

Tut 01

EE 2000

1. Simplify the following expressions using Boolean algebra

$$F(A, B, C) = A\bar{B}(\bar{B} + \bar{C})(A + C)$$

$$f(x, y, z) = \bar{x}z + y\bar{z} + \bar{x}y\bar{z} + xy$$

$$F(A, B, C) = A\bar{B}(\bar{B} + \bar{C})(A + C)$$

$$= AB'(AB' + AC' + B'C + CC')$$

$$= AB' + AB'C' + AB'C$$

$$= AB'(1 + C' + C)$$

$$= AB'$$

$$f(x, y, z) = \bar{x}z + y\bar{z} + \bar{x}y\bar{z} + xy$$

$$= x'z(1+y') + yz' + xy$$

Absorption

$$= x'z(1) + (x+x')yz' + xy$$

Distribution

$$= x'z + (xyz' + x'y\bar{z}') + xy$$

$$= x'(z + yz') + xy(1 + z')$$

$$= x'(z + y) + xy$$

$$= x'z + x'y + xy$$

$$= x'z + y(x + x')$$

$$= x'z + y$$

$$a + a'b = a + b$$

Proof:

$$a + b = (a + a')(a + b) \quad \{\text{distribution rule}\}$$

$$= aa + ab + a'a + a'b$$

$$= a + ab + a'b$$

$$= a(1+b) + a'b$$

$$= a + a'b$$

Quick Check

$$\text{If } a = 1, \text{ then } a + a'b = 1$$

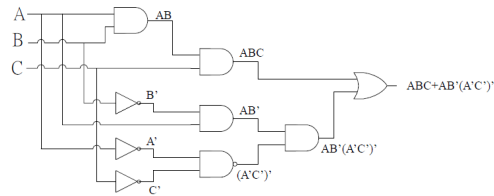
$$\text{If } a = 0, \text{ then } a + a'b = b$$

$$\text{So } a + a'b = a + b$$

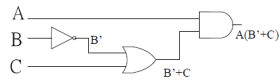
- 2 (a) Draw the logic circuit of the following Boolean function without simplification:

$$Y(A, B, C) = ABC + AB'(A'C')'$$

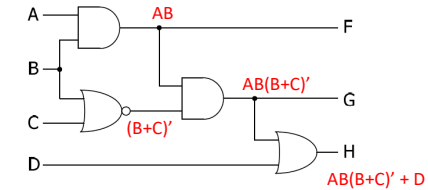
- (b) Simplify the logic function in (a) and, hence, re-draw the logic circuit.



$$\begin{aligned} Y(A, B, C) &= ABC + AB'[A'C']' = ABC + AB'(A+C) = ABC + AB' + AB'C \\ &= A(B+B')C + AB' = AC + AB' = A(B'+C) \end{aligned}$$



- 3 (a) Derive Boolean functions to describe the operations of the logic circuit as follow:

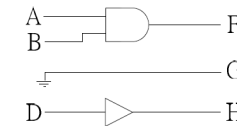


- (b) Hence, simply and draw a new logic circuit.

$$\begin{aligned} \text{Therefore,} \\ F &= AB \end{aligned}$$

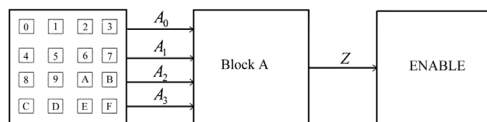
$$G = AB(B+C)' = ABB'C' = 0$$

$$H = AB(B+C)' + D = D$$



4. A simple locker system that has a output signal $Z = 1$ when the hexadecimal keypad input is either 0, 1, 2, 3, 4, 5, 8, 9, or A; otherwise $Z = 0$. Assume that A_0, A_1, A_2, A_3 represent a 4-digit binary number output from the keypad. Block A decodes these signals and outputs the signal Z.

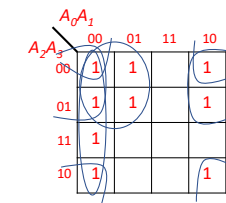
- (i) Write down the truth table of Block A.
(ii) Find the SOP and POS expression of Block A.
(iii) Design the circuit of Block A using minimum number of 2-input AND, OR gates and NOT gates.



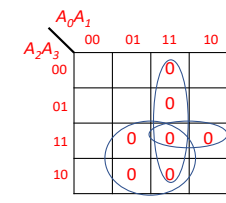
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• Assume a conventional BCD for A_0, A_1, A_2, A_3 .

Hexadecima l	A_0	A_1	A_2	A_3	Z
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
A	1	0	1	0	1
B	1	0	1	1	0
C	1	1	0	0	0
D	1	1	0	1	0
E	1	1	1	0	0
F	1	1	1	1	0



$$\text{SOP: } f = A'_0A'_1 + A'_0A'_2 + A'_1A'_2 + A'_1A'_3$$



$$f' = A_0A_1 + A_1A_2 + A_0A_2A_3$$

$$\text{POS: } f = (A'_0 + A'_1)(A'_1 + A'_2)(A'_0 + A'_2 + A'_3)$$

Minimum number of AND, OR and NOT gates

$$\begin{aligned} f &= A'_0A'_1 + A'_0A'_2 + A'_1A'_2 + A'_1A'_3 \\ &= A'_0(A'_1 + A'_2) + A'_1(A'_2 + A'_3) \\ &= A'_0(A_1A_2)' + A'_1(A_2A_3)' \end{aligned}$$

(Please draw the circuit.)