

# EE3123 Tutorial 5 (Solution)

## Power Flow Analysis

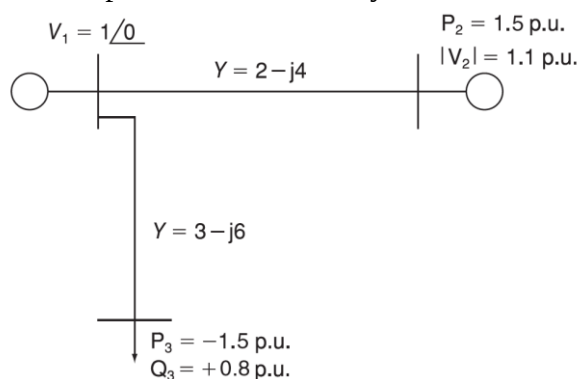
Name:

Student No.:

### Q1

Consider the simplified electric power system shown below for which the power flow solution can be obtained without resorting to iterative techniques.

- (a) Compute the elements of the bus admittance matrix  $Y_{bus}$ .
- (b) Calculate the phase angle  $\delta_2$  by using the real power equation at bus 2 (voltage-controlled bus).
- (c) Determine  $|V_3|$  and  $\delta_3$  by using both the real and reactive power equations at bus 3 (load bus).
- (d) Find the real power generated at bus 1 (swing bus).
- (e) Evaluate the total real power losses in the system.



### Solution

(a)  $Y_{11} = 2 - j4 + 3 - j6 = 5 - j10 = 11.18 \angle -63.43^\circ$

$Y_{22} = 2 - j4 = 4.47 \angle -63.43^\circ$

$Y_{33} = 3 - j6 = 6.71 \angle -63.43^\circ$

$Y_{12} = Y_{21} = -2 + j4 = 4.47 \angle 116.57^\circ$

$Y_{13} = Y_{31} = -3 + j6 = 6.71 \angle 116.57^\circ$

$Y_{23} = Y_{32} = 0$

(b)  $P_k = V_k \sum_{n=1}^N Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn})$

At bus 2,

$$P_2 = V_2 \left[ Y_{21} V_1 \cos(\delta_2 - \delta_1 + \theta_{21}) + Y_{22} V_2 \cos(\delta_2 - \delta_2 - \theta_{22}) \right. \\ \left. + Y_{23} V_3 \cos(\delta_2 - \delta_3 - \theta_{23}) \right]$$

$$1.5 = 1.1 [4.47 \cos(\delta_2 - 116.57^\circ) + 4.47 \times 1.1 \cos(63.43^\circ)]$$

Solving for  $\delta_2$

$$\delta_2 = 15.795^\circ$$

(c) At bus 3

$$P_3 = V_3 \left[ Y_{31} V_1 \cos(\delta_3 - \delta_1 - \theta_{31}) + Y_{33} V_3 \cos(-\theta_{33}) \right]$$

$$-1.5 = V_3 [6.71 \cos(\delta_3 - 116.57^\circ) + 6.71 V_3 \cos(63.43^\circ)] \quad \text{Eq. 1}$$

Also

$$Q_3 = V_3 \left[ Y_{31} V_1 \sin(\delta_3 - \delta_1 - \theta_{31}) + Y_{33} V_3 \sin(-\theta_{33}) \right]$$

$$0.8 = V_3 [6.71 \sin(\delta_3 - 116.57^\circ) + 6.71 V_3 \sin(63.43^\circ)] \quad \text{Eq. 2}$$

Solving for  $V_3$  and  $\delta_3$  using Eq. 1 and Eq. 2:

First, rearrange Eq. 1 and Eq. 2 to separate  $V_3$  and  $\delta_3$

$$-\frac{1.5}{6.71 V_3} - V_3 \cos(63.43^\circ) = \cos(\delta_3 - 116.57^\circ)$$

$$\frac{0.8}{6.71 V_3} - V_3 \sin(63.43^\circ) = \sin(\delta_3 - 116.57^\circ)$$

Next, square both sides of each equation, then add the two equations. The trigonometric identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  can be used to simplify the right hand side.

$$\left[ \frac{1.5}{6.71 V_3} - V_3 \cos(63.43^\circ) \right]^2 + \left[ \frac{0.8}{6.71 V_3} - V_3 \sin(63.43^\circ) \right]^2 = 1$$

Solving, we get  $V_3 = 0.9723$  p.u.

Substitute  $V_3$  back into Eq. 1 or Eq. 2 to get  $\delta_3 = -15.10^\circ$ .

(d) At bus 1

$$P_1 = V_1 \left[ Y_{11} V_1 \cos(-\theta_{11}) + Y_{12} V_2 \cos(-\delta_2 - \theta_{12}) + Y_{13} V_3 \cos(-\delta_3 - \theta_{13}) \right]$$

$$= 11.18 \cos(63.43^\circ) + 4.47 \times 1.1 \cos(-15.795^\circ - 116.57^\circ) \\ + 6.71 \times 0.9723 \cos(15.10^\circ - 116.57^\circ)$$

$$= 0.39 \text{ p.u.}$$

(e)

$$P_{\text{loss}} = P_{\text{gen}} - P_{\text{load}} \\ = P_1 + P_2 + P_3 \\ = 0.39 + 1.5 - 1.5 \\ = 0.39 \text{ p.u.}$$

## Q2

Determine the bus admittance matrix ( $Y_{bus}$ ) for the three-phase power system shown below with input data given in below Table 1 and partial results in below Table 2. Assume a three-phase 100 MVA per unit base.

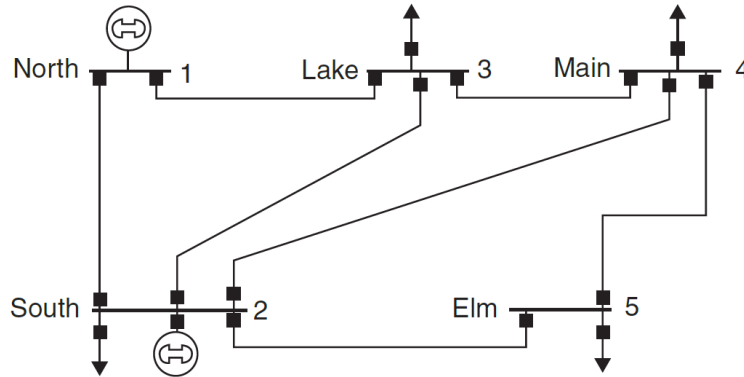


Table 1 Bus input data

| Bus-to-Bus | R per unit | X per unit | B per unit |
|------------|------------|------------|------------|
| 1-2        | 0.02       | 0.06       | 0.06       |
| 1-3        | 0.08       | 0.24       | 0.05       |
| 2-3        | 0.06       | 0.18       | 0.04       |
| 2-4        | 0.08       | 0.24       | 0.05       |
| 2-5        | 0.02       | 0.06       | 0.02       |
| 3-4        | 0.01       | 0.04       | 0.01       |
| 4-5        | 0.03       | 0.10       | 0.04       |

Table 2 Partially Completed Bus Admittance Matrix

|                  |                  |                 |   |   |
|------------------|------------------|-----------------|---|---|
| $6.25 - j18.695$ | $-5.00 + j15.00$ | $-1.25 + j3.75$ | 0 | 0 |
| $-5.00 + j15.00$ |                  |                 |   |   |

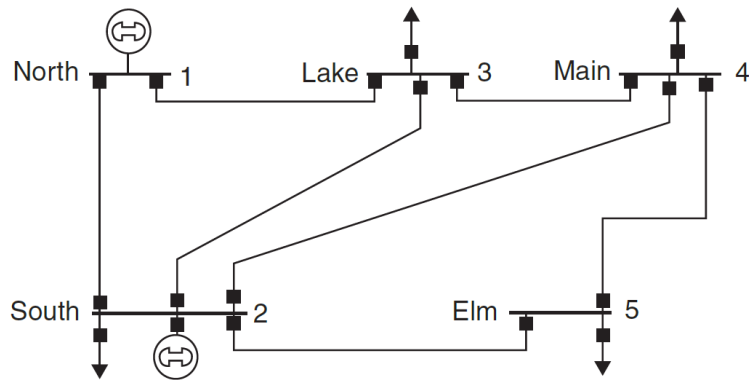
## Solution

$$\begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & -Y_{1n} \\ \vdots & \ddots & \vdots \\ -Y_{n1} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_n \end{bmatrix}$$

|                  |                     |                      |                      |                     |
|------------------|---------------------|----------------------|----------------------|---------------------|
| $6.25 - j18.695$ | $-5.00 + j15.00$    | $-1.25 + j3.75$      | 0                    | 0                   |
| $-5.00 + j15.00$ | $12.9167 - j38.665$ | $-1.6667 + j5.00$    | $-1.25 + j3.75$      | $-5.00 + j15.00$    |
| $-1.25 + j3.75$  | $-1.6667 + j5.00$   | $8.7990 - j32.2294$  | $-5.8824 + j23.5294$ | 0                   |
| 0                | $-1.25 + j3.75$     | $-5.8824 + j23.5294$ | $9.8846 - j36.4037$  | $-2.7523 + j9.1743$ |
| 0                | $-5.00 + j15.00$    | 0                    | $-2.7523 + j9.1743$  | $7.7523 - j24.1443$ |

### Q3

For the system from Q2, assume that a 75-Mvar shunt capacitance (three phase assuming one per unit bus voltage) is added at bus 4. Calculate the new value of  $Y_{44}$ .



### Solution

First, we need to find the per-unit shunt admittance of the added capacitor.

$$75 \text{ Mvar} = 0.75 \text{ p.u.}$$

To find  $Y$ , we use the relation

$$S = V^2 Y$$

$$Y = \frac{S}{V^2} = \frac{0.75}{1^2} = 0.75 \text{ p.u.}$$

Now, we can find  $Y_{44}$

$$\begin{aligned} Y_{44} &= \frac{1}{0.08 + j0.24} + \frac{1}{0.01 + j0.04} + \frac{1}{0.03 + j0.10} \\ &\quad + j \frac{0.05 + 0.01 + 0.04}{2} + j0.75 \\ &= 9.8846 - j35.6537 \end{aligned}$$