

Write your answers on **paper** using the **following format**:

Answer to Question X

Scan your answers **or** take **photos** of them.

Submit **scanned** answers **or** their **photos** in a **single file** to Canvas **before 4:10 pm**.

Name your file in following format: **Quiz1_SID_LastName_FirstName**

Part 1: Short questions:

40 marks

Write down all the steps in your calculations.

Q1) Find the floating-point representation of the following number with 5 decimal points.

0.00126678

Q2) Consider the function $f(x) = x^3 + 3x^2 + 2x + 4$. Compute the first derivative of $f(x)$ around $x_i = 0.5$ using the forward, backward, and centered methods. Use the step-size $h = 0.01$.

Q3) Suppose that $\tilde{x} = 1.5$ is an approximation of x with an error of $\Delta\tilde{x} = 0.02$. We would like to approximate the value of $f(x) = x^2 + 2x + 2$ using $f(\tilde{x})$. Compute the approximation error.

Q4) Consider the following vectors v_1 and v_2 . Can v_1 and v_2 be eigenvectors of a symmetric matrix?

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

Q5) Find the determinant of the following matrix.

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 1 & -1 & 1 \\ 2 & 4 & 3 \end{bmatrix}$$

Q6) Does the following matrix have any eigenvalues equal to zero? Why?

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -3 & 3 & 3 \\ 1 & -1 & -1 \end{bmatrix}$$

Q7) Consider matrix A and vector v below. Is v , an eigenvector of A ? Why?

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 1 & -1 & 1 \\ 2 & 4 & 3 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Q8) Can the polynomial $\lambda^2 + 2\lambda + 1$ be the characteristic polynomial of a symmetric 2-by-2 matrix? Why?

Part 2: Long questions:

Write down all the steps in your calculations.

Q1) Consider the function $f(x) = 1 + x^2 + x^3 + x^4 + 3x^5$.

20 marks

- Find the first-order Taylor approximation of $f(x)$ around $x_i=1$.
- Find the percentage of the relative error when the first-order Taylor approximation of $f(x)$ is used to evaluate $f(2)$.
- Find the second-order Taylor approximation of $f(x)$ around $x_i=1$.
- Find the percentage of the relative error when the second-order Taylor approximation of $f(x)$ is used to evaluate $f(2)$.
- Find the third-order Taylor approximation of $f(x)$ around $x_i=1$.
- Find the percentage of the relative error when the third-order Taylor approximation of $f(x)$ is used to evaluate $f(2)$.
- Find the fourth-order Taylor approximation of $f(x)$ around $x_i=1$.
- Find the percentage of the relative error when the fourth-order Taylor approximation of $f(x)$ is used to evaluate $f(2)$.

Write down all the steps in your calculations.

Q1) Consider the following matrix.

20 marks

$$A = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ -3 & 4 & 6 \end{bmatrix}$$

a) Find the characteristic polynomial of A .

b) The vector $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ is an eigenvector of A . Find its corresponding eigenvalue.

c) Find the other two eigenvalues of A

Write down all the steps in your calculations.

Q3)

Let A be an invertible $n \times n$ matrix. Suppose that v is an eigenvector of A and λ is the eigenvalue corresponding to v . Assume that λ is non-zero.

20 marks

a) Show that v is an eigenvector of $(A^{-1})^n$ and $(\frac{1}{\lambda})^n$ is its corresponding eigenvalue.

b) Show that λ^n is an eigenvalue of the matrix A^n and v is its corresponding eigenvector.