

1. The mean μ is $\frac{5}{4}$. The RMS is $\frac{3}{2}$. The standard deviation σ is $\frac{\sqrt{11}}{4}$.
They are related by $\mathbf{rms}(x)^2 = \mu^2 + \sigma^2$.

2. Yes, it is.

Let $a = (a_1, a_2, \dots, a_n)$.

The average of the entries with odd indices is given by $\frac{x_1 + x_3 + \dots + x_{n-1}}{n/2}$, which can be written as $\frac{2}{n}(x_1 + x_3 + \dots + x_{n-1})$.

Similarly, the average of the entries with even indices can be written as $\frac{2}{n}(x_2 + x_4 + \dots + x_n)$.

Therefore, $f(x) = \frac{2}{n}x_1 - \frac{2}{n}x_2 + \frac{2}{n}x_3 - \frac{2}{n}x_4 + \dots + \frac{2}{n}x_{n-1} - \frac{2}{n}x_n$.

Hence, $a_i = \begin{cases} \frac{2}{n} & \text{if } i \text{ is odd,} \\ -\frac{2}{n} & \text{if } i \text{ is even.} \end{cases}$

3. The rotation matrix is

$$R = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$$

The resultant vector is

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 2\sqrt{3} - 3 \\ 2 + 3\sqrt{3} \end{bmatrix}.$$

4. The following equation would hold if a line could go through all four points:

$$A\hat{x} = b \Rightarrow \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} v \\ \beta \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

Then we form the normal equation:

$$A^T A \hat{x} = A^T b \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} v \\ \beta \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 \\ -1 & 9 \end{bmatrix} \begin{bmatrix} v \\ \beta \end{bmatrix} = \begin{bmatrix} 8 \\ -11 \end{bmatrix}$$

Solving the equations, we obtain $v = \frac{61}{35}$ and $\beta = -\frac{36}{35}$.

5. No, it is not a group because 0 has no inverse.

6.

a) The elements are 1, 3, 7, 9.

b) 1 is the identity element.

The inverse of 1 is 1.

The inverse of 3 is 7.

The inverse of 7 is 3.

The inverse of 9 is 9.

c) Yes, it is a cyclic group. The generators are 3 and 7.

7.

a) The codewords are

0 0 0 0 0

1 0 1 1 0

0 1 0 1 1

1 1 1 0 1

The code rate is 2/5 or 0.4.

The minimum distance d_{min} is 3.

It can correct one error bit.

b) The generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$.

The parity check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$.

c) The syndrome is $s = yH^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$.

The first component is non-zero and the last two components are zero, which indicate that the first parity-check equation is in error while the last two parity-check equations are without error.

Since only c_3 occurs in the first parity-check equation but not in the last two equations, the error bit is c_3 .