

EE1001

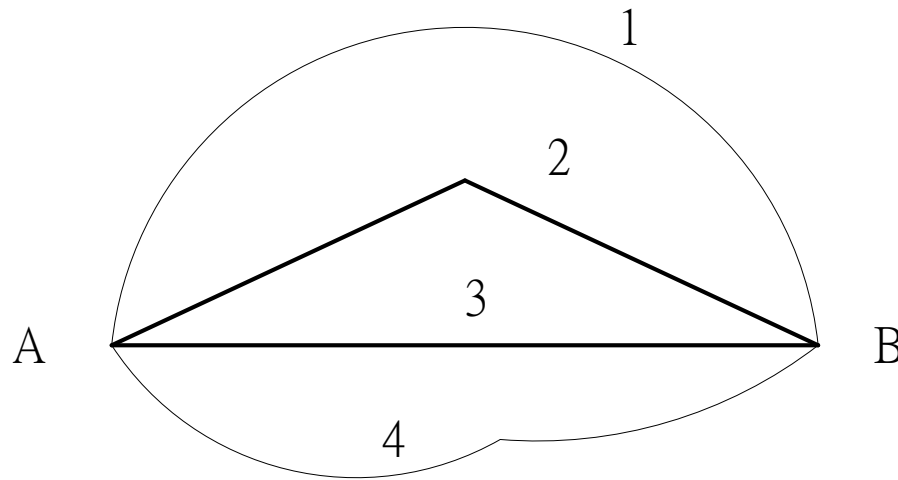
Foundations of Digital Techniques and Data Analysis

Essence of Optimization – Introduction



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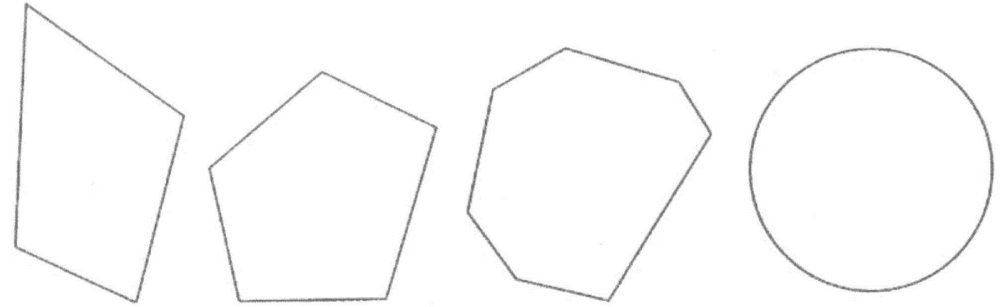
The Wisdom of The Ancient Greeks



Euclid of Alexandria
325 BC- 265 BC

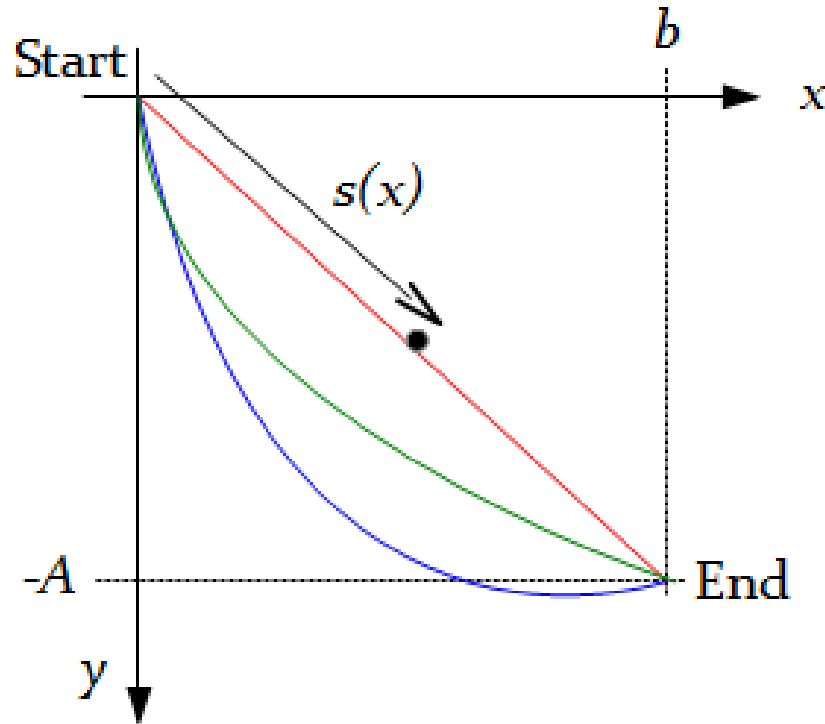
- What is the minimum distance between two places A and B?
- STRAIGHT LINE.

Dido's Problem



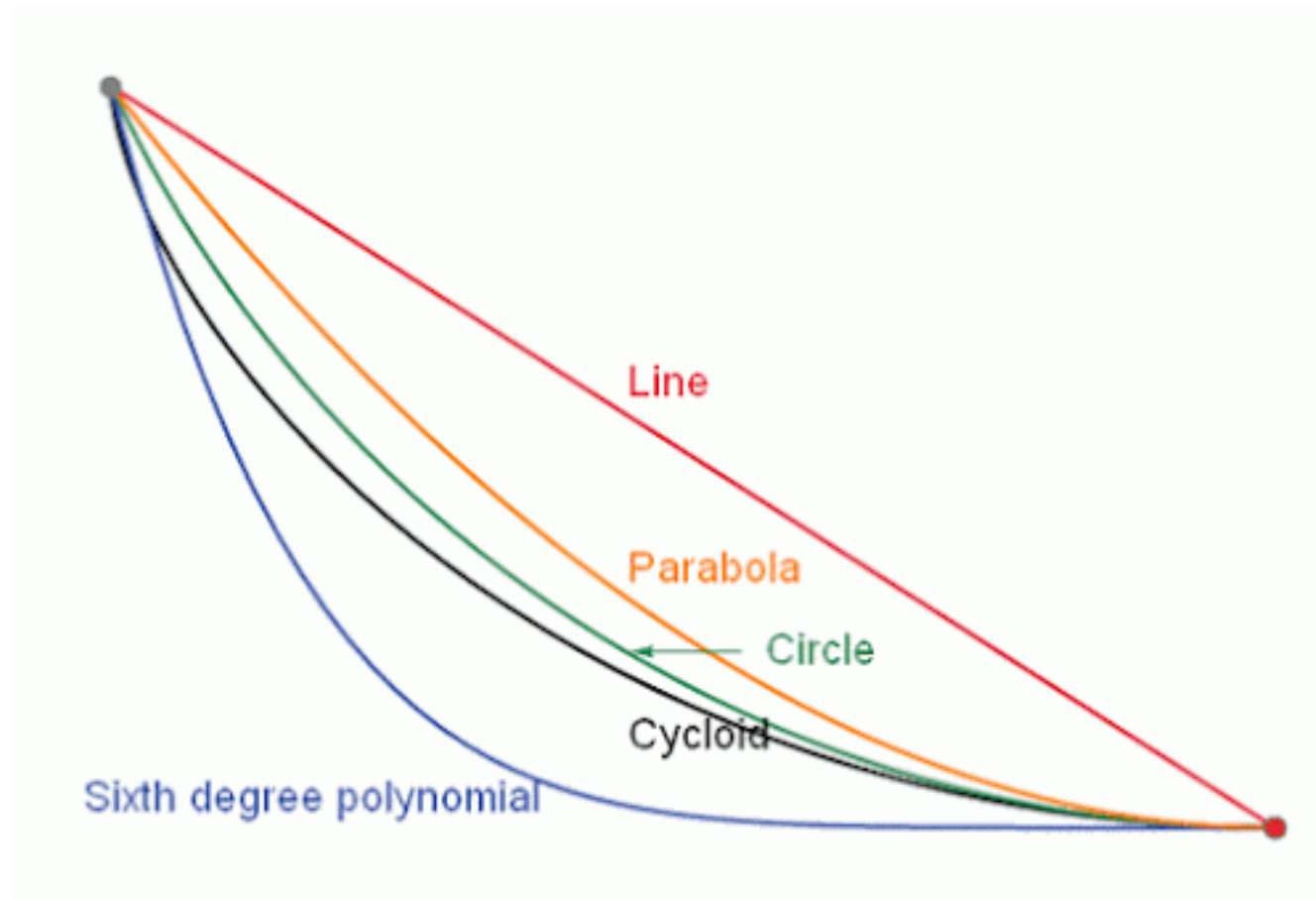
- Dido founded the city of Carthage, in Tunisia.
- She asked the locals for as much land as could be bound by a bulls hide.
- She cut the hide into a long thin strip and bounded the **maximum possible area** with this.
- The maximum possible area bounded by a curve of fixed length is a **circle**.

Brachistochrone Problem

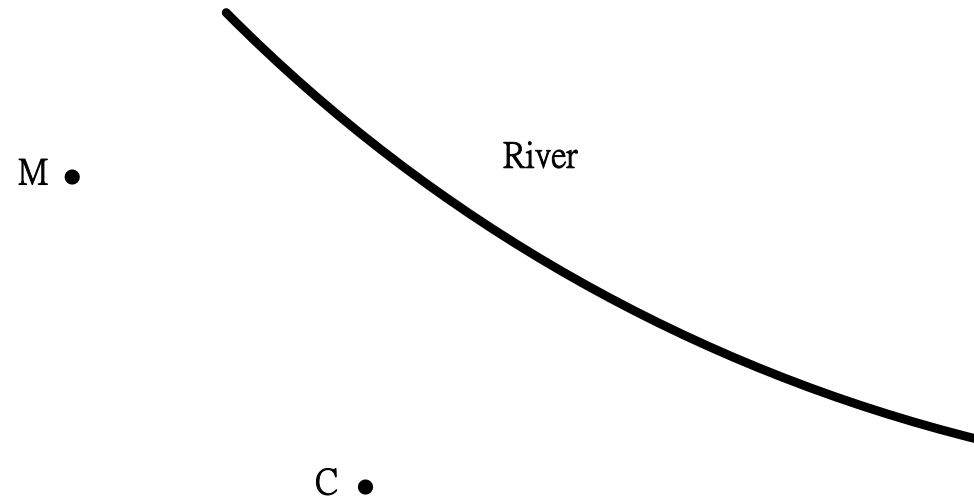


- $\beta\rho\alpha\chi\tau\omicron\varsigma$ (brachistos) "the shortest" + $\chi\rho\omicron\nu\omicron\varsigma$ (chronos) "time, delay"
- The only force acting on the ball is gravity.
- What must be the shape of the ramps curve be, such that the descent time is a minimum?

Solution of Brachistochrone: Cycloid

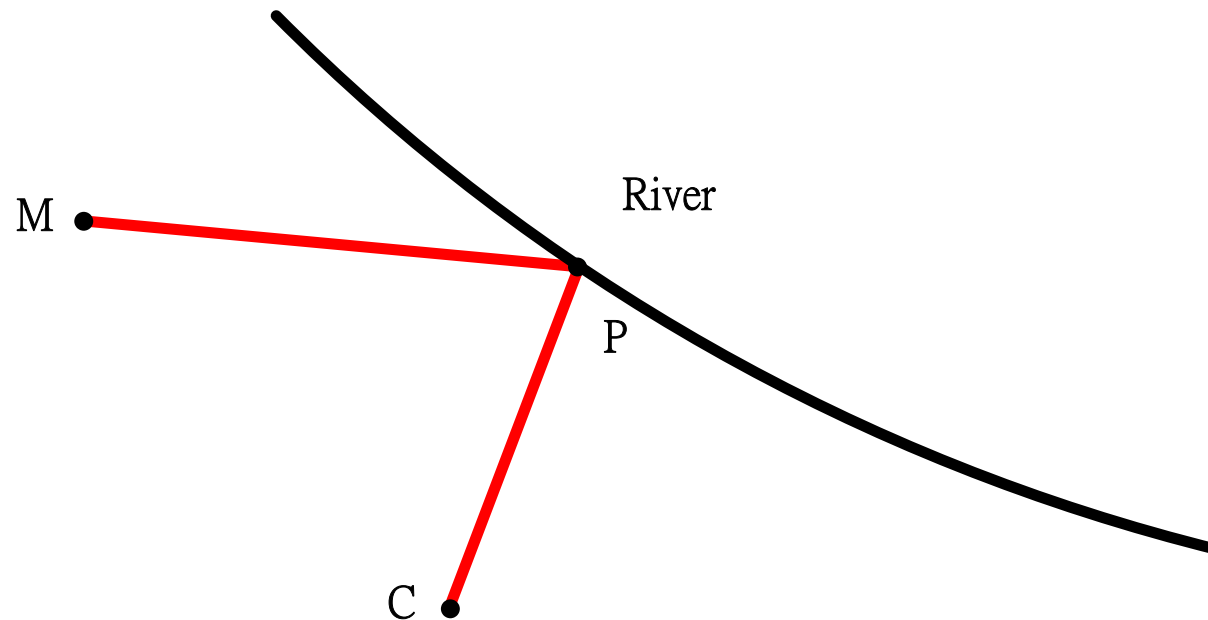


The Milkmaid Problem



- The milkmaid is at point M , the cow is at point C
- Before gathering the milk, she has to rinse out her bucket in the river
- Field is flat and uniform, all points on the river bank are equally good
- She's in a hurry to get back for a date with a handsome young goatherd

Shortest Path



- What is the best point P on the riverbank for her to rinse the bucket?
- What is the shortest path $MP + PC$?
- Your guesses? Why?

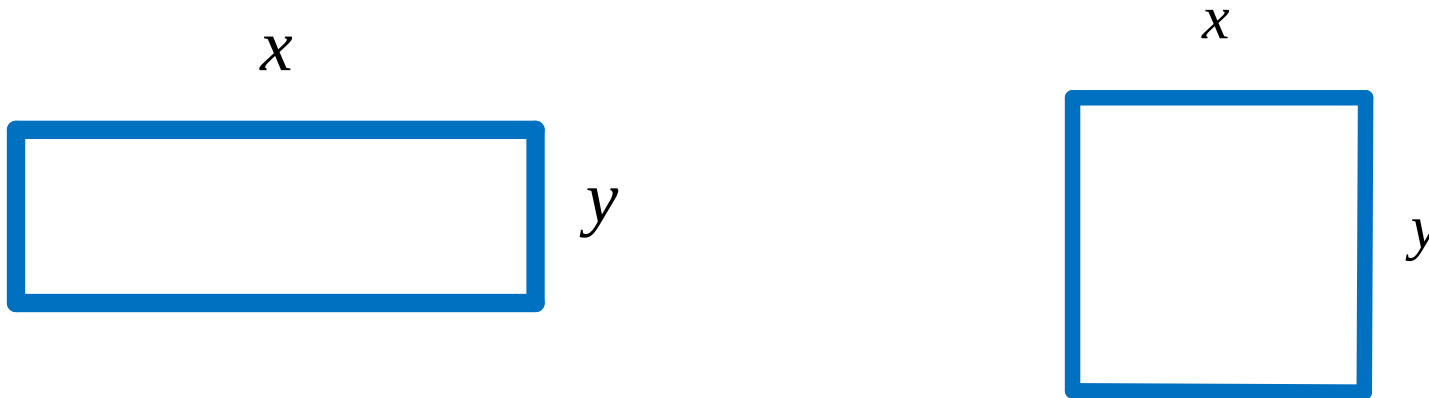
Maximum Production Level

- A manufacturer's production is modeled by the Cobb-Douglas function:

$$f(x, y) = 100x^{\frac{4}{3}}y^{\frac{1}{4}}$$

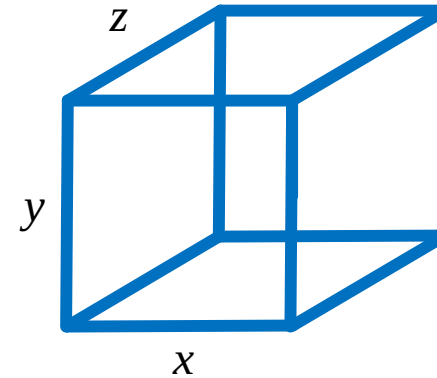
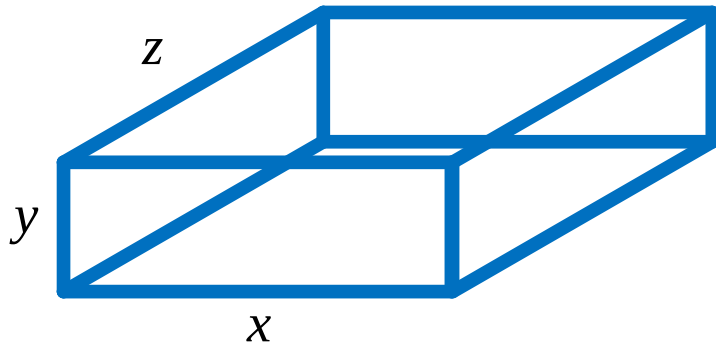
- x represents the units of labor, y represents the units of capital
- Each labor unit costs 200 HKD and each capital unit costs 250 HKD. The total expenses for labor and capital cannot exceed 50,000 HKD .
- Find the maximum production level

Rectangular with a Fixed Perimeter



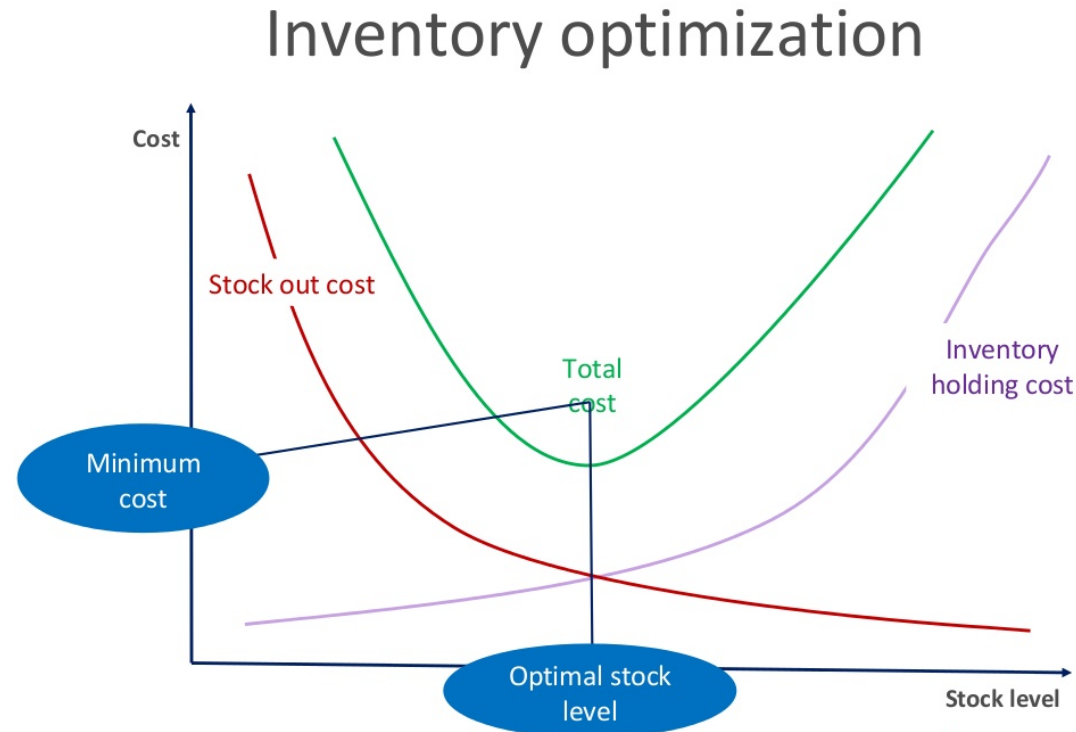
- Given a rectangular with a fixed perimeter $2(x + y) = c$
- What is the largest area xy ?
- What is your guess?
- It is a square!

Cuboid with a Fixed Surface Area



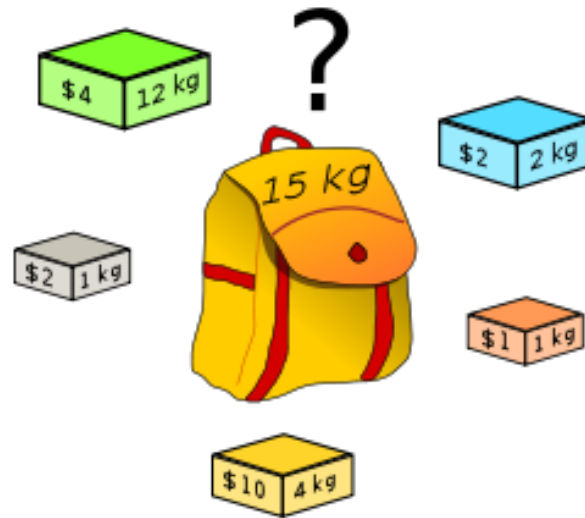
- Given a cuboid with a fixed surface area $2(xy + yz + zx) = c$
- What is the largest volume xyz ?
- It is a cube!

Inventory Optimization



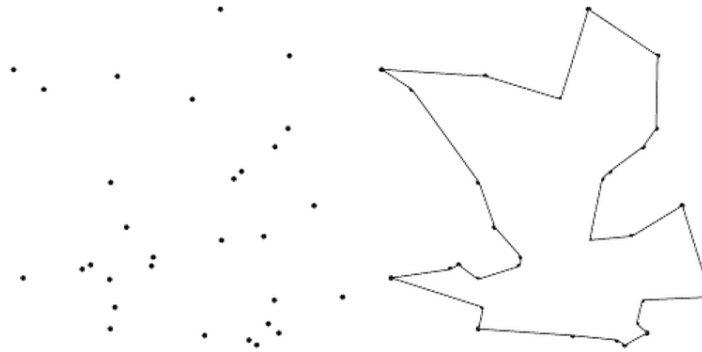
- Inventory cost increases as stock level increases
- Stock out decreases as stock level increases
- At what stock level, the total cost reaches the minimum?

Knapsack Problem



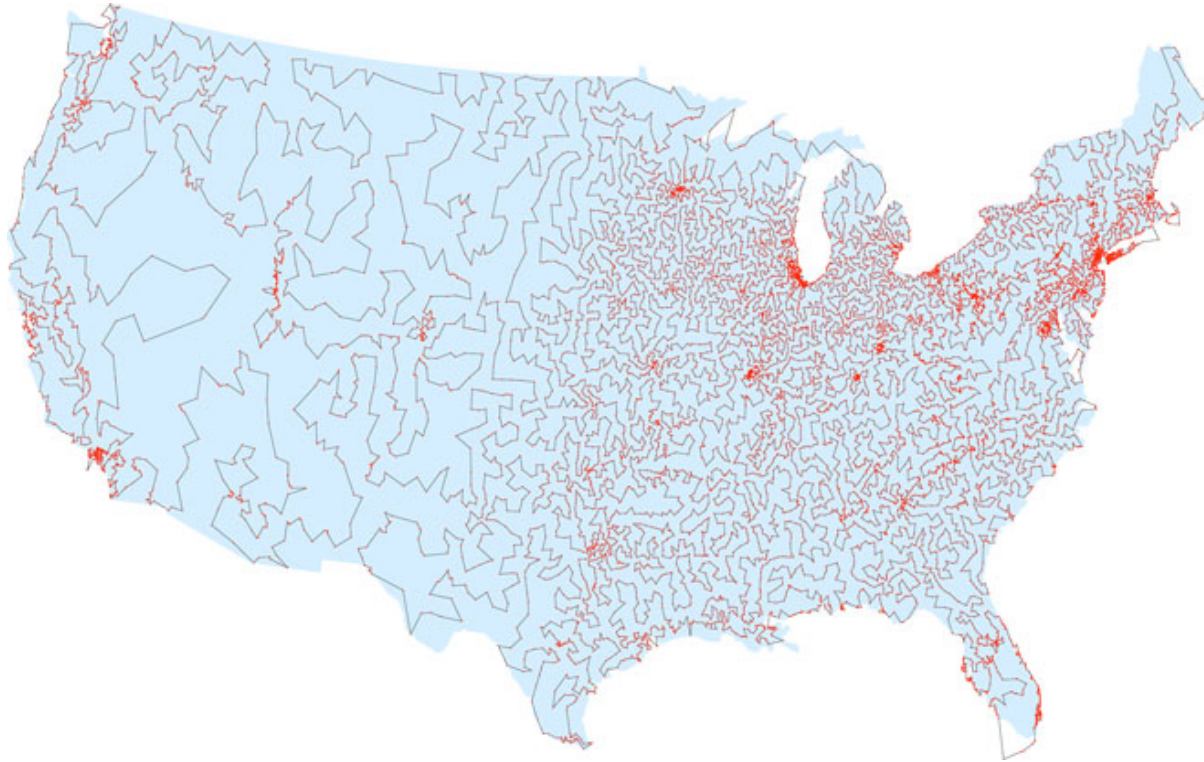
- Given a set of items, each with a mass and a value
- Determine the number of each item to include in a collection
- The total weight is less than or equal to a given limit
- The total value is as large as possible

Traveling Salesman Problem



- Finding the most efficient cycle a salesman can take through each of the given cities
- For instance, least total distance

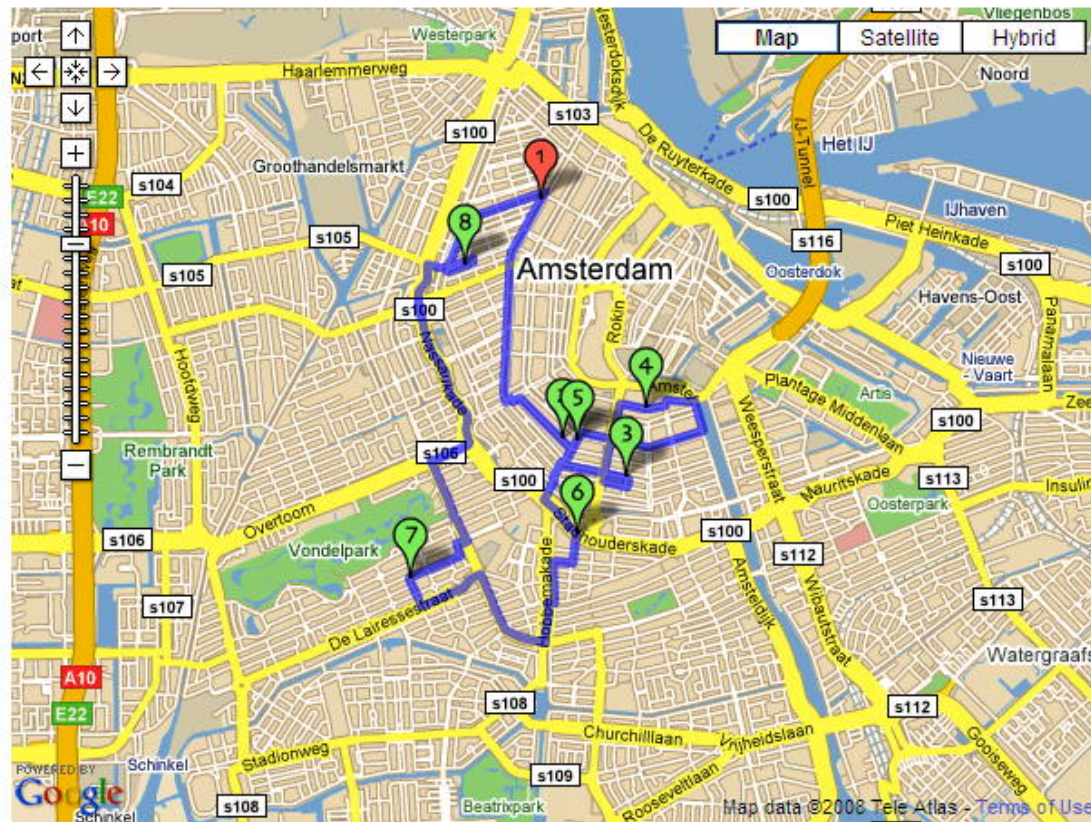
Traveling Salesman Problem



- The shortest traveling salesman route going through all 13,509 cities in the United States with a population of at least 500 (as of 1998)
- Illustration: Courtesy of David Applegate, Robert Bixby, Vasek Chvatal and William Cook

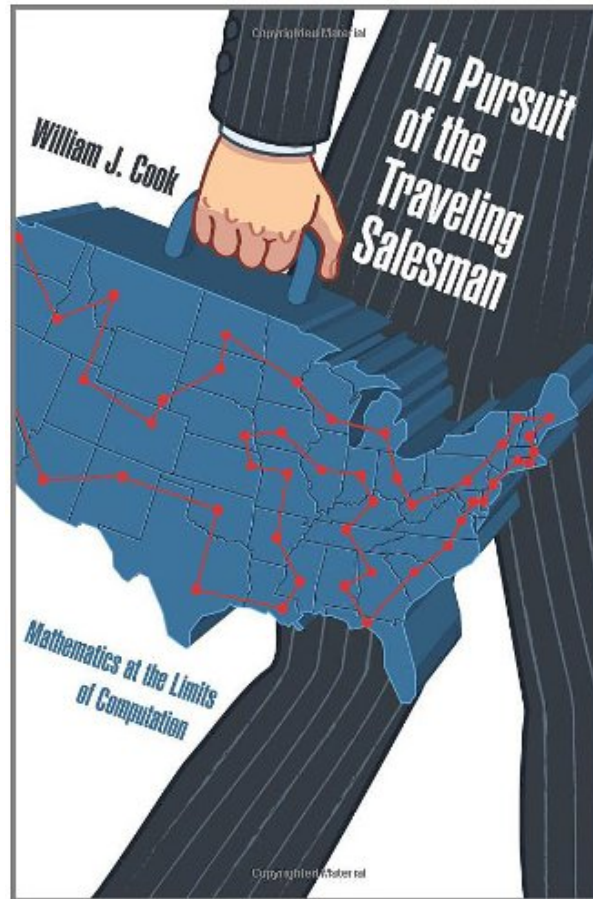
Traveling Salesman Problem

Google Maps Fastest Roundtrip Solver



- Fastest roundtrip using Google Maps

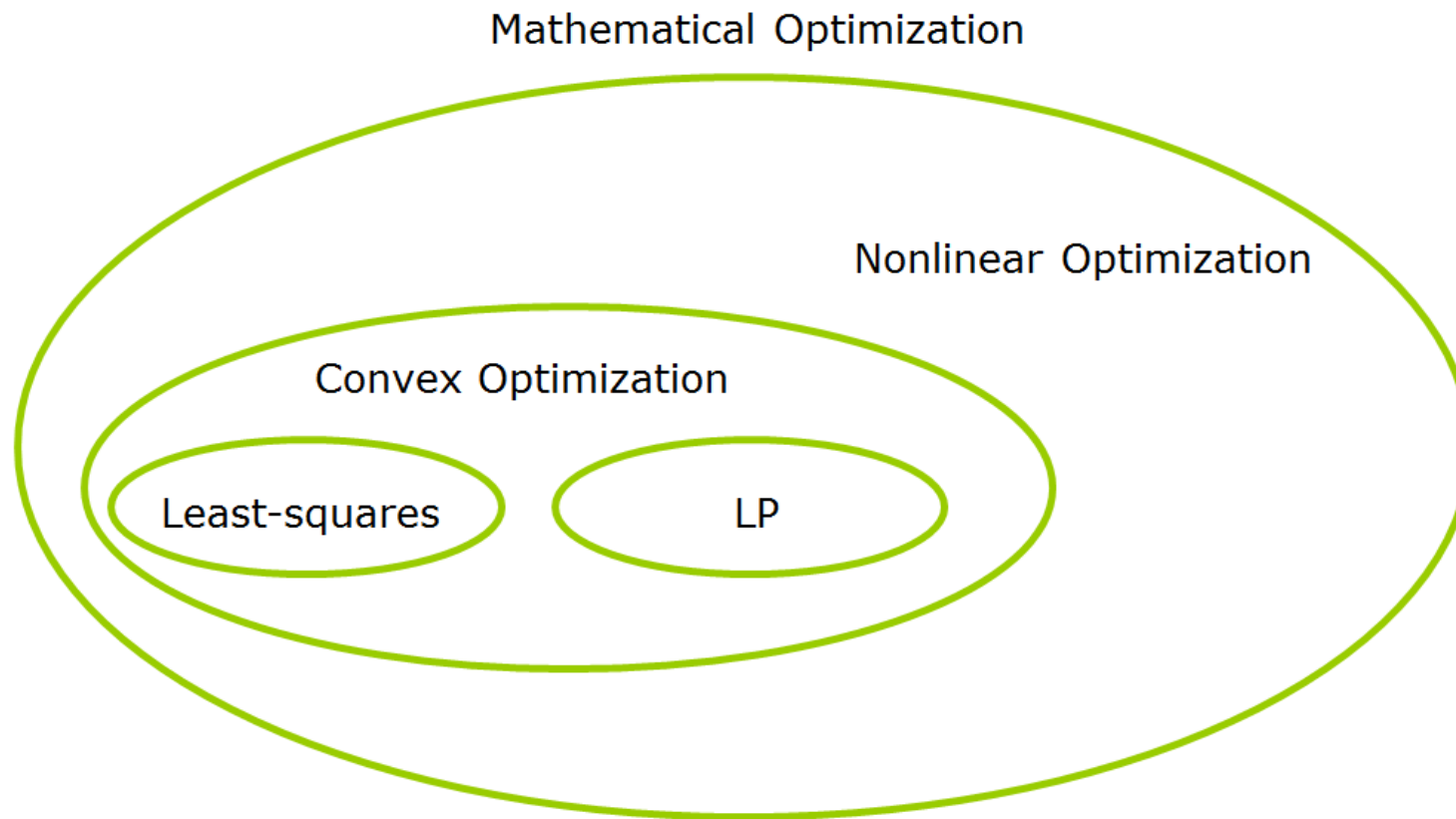
Traveling Salesman Problem



- Strongly recommend this book by William J. Cook
- In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation

What is Optimization

- Optimization: Selection of a *best element* from some set of available alternatives.



What is an Optimization Problem

- An Objective: What you want to minimize or maximize.
 - Dido's problem: Area encircled.
 - Brachistochrone Problem: Time spent.
 - Milkmaid problem: Path traveled.
- A Set of Constraints: What you cannot violate nor go beyond, because of physical, resource boundaries.
 - Dido's problem: Length of Dido's strip.
 - Brachistochrone Problem: Fixed end points.
 - Milkmaid problem: Must walk to the river before milking the cow.