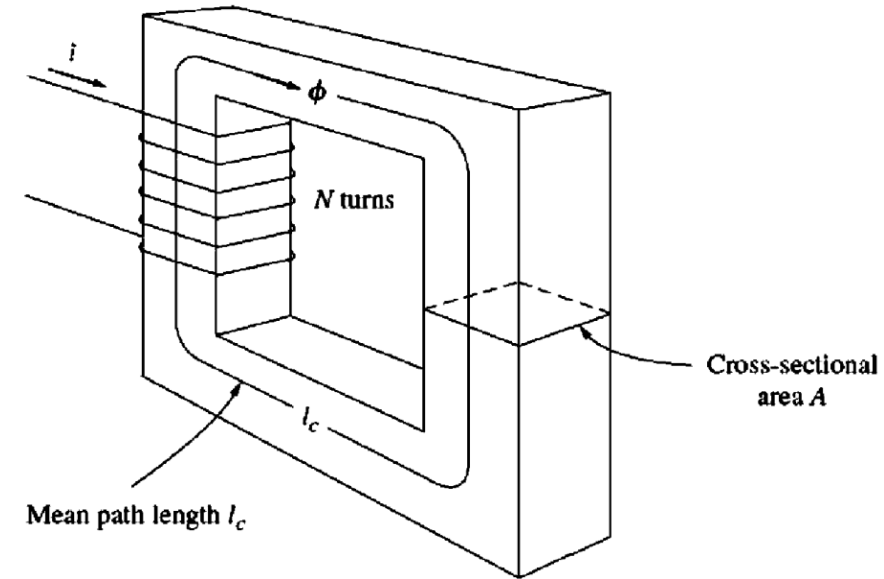
Inside a Ferromagnetic Material

- The total flux in a given area is given by

$$\phi = \int_A \mathbf{B} \cdot d\mathbf{A}$$

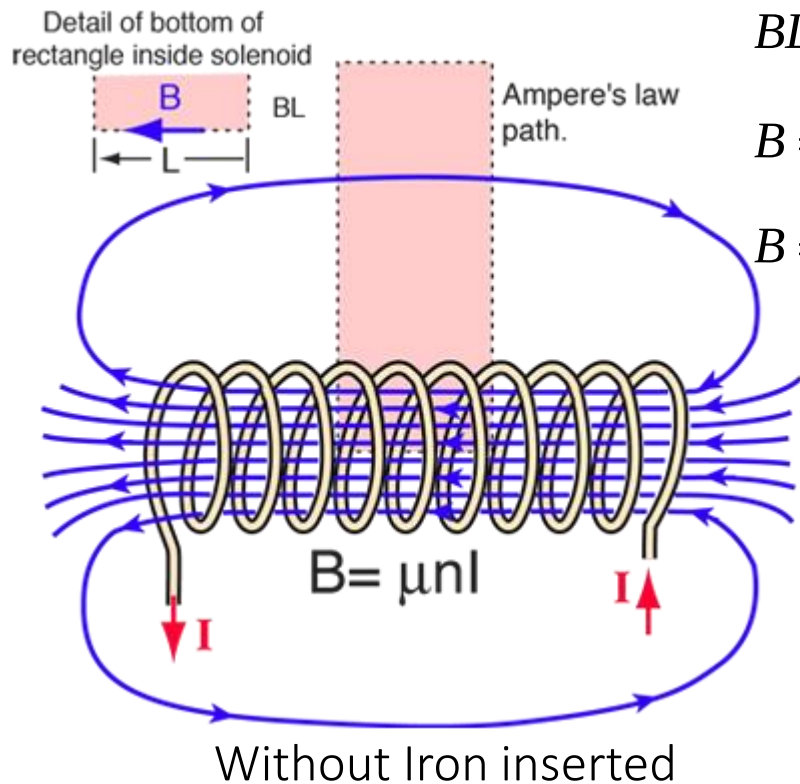
- where dA is the differential unit of area. If the flux density vector is perpendicular to a plane of area A , and if the flux density is constant throughout the area, then this equation reduces to

$$\begin{aligned}\phi &= BA \\ &= \frac{\mu N i A}{l_c}\end{aligned}$$



Example 1: Magnetic Flux Density Calculation

- Solenoid Field from Ampere's Law



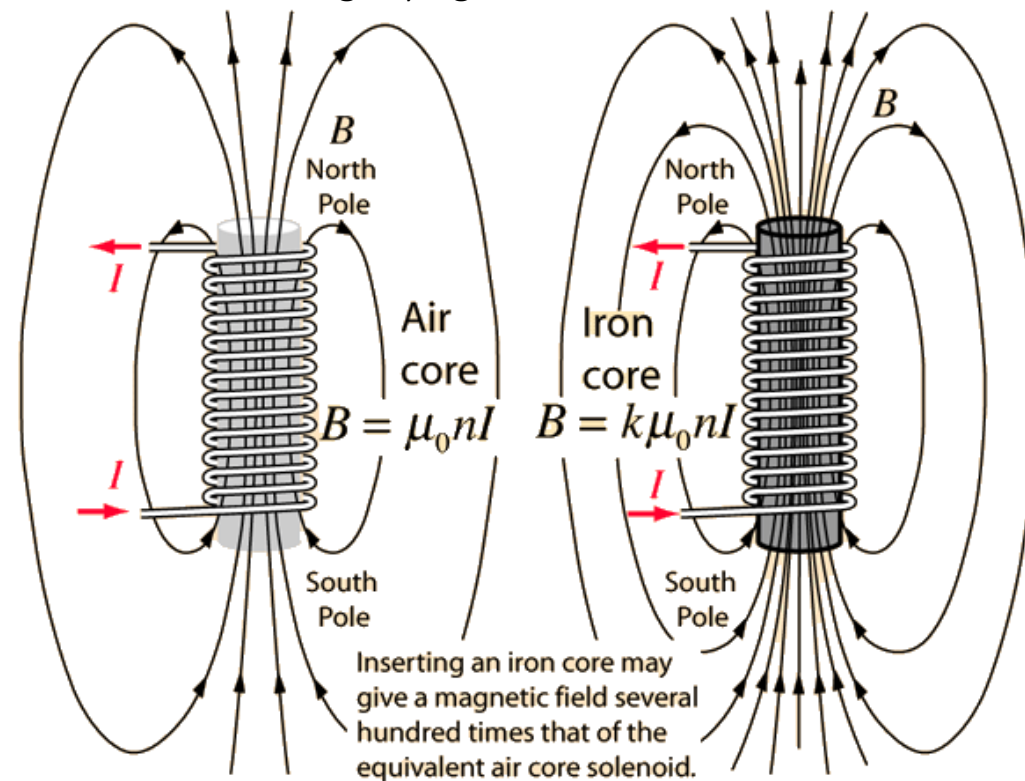
$$BL = \mu NI$$

$$B = \mu \frac{N}{L} I$$

$$B = \mu n I$$

$$B = k\mu_0 n I$$

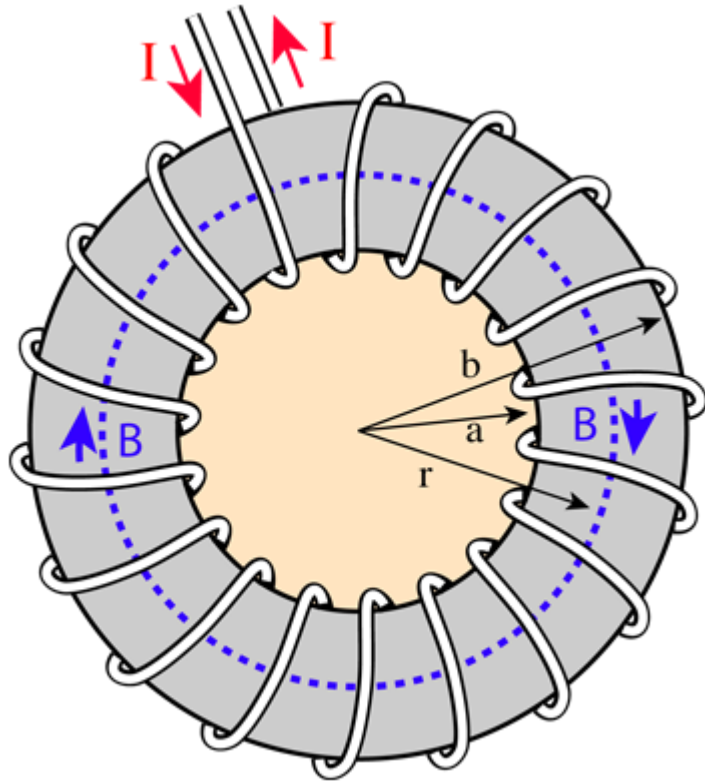
k is the relative permeability of the iron, shows the magnifying effect of the iron core.



With Iron inserted

Example 2: Magnetic Flux Density Calculation

- Magnetic Field of Toroid



With Iron inserted

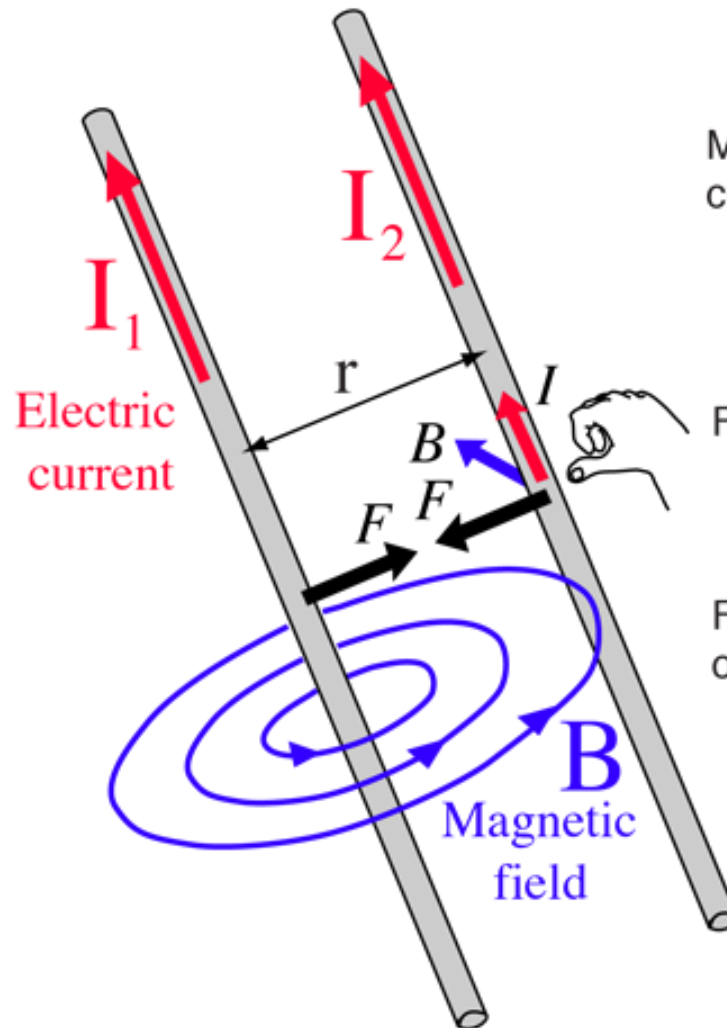
The current enclosed by the dashed line is just the number of loops times the current in each loop. Amperes law then gives the magnetic field by

$$B2\pi r = \mu NI$$

$$B = \frac{\mu NI}{2\pi r}$$

Example 3: Magnetic Flux Density Calculation

- Magnetic Force Between Wires



Magnetic field at wire 2 from current in wire 1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

Force on a length ΔL of wire 2:

$$F = I_2 \Delta L B$$

Force per unit length in terms of the currents:

$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Magnetic Circuit - Reluctance of Magnetic Bar

We've already known

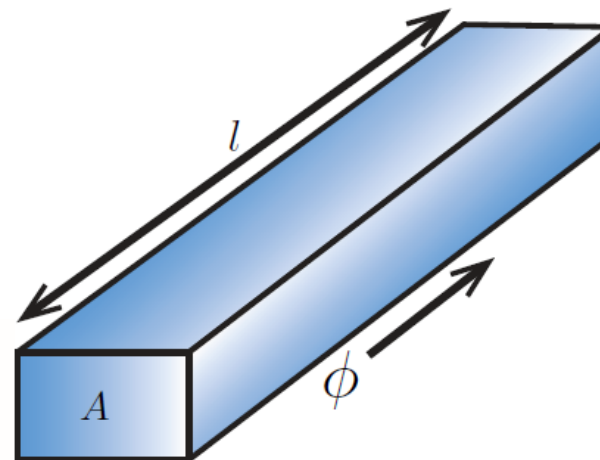
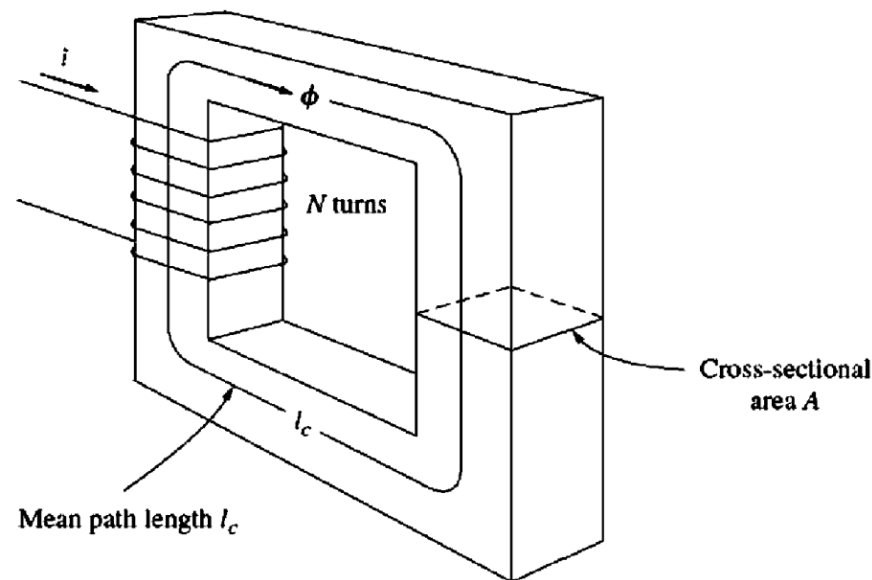
$$\begin{aligned}\phi &= BA \\ &= \frac{\mu NiA}{l_c} \end{aligned} \quad \begin{array}{c} \longrightarrow \\ \longrightarrow \end{array} \quad \begin{aligned}\phi &= BA \\ &= \frac{\mu \mathcal{F} A}{l_c} \end{aligned}$$

○ Magnetic “OHM’s LAW”

$$\text{mmf: } \mathcal{F} = Ni = \Phi \mathfrak{R}$$

$$\mathfrak{R} = \frac{l}{\mu A}$$

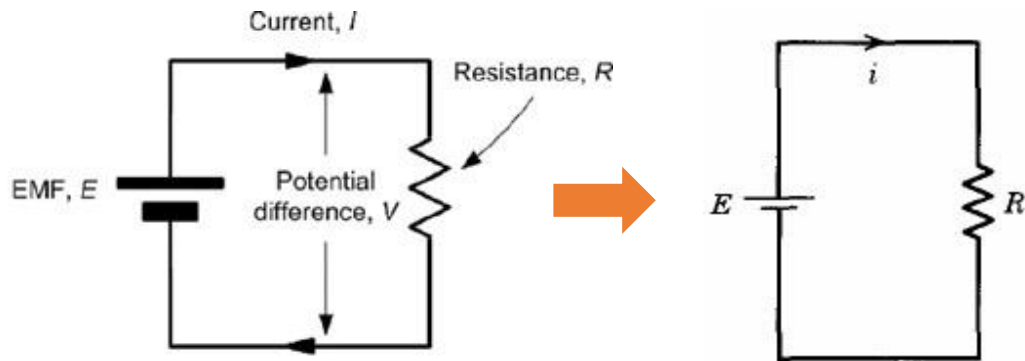
- High material permeability can result in low core reluctance.



The reluctance $\mathfrak{R}$ of a magnetic path depends on the mean length l , the area A , and the permeability μ of the material.

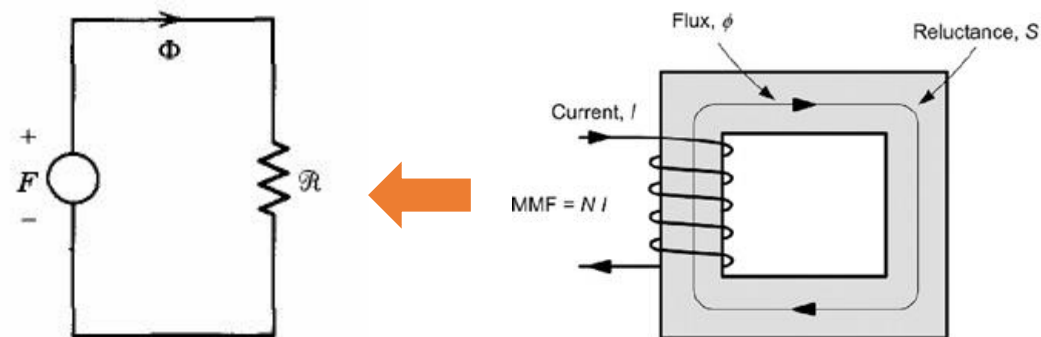
Magnetic Circuit - Electrical Circuit Analogy

○ Electric circuit



$$i = \frac{E}{R}$$

○ Magnetic circuit

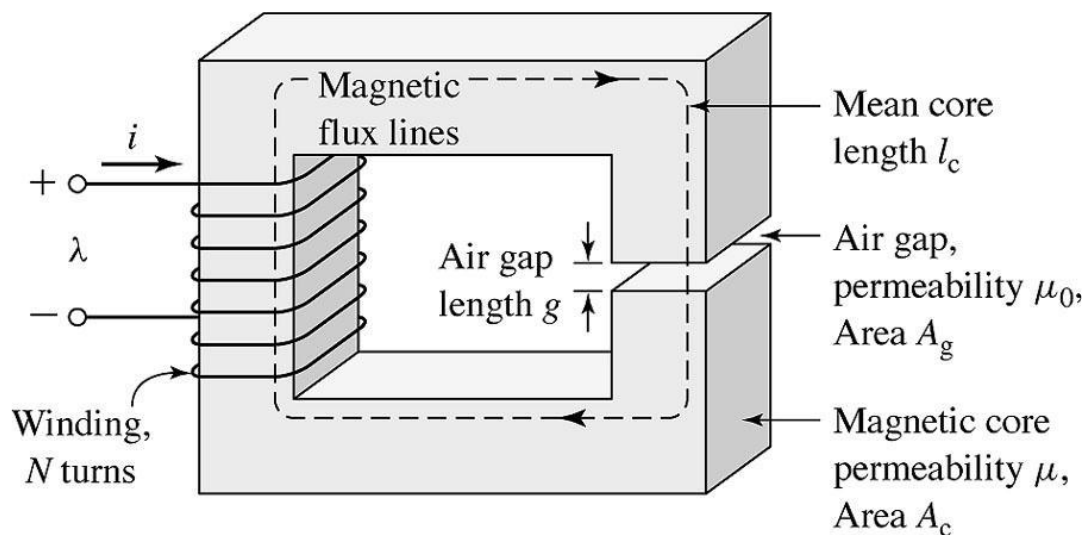


$$\phi = \frac{\mathcal{F}}{\mathcal{R}}$$

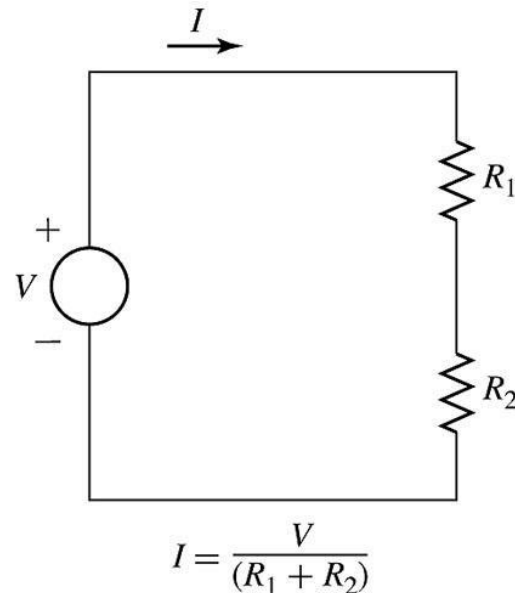
Electrical	Magnetic
Voltage v	Magnetomotive Force $\mathfrak{F} = Ni$
Current i	Magnetic Flux ϕ
Resistance R	Reluctance $\mathfrak{R}$
Conductivity $1/\rho$	Permeability μ
Current Density J	Magnetic Flux Density B
Electric Field E	Magnetic Field Intensity H

Magnetic Circuit with Air Gap

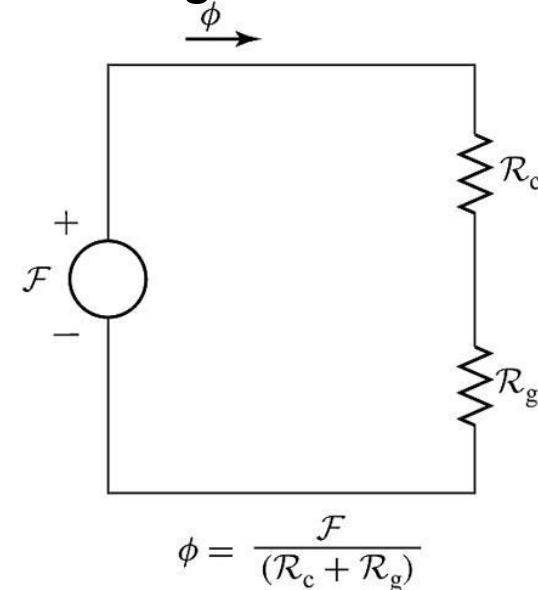
Provided the air-gap length g is sufficiently small, the configuration can be analyzed as a magnetic circuit with two series components both carrying the same flux.



Electric circuit



Magnetic circuit



$$\mathcal{R}_c = \frac{l_c}{\mu A_c}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g}$$

Reluctance

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} \rightarrow \phi = \frac{\mathcal{F}}{\mathcal{R}_c + \mathcal{R}_g}$$

Flux

$$\phi = \frac{\mathcal{F}}{\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g}}$$

$$B_c = \frac{\phi}{A_c}$$

Flux density in Core

Magnetic Circuit with Air Gap

$$\phi = \frac{\mathcal{F}}{\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g}}$$

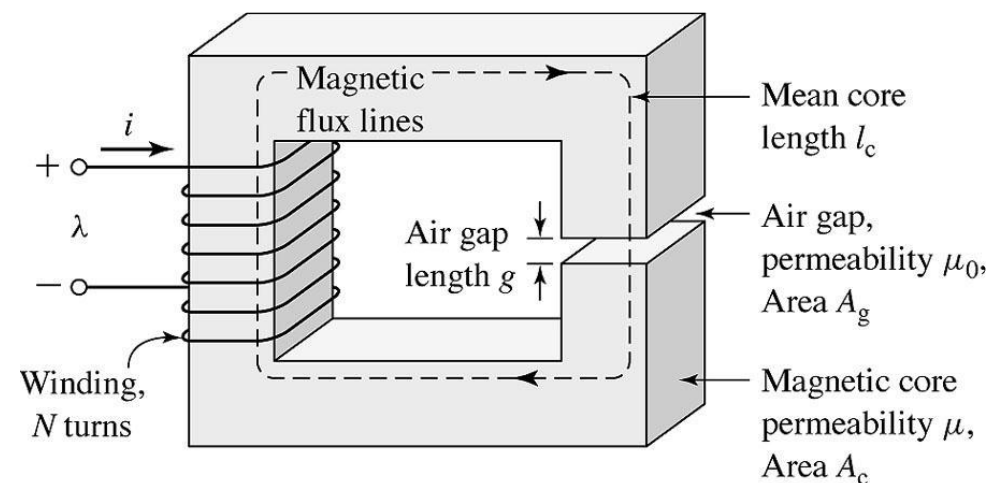
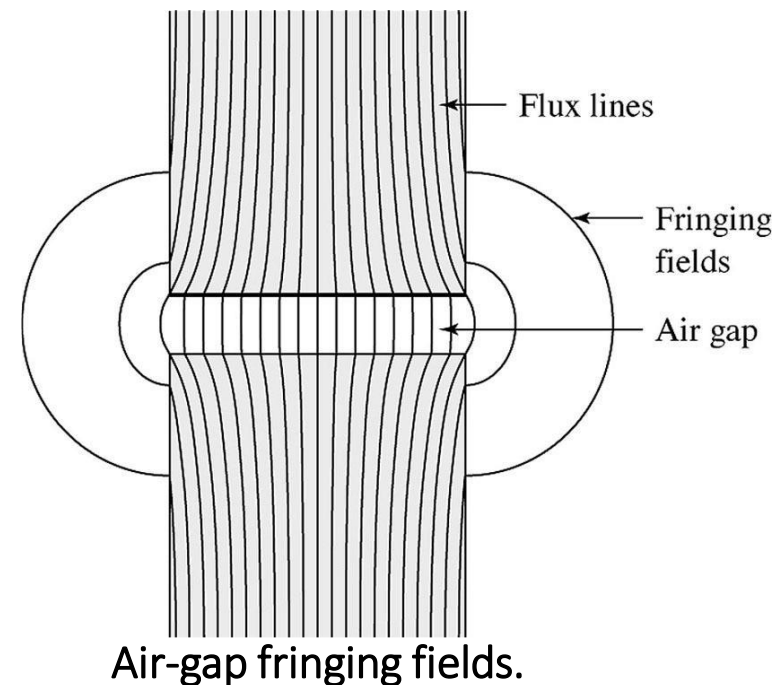
We see that high material permeability can result in low core reluctance, which can often be made much smaller than that of the air gap, thus

$$(\mu A_c / l_c) \gg (\mu_0 A_g / g), \mathcal{R}_c \ll \mathcal{R}_g$$

In this case, the reluctance of the core can be neglected and the flux can be found

$$\phi \approx \frac{\mathcal{F}}{\mathcal{R}_g} = \frac{\mathcal{F} \mu_0 A_g}{g} = Ni \frac{\mu_0 A_g}{g}$$

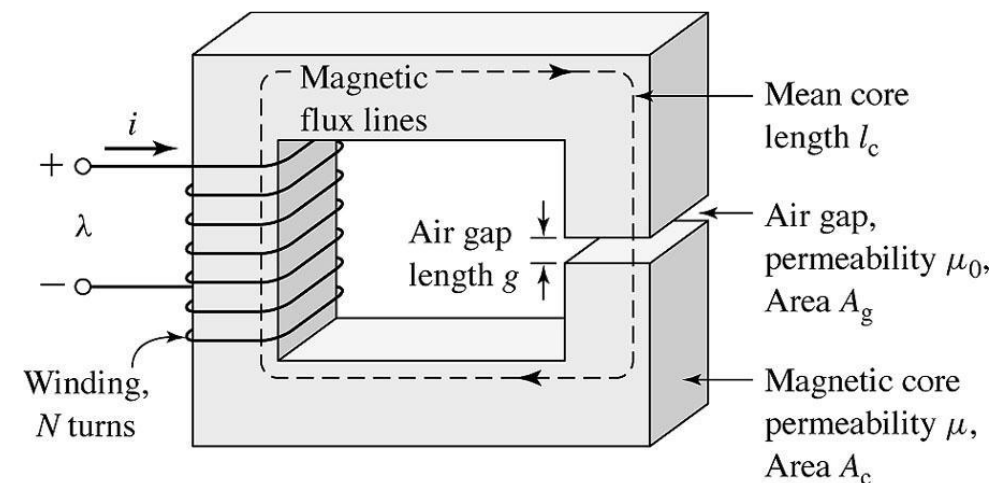
In practical systems, the magnetic field lines “fringe” outward somewhat as they cross the air gap. Provided this fringing effect is not excessive, the magnetic-circuit concept remains applicable. The effect of these fringing fields is to increase the effective cross-sectional area A_g of the air gap. In this course, the effect of fringing fields is ignored, thus $A_g = A_c$.



Example 4: Magnetic Circuit with Air Gap

The magnetic circuit shown in right figure has dimensions $A_c = A_g = 9 \text{ cm}^2$, $g = 0.050 \text{ cm}$, $l_c = 30 \text{ cm}$, and $N = 500$ turns. Assume the value $\mu_r = 70,000$ for core material.

- (a) Find the reluctances $\mathcal{R}_c$ and $\mathcal{R}_g$.
- (b) For the condition that the magnetic circuit is operating with $B_c = 1.0 \text{ T}$, find the flux ϕ
- (c) Find the current i based on (b).



□ Solution

$$(a) \quad \mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{0.3}{70,000 (4\pi \times 10^{-7})(9 \times 10^{-4})} = 3.79 \times 10^3 \frac{\text{A} \cdot \text{turns}}{\text{Wb}}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g} = \frac{5 \times 10^{-4}}{(4\pi \times 10^{-7})(9 \times 10^{-4})} = 4.42 \times 10^5 \frac{\text{A} \cdot \text{turns}}{\text{Wb}}$$

$$(b) \quad \phi = B_c A_c = 1.0(9 \times 10^{-4}) = 9 \times 10^{-4} \text{ Wb}$$

$$(c) \quad i = \frac{\mathcal{F}}{N} = \frac{\phi(\mathcal{R}_c + \mathcal{R}_g)}{N} = \frac{9 \times 10^{-4}(4.46 \times 10^5)}{500} = 0.80 \text{ A}$$

Be careful with the Units

Flux Linkage and Inductance

Based on Faraday's Law of Induction, the time-varying flux will induce a voltage (*emf*) on the coil.

$$emf = -N \frac{d\Phi_{mag}}{dt}$$

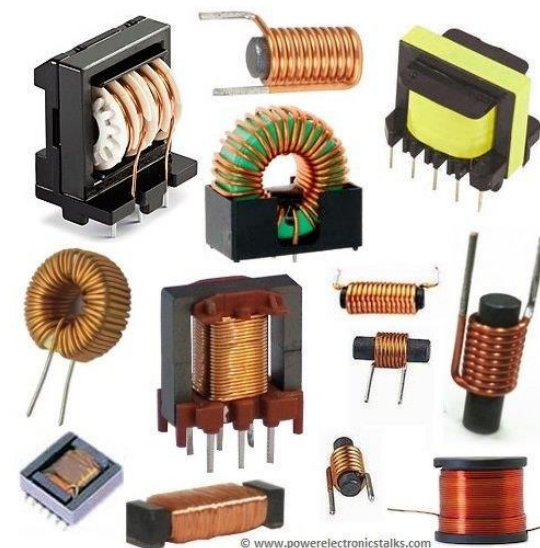
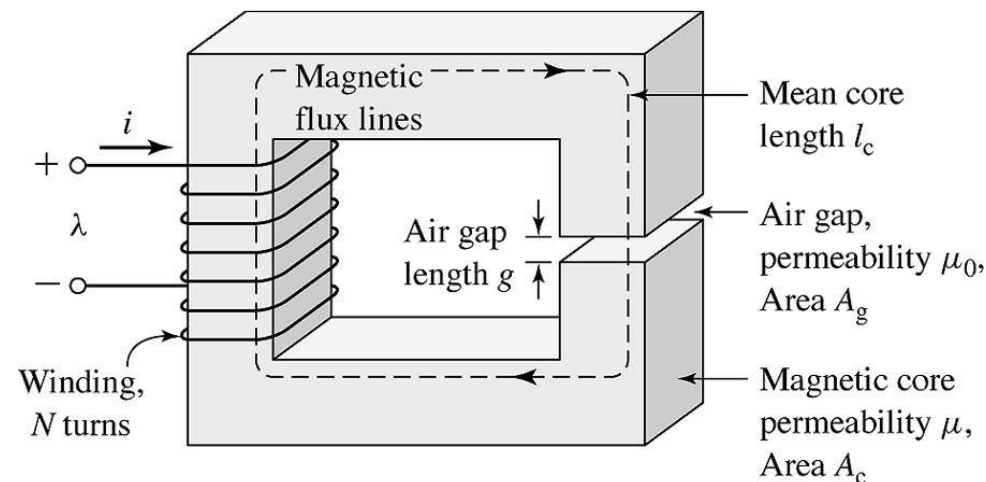
Change the pattern and then use induced voltage e to represent the *emf*.

$$e = N \frac{d\varphi}{dt} = \frac{d\lambda}{dt}$$

where λ is the **flux linkage** of the winding and is defined as

$$\lambda = N\varphi$$

Flux linkage is measured in units of webers (or equivalently weber-turns). Note that we have chosen the symbol φ to indicate the instantaneous value of a time-varying flux ϕ .



Flux Linkage and Inductance

For a magnetic circuit composed of magnetic material of constant magnetic permeability or which includes a dominating air gap, the relationship between λ and i will be linear and we can define the *inductance* L as

$$L = \frac{\lambda}{i}$$

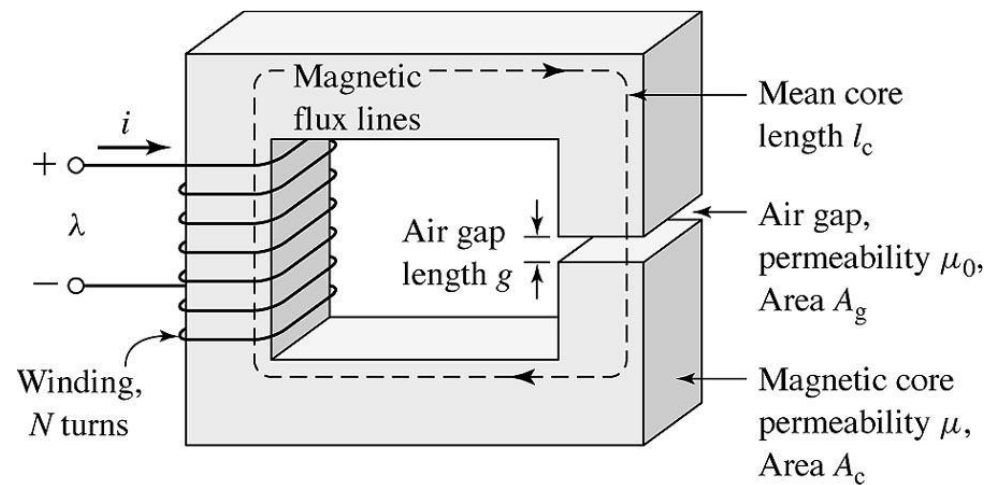
For the magnetic circuit with an air gap, we have

$$\phi \approx \frac{\mathcal{F}}{\mathcal{R}_g} = \frac{\mathcal{F} \mu_0 A_g}{g} = Ni \frac{\mu_0 A_g}{g} \quad \lambda = N\phi$$

Under the assumption that the reluctance of the core is negligible as compared to that of the air gap, the inductance of the winding

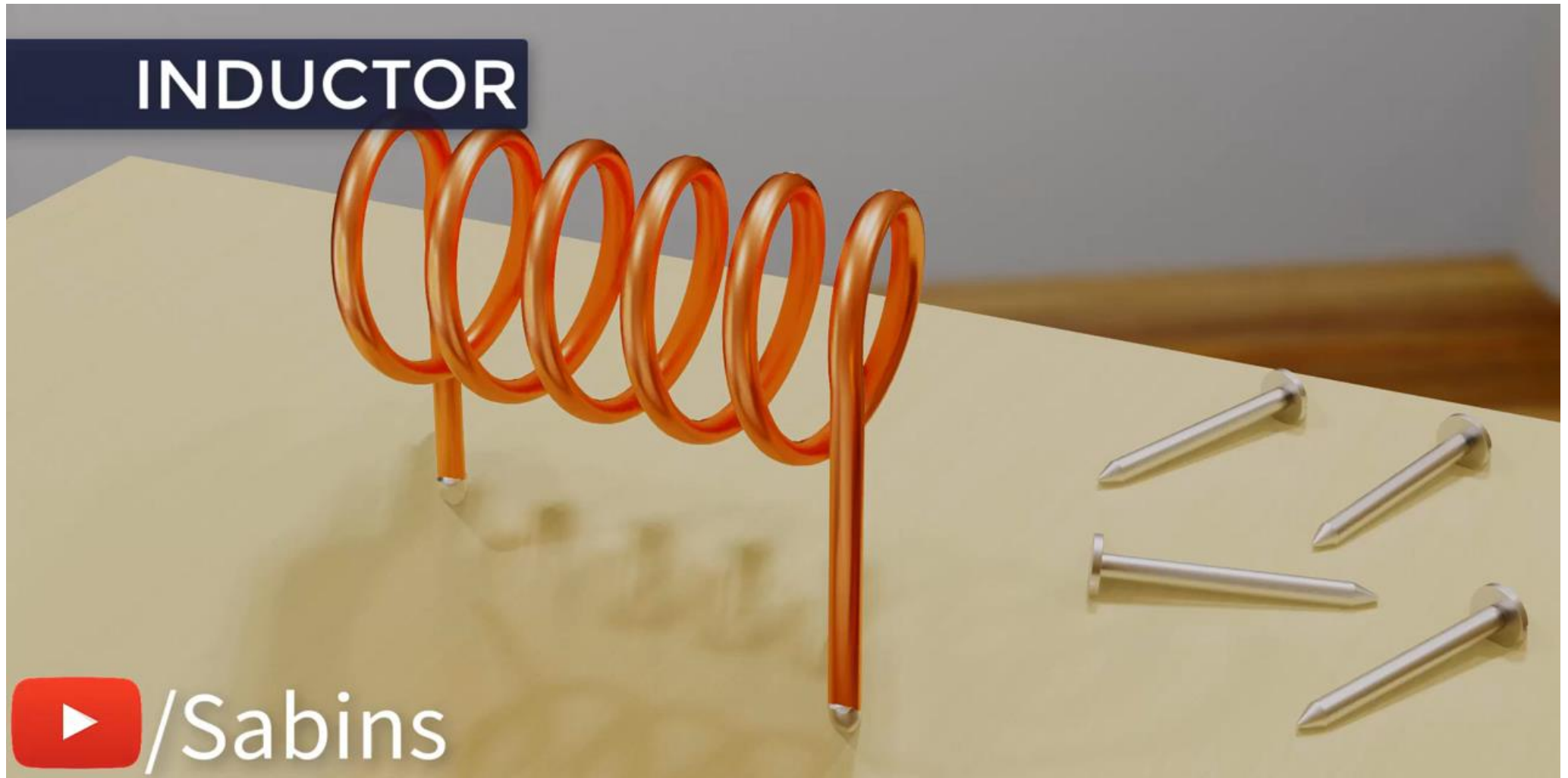
$$L = \frac{N^2}{(g/\mu_0 A_g)} = \frac{N^2 \mu_0 A_g}{g}$$

Inductance is measured in *henrys* (H) or *weber-turns per ampere*.



Inductance is proportional to the square of the number of turns, to a magnetic permeability and to a cross-sectional area and is inversely proportional to a length. L is irrelevant to current i even defined by i .

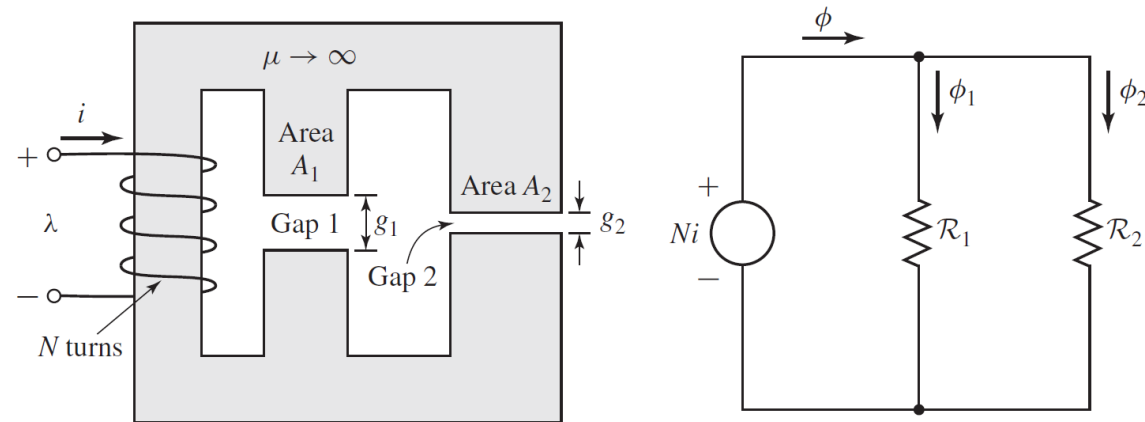
What is Inductor



Example 5: Inductance Calculation with Air Gaps

The magnetic circuit of right figure consists of an N -turn winding on a magnetic core of infinite permeability with two parallel air gaps of lengths g_1 and g_2 and areas A_1 and A_2 , respectively.

- (a) Find the inductance of the winding
- (b) Find the flux density B_1 in gap 1 when the winding is carrying a current i . Neglect fringing effects at the air gap.



□ Solution

- (a) The permeability of the magnetic core is infinite, then the core reluctance can be neglected. Thus, the total reluctance is equal to the parallel combination of the two gap reluctances.

$$\mathcal{R}_1 = \frac{g_1}{\mu_0 A_1} \quad \mathcal{R}_2 = \frac{g_2}{\mu_0 A_2} \quad \phi = \frac{Ni}{\frac{\mathcal{R}_1 \mathcal{R}_2}{\mathcal{R}_1 + \mathcal{R}_2}} \quad \longrightarrow \quad L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N^2(\mathcal{R}_1 + \mathcal{R}_2)}{\mathcal{R}_1 \mathcal{R}_2}$$
$$= \mu_0 N^2 \left(\frac{A_1}{g_1} + \frac{A_2}{g_2} \right)$$

- (b) From the equivalent circuit, one can see that

$$\phi_1 = \frac{Ni}{\mathcal{R}_1} = \frac{\mu_0 A_1 Ni}{g_1} \quad \longrightarrow \quad B_1 = \frac{\phi_1}{A_1} = \frac{\mu_0 Ni}{g_1}$$