

Lab 5 Flow of Control (II): Looping Statements

Please test the correctness of your programs on **PASS**.

Q-4. [Optional]

continue ...

It is easy to find out small Pythagorean triples but takes quite a lot of time to find large Pythagorean triples with the above brutal-force method. Fortunately, mathematicians have found out many other more efficient algorithmic approaches to search for Pythagorean triples. Below we show a method named the **Dickson's method**, invented in 1920. We only need to write two for-loops with this method and can easily find large Pythagorean triples.

Dickson's method:

To find positive integer solutions to $a^2 + b^2 = c^2$, find **positive integers** r , s , and t such that $r^2 = 2*s*t$. Then, $(a = r + s, b = r + t, c = r + s + t)$ form Pythagorean triples. In fact, **all Pythagorean triples can be found by this method** (by setting r to every possible positive number).

Here is an intuitive example:

Choose $r = 6$. Then $r^2/2 = 18$. The three factor-pairs of 18 are: (1, 18), (2, 9), and (3, 6). All three factor-pairs will produce triples using the above equations.

$s = 1, t = 18$ gives the triple (7, 24, 25) because $a = 6 + 1, b = 6 + 18, c = 6 + 1 + 18$.

$s = 2, t = 9$ gives the triple (8, 15, 17) because $a = 6 + 2, b = 6 + 9, c = 6 + 2 + 9$.

$s = 3, t = 6$ gives the triple (9, 12, 15) because $a = 6 + 3, b = 6 + 6, c = 6 + 3 + 6$.

Expected Output:

Example 1
Please input the maximal length allowed for the edge: <u>10000</u> Please input the minimal length allowed for the edge: <u>1000</u> The number of all Pythagorean triplets between 1000 and 10000 is 8838.
Example 2
Please input the maximal length allowed for the edge: <u>50000</u> Please input the minimal length allowed for the edge: <u>10000</u> The number of all Pythagorean triplets between 10000 and 50000 is 34553.

The above two examples cannot be efficiently derived using the brutal-force method introduced in Q3. Please use **Dickson's method** to rewrite.

Hint 1: We can iterate the **positive integer** r from the smallest possible value to the **upper bound** in the outer loop first, as a , b , and c are always larger than r . Note that the step size for iteration could be set to, for example, 2 (the intention is to make sure r is always even and hence $r^2/2$ is an integer). Then compute $r^2/2$ and find all possible factors of this value with an inner loop (think about how to minimize the number of iterations). Lastly, use if-condition to bound $(a = r + s, b = r + t, c = r + s + t)$ such that $a \geq$ **lower bound**, $b \geq$ **lower bound** and $c \leq$ **upper bound**.

Hint 2: First try triples of small ranges as in a) to see if the results stay the same. For large triple based right-angled triangles, the maximal length in test cases will not exceed 50000, as 50000^2 reaches the limit of what **int** can represent ($\leq 2^{31}-1$).