

EF3441: Intermediate Macroeconomics

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Lecture 2

The Main Goal of this Lecture

- ▶ Understand the consumer's/household's consumption and leisure decisions in a one-period model. Particularly the tradeoff between consumption and leisure.
- ▶ To do that, we need to
 - ▶ list the properties of the consumer's preferences,
 - ▶ construct the consumer's budget constraints,
 - ▶ show how the consumer optimizes given preferences and budget constraints.

The Household

- ▶ The household (the representative consumer) cares about:
 - ▶ Consumption: c (Recall: in GDP consumption is C ; here is c)
 - ▶ Think of c as a bundle/basket of goods. With the loss of any generality (If you are interested in the proof, please see Walras' law. We will not cover this topic in this course.), we normalize the price of c to 1 (c is called *numeraire* good.).
 - ▶ Leisure: ℓ
- ▶ The household (HH) has a goal: be happy.
- ▶ Assumptions
 - ▶ All the households have the same tastes/preferences (Is it a realistic assumption?).
 - ▶ Households are very smart (optimization).
 - ▶ Households live for *one* period (static model for now).

The Decision of the Household

- ▶ Be happy: maximize utility.
 - ▶ For every bundle (c, ℓ) , $U(c, \ell)$ associates a level of utility
 - ▶ Remember: c and ℓ can *not* be negative.
 - ▶ We say that (c_1, ℓ_1) is preferred to (c_2, ℓ_2) if and only if

$$U(c_1, \ell_1) > U(c_2, \ell_2)$$

- ▶ The objective of the HH is to maximize $U(c, \ell)$ on every pair (c, ℓ) that it has available (feasible and affordable)

Properties of the Utility Function

- ▶ Properties of the utility function:
 - ▶ Utility is increasing (more is preferred to less)
 - ▶ The slopes of the utilities are *non-negative*. *i.e.*, both the *marginal utility of consumption*, $\frac{\partial U}{\partial c}$, and the *marginal utility of leisure*, $\frac{\partial U}{\partial \ell}$, are *non-negative*.
 - ▶ Concavity: each additional unit of consumption and leisure adds less utility (eating twice as much does not make you twice as happy)
 - ▶ *i.e.*, the second-order derivatives are *non-positive*, or the slopes of the marginal utility are *non-positive*.
 - ▶ Consumption and leisure are normal goods (leisure is a consumption good, too!)

Properties of the Utility Function, Con't

- ▶ More preferred to less:

- ▶ Marginal utility of leisure

$$\frac{\partial U}{\partial \ell} = MU_{\ell} > 0$$

- ▶ Marginal utility of labor (supply)

$$\frac{\partial U}{\partial N^s} = MU_{N^s} < 0$$

- ▶ Marginal utility of consumption

$$\frac{\partial U}{\partial c} = MU_c > 0$$

- ▶ Concavity:

- ▶ Declining MU_{ℓ}

$$\frac{\partial MU_{\ell}}{\partial \ell} < 0$$

- ▶ Declining MU_c

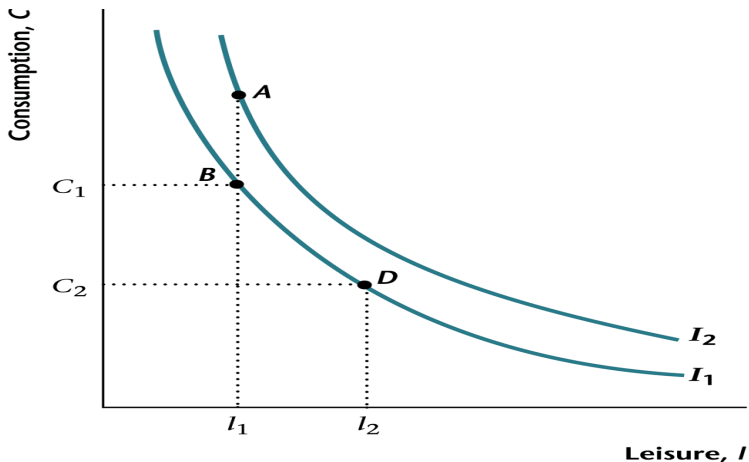
$$\frac{\partial MU_c}{\partial c} < 0$$

From U to Indifference Curves

An indifference curve is a curve connecting all values of consumption and leisure, among which the consumer is indifferent.

- ▶ Indifference curves
 - ▶ Downward sloping (more preferred to less)
 - ▶ Convex to the origin (concavity or preference for diversity)

Indifference Curves



- ▶ A is preferred to B captures the idea that more is preferred to less
- ▶ B to D reflects a preference for diversity

From Utility to Indifference Curves

An indifference curve is a curve connecting all values of consumption and leisure, among which the consumer is indifferent.

- ▶ Indifference curves
 - ▶ Downward sloping (more preferred to less)
 - ▶ Convex to the origin (preference for diversity)
- ▶ Marginal rate of substitution (**MRS**): the marginal rate of substitution is the rate at which a consumer is willing to give up one good in exchange for another good while keeping utility *constant*.
 - ▶ **Negative** slope of the indifference curve:

$$\frac{dc}{d\ell} = -\frac{\frac{\partial U}{\partial \ell}}{\frac{\partial U}{\partial c}} = -MRS_{\ell,c}$$

MRS

- $U = U(c, \ell)$, taking total differential, we get

$$dU = \frac{\partial U}{\partial c} dc + \frac{\partial U}{\partial \ell} d\ell$$

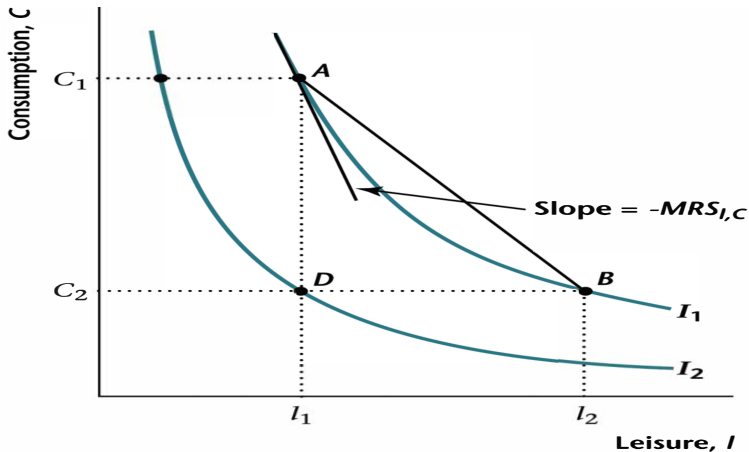
- $dU = 0$ at the indifference curve, so we get

$$0 = \frac{\partial U}{\partial c} dc + \frac{\partial U}{\partial \ell} d\ell$$

or

$$\frac{dc}{d\ell} = -\frac{\frac{\partial U}{\partial \ell}}{\frac{\partial U}{\partial c}} = -\frac{U_\ell}{U_c} = -MRS_{\ell,c}$$

Indifference Curves



- In the figure, a preference for diversity implies that any point on the line segment AB (other than A and B) is strictly preferred to A or B .

HH Constraints

- ▶ HH has a limited amount of time available (time constraints):
let N^s be time working

$$\ell + N^s = h$$

- ▶ Limited disposable income: a budget constraint. Let:
 - ▶ c = consumption (c is *numeraire* by assumption)
 - ▶ h = time endowment. For example, $h = 24$ hours per day
 - ▶ ℓ = leisure time
 - ▶ w = wage
 - ▶ π = dividend income
 - ▶ T = lump-sum taxes (rebates if $T < 0$)

A budget constraint

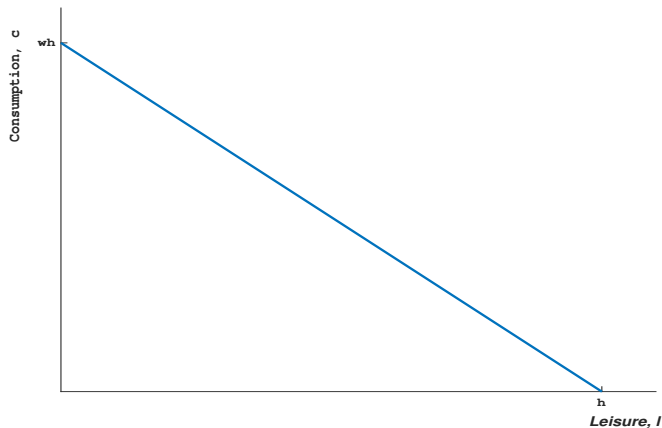
$$c = wN^s + \pi - T$$

or combining the time constraint

$$c + w\ell = wh + \pi - T$$

The Budget Constraint

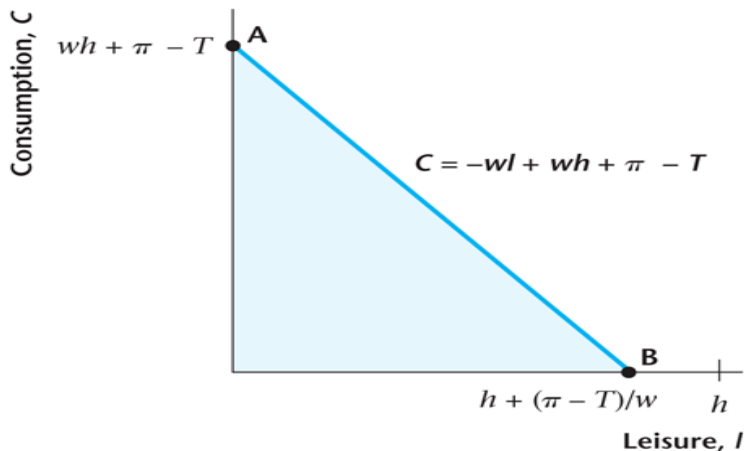
- Suppose $T = \pi$



- Wages determines the slope of the budget constraint. *i.e.*, rotate the budget line.

The Budget Constraint

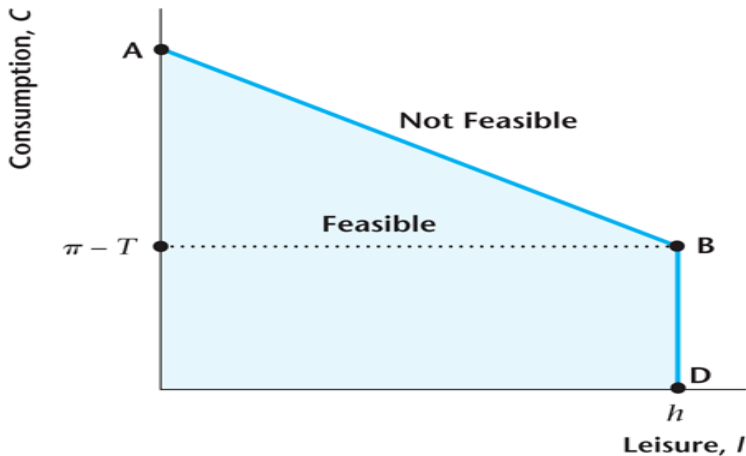
- Suppose $T > \pi$



- $\pi - T$ determine the intercept. *i.e.*, (parallelly) shift the budget line up/down.

The Budget Constraint

- Suppose now $T < \pi$ positive consumption even with no work



Example 1

Originally a consumer faces the budget line $p_1x_1 + p_2x_2 = m$. Then the price of good 1 doubles, the price of good 2 becomes 8 times larger, and income becomes 4 times larger. Write down an equation for the new budget line in terms of the original prices and income. Graph these two budget lines into the same figure.

Consumer Maximization

A consumption-leisure bundle is:

- ▶ Feasible: if it lies on or within the budget set.
- ▶ Optimal: if it is affordable and is on the highest indifference curve.

Consumer Maximization

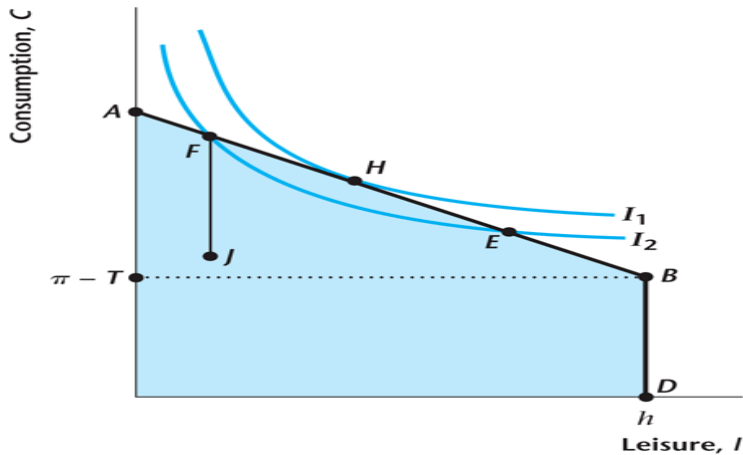
- ▶ Rational behavior: can make informed

$$\max_{c, \ell} U(c, \ell)$$

$$s.t. \quad c + w\ell = wh + \pi - T$$

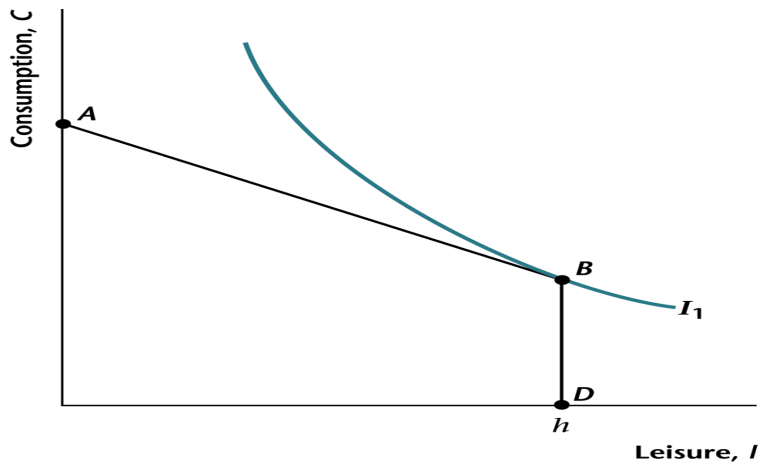
- ▶ Optimization condition: $MRS_{\ell, c} = \text{Relative Price } (w)$.
- ▶ A graphical representation

Consumer Maximization



Only H is optimal.

Choosing Unemployment



Only B is optimal.

Consumer Maximization

$$\begin{aligned} \max_{c, \ell} U(c, l) \\ \text{s.t. } c + w\ell = wh + \pi - T \end{aligned}$$

Rewrite:

$$\max_{\ell} U(c(\ell), \ell)$$

FOC:

$$\frac{\partial U(c, \ell)}{\partial c} \times \frac{dc(\ell)}{d\ell} + \frac{\partial U(c, \ell)}{\partial \ell} = 0$$

$$\frac{\partial U(c, \ell)}{\partial c} \times (-w) + \frac{\partial U(c, \ell)}{\partial \ell} = 0$$

$$\frac{\frac{\partial U(c, \ell)}{\partial \ell}}{\frac{\partial U(c, \ell)}{\partial c}} = MRS_{\ell, c} = w$$

Income and Substitution Effects

Effect of changing parameters of the model (comparative static) can be decomposed into:

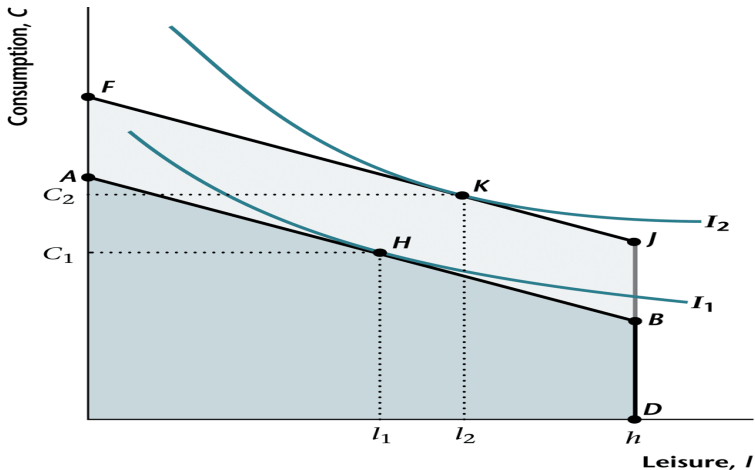
- ▶ **Income effect:** the effect on quantities as a result of having different income **holding prices constant**.
- ▶ **Substitution effect:** the effect on quantities given a price change **holding utility constant**.

Income and Substitution Effects: Example

- ▶ Pure income effect: more income $\rightarrow$ more leisure.
- ▶ Pure substitution effect: higher wage for a day $\rightarrow$ less leisure.
- ▶ Income + substitution effect: permanent high wage $\rightarrow$ leisure?

Comparative Static: Changing $\pi - T$

- Suppose π increases or T decreases. Since c and ℓ are normal goods

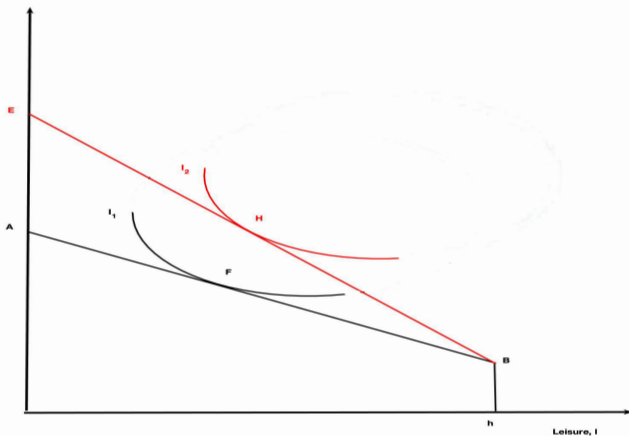


Comparative Static: Increase in the Real Wage w

- ▶ This has income and substitution effects.
- ▶ Substitution effect: the price of leisure rises, so the consumer substitutes leisure for consumption.
- ▶ Income effect: the consumer is effectively more wealthy and, since both goods are normal, consumption increases, and leisure increases.
- ▶ Conclusion: consumption must rise, but leisure may rise or fall.

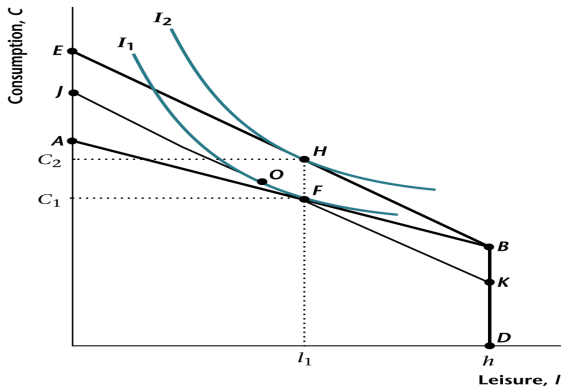
Comparative Static: Increase in the Real Wage w

- Suppose your wage goes up. Graph the change of optimal allocation.



Comparative Static: Increase in the Real Wage w

- Draw line JK (tangent to indifference curve I_1 at point O) to separate income effect and substitution effect.



- Remember: leisure is more expensive.
- In general, this case features both income and substitution effects $\rightarrow$ leisure changes are undetermined.

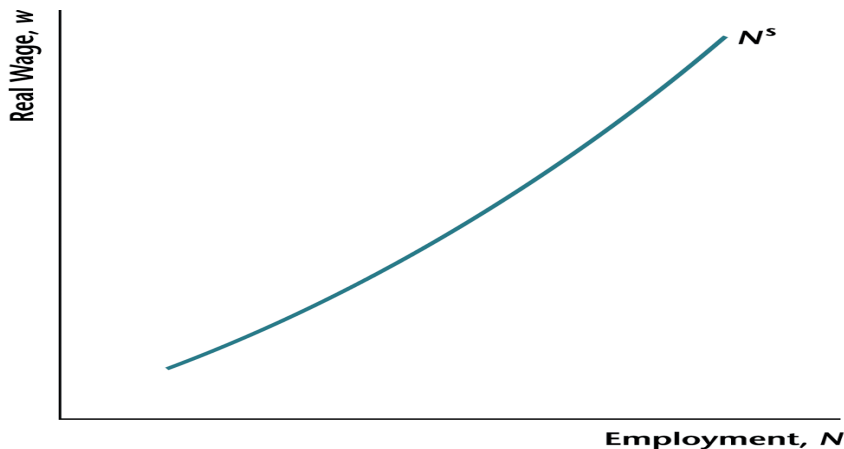
Comparative Static: Increase in the Real Wage w

Comparative Static		
	c	ℓ
Income effect $O \rightarrow H$	$\uparrow$	$\uparrow$
Substitution effect $F \rightarrow O$	$\uparrow$	$\downarrow$
Net effects $F \rightarrow H$	$\uparrow$	$\uparrow\downarrow$

Labor Supply

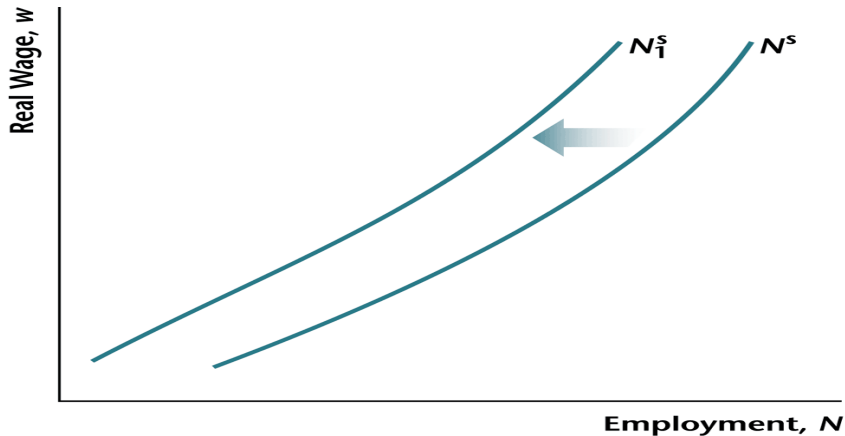
- ▶ Labor supply curve (function): $N^s(w) = h - \ell(w)$. It represents the household's optimal labor supply decision given wage.
 - ▶ Substitution effect $>$ income effect the upward-sloping labor supply curve
 - ▶ Substitution effect $<$ income effect downward-sloping labor supply

Labor Supply Curve



- ▶ Assuming the substitution effect dominates the income effect.
- ▶ Is it true in the data? Read Pg 140-141.

Effect of an Increase in Dividend or a Decrease in Taxes on Labor Supply



Example 1

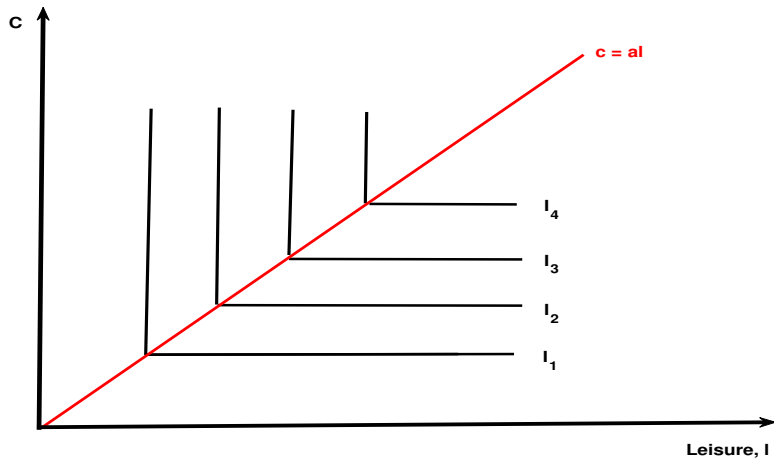
Leontief preference: consumption and leisure are perfect complements. *i.e.*, there is no substitution effect.

$$\max \{ \min(c, a\ell) \}$$

$$s.t. \quad c + w\ell = wh + \pi - T$$

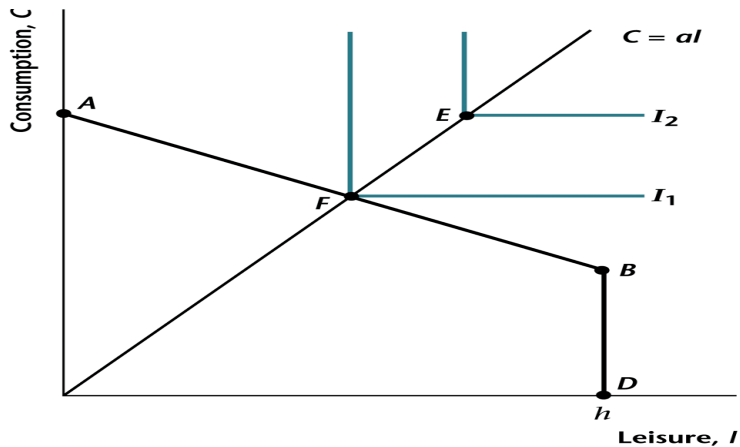
where $a > 0$.

Example 1: Indifference Curves



Example 1: Optimal Allocation

Assume $\pi > T$, graph the optimal allocation:



Aggregation: From One to Many Housholds

- ▶ So far, we have studied the problem of a single household.
- ▶ We need to compare aggregate quantities ($GDP, C, \dots$) and household decision (called c_i and ℓ_i).
- ▶ In Macroeconomics, the problem of going from one to many households is referred to as an aggregation problem.

The issue:

- ▶ In the data, we have households with disparate income levels.
- ▶ In the data, we have households with different tastes.

The Solution: Assumptions

- ▶ Homogeneous function of degree 1: if for all $n \in \mathbb{R}$:

$$n \cdot f(x) = f(n \cdot x)$$

- ▶ Key assumptions:
 - ▶ Utility function is homogeneous of degree one. *i.e.*,

$$N \times U(c, \ell) = U(N \times c, N \times \ell).$$

- ▶ Households have similar preferences.

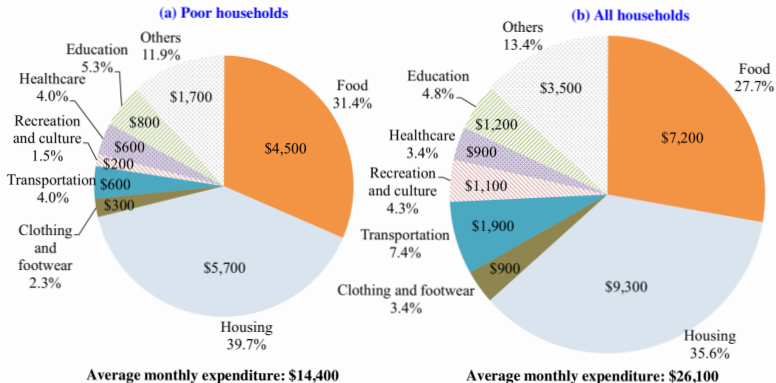
Aggregation

- ▶ We have:

$$\sum_{i=1}^N \max_{c_i, \ell_i} U(c_i, \ell_i) = N \max_{c, l} U(c, l) = \max_{c, \ell} U(N \cdot c, N \cdot \ell)$$

- ▶ $C = N \cdot c$: aggregate consumption;
- ▶ $L = N \cdot \ell$: aggregate leisure.

Test Similar Preference Assumption: How good is the assumption that c_i is the same for everybody?



Source: 2014/15 Household Expenditure Survey, Census and Statistics Department.