

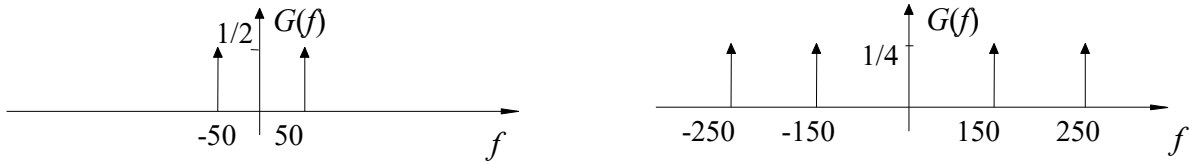
EE 3008 Chapter 7 Tutorial Solutions

AM Solutions

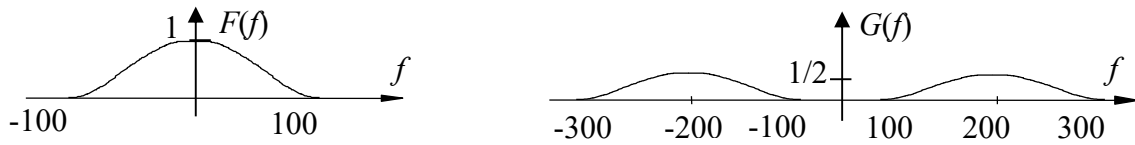
Q1) (i) Now $f_c = 200$ Hz. The modulated signal is given by

$$g(t) = f(t)\cos(2\pi f_c t) = \cos(100\pi t)\cos(400\pi t) = 0.5\cos(300\pi t) + 0.5\cos(500\pi t)$$

$$G(f) = (1/4)[\delta(f-150) + \delta(f+150) + \delta(f-250) + \delta(f+250)]$$



(ii) $g(t) = f(t)\cos(2\pi f_c t)$,
 $G(f) = (1/2)F(f+f_c) + (1/2)F(f-f_c)$, $f_c = 200$



Q2) (i) $m = (125-25)/(125+25) = 2/3 = 66.7\%$.

(ii) Consider an AM-DSB-C signal.

$$\phi_{AM}(t) = A(1 + \beta s(t))\cos(2\pi f_c t) \quad (1)$$

Note that for this question $\max|s(t)| = 1$. Therefore $m = \beta$ and so

$$\phi_{AM}(t) = A(1 + m\cos(2\pi f_m t))\cos(2\pi f_c t).$$

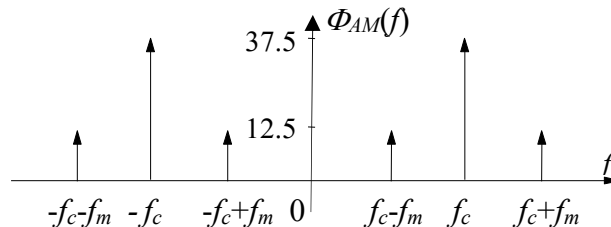
Note that at $t=0$, we can write

$$\phi_{AM}(0) = A(1+m) = 125.$$

As $m=2/3$, we have $A=75$, and so

$$\phi_{AM}(t) = 75(1 + 0.667\cos 2\pi f_m t)\cos(2\pi f_c t)$$

(iii)
$$\begin{aligned} \phi_{AM}(t) &= 75(1 + 0.667\cos 2\pi f_m t)\cos(2\pi f_c t) \\ &= 75\cos(2\pi f_c t) + 50\cos 2\pi f_m t \cos(2\pi f_c t) \\ &= 75\cos(2\pi f_c t) + 25(\cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t)) \\ \Phi_{AM}(f) &= 37.5[\delta(f-f_c) + \delta(f+f_c)] + 12.5[\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m)) + \delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m))] \end{aligned}$$



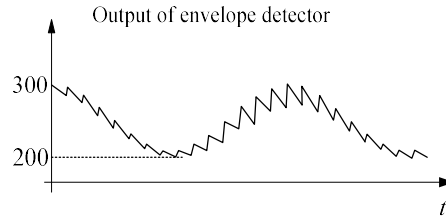
(iv) $(12.5+12.5)/37.5 = 25/37.5 = 0.667 \Rightarrow m$

(v) Let B be the amplitude of the extra carrier component. Adding extra carrier to $\phi_{AM}(t)$, we have

$$\begin{aligned}\phi(t) &= \phi_{AM}(t) + B\cos(2\pi f_c t) \\ &= 75(1+0.667\cos(2\pi f_m t))\cos(2\pi f_c t) + B\cos(2\pi f_c t) \\ &= (75+B)[1+(50/(75+B))\cos(2\pi f_m t)] \cos(2\pi f_c t)\end{aligned}$$

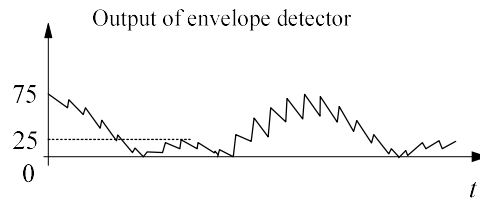
The new modulation index is $50/(75+B)=0.2$. Therefore $B=175$.

(vi)



(vii) $50/(75+B) = 2$. Therefore $B = -50$.

(viii)



Q3 (a) The spectrum of $s_{AM}(t)$ is

$$S_{AM}(f) = A_0/2 \times [\delta(f-f_c) + \delta(f+f_c)] + A_m/4 \times [\delta(f-f_m-f_c) + \delta(f-f_m+f_c) + \delta(f+f_m-f_c) + \delta(f+f_m+f_c)].$$

(b) We can write $s_{AM}(t)$ as

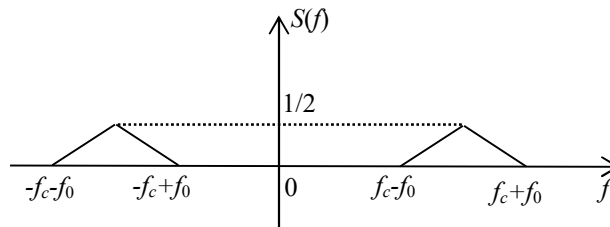
$$s_{AM}(t) = A_0 (1 + (A_m/A_0)\cos(2\pi f_m t))\cos(2\pi f_c t),$$

Therefore modulation index $m = A_m/A_0$.

The power efficiency is $\eta = \frac{m^2}{2+m^2} = \frac{A_m^2}{2A_0^2 + A_m^2}.$

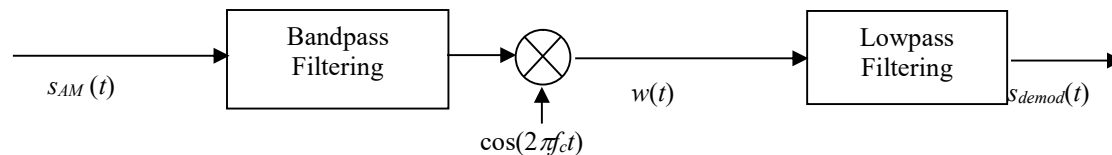
(c) Now $m = A_m/A_0 \leq 1$, therefore an envelope detector can be used.

Q4 (a)



(b) $2f_0$.

(c)



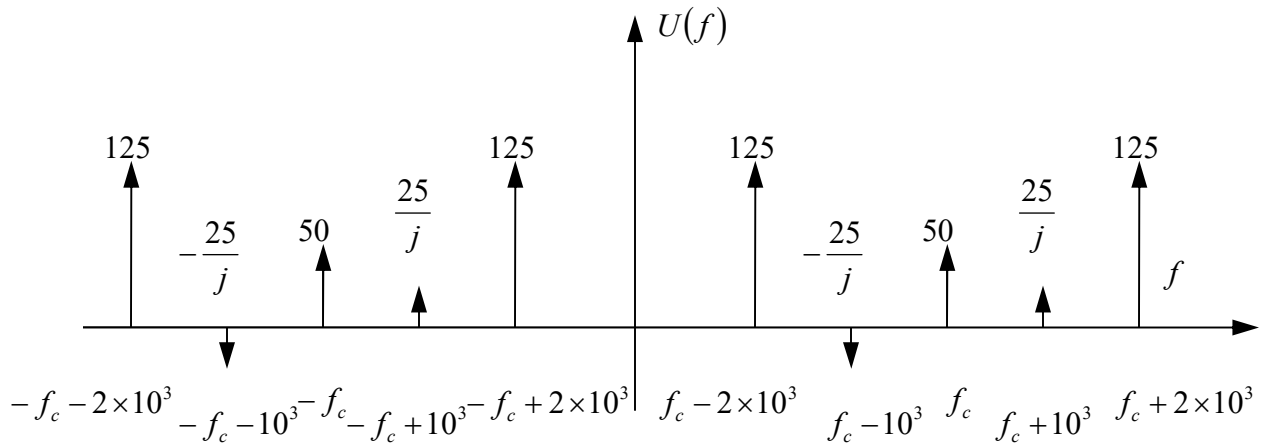
Q5) 1) The modulated signal is

$$\begin{aligned}
 u(t) &= 100(1+s(t))\cos(2\pi 8 \times 10^5 t) \\
 &= 100\cos(2\pi 8 \times 10^5 t) + 100\sin(2\pi 10^3 t)\cos(2\pi 8 \times 10^5 t) \\
 &\quad + 500\cos(2\pi 2 \times 10^3 t)\cos(2\pi 8 \times 10^5 t) \\
 &= 100\cos(2\pi 8 \times 10^5 t) + 50[\sin(2\pi(10^3 + 8 \times 10^5)t) - \sin(2\pi(8 \times 10^5 - 10^3)t)] \\
 &\quad + 250[\cos(2\pi(2 \times 10^3 + 8 \times 10^5)t) + \cos(2\pi(8 \times 10^5 - 2 \times 10^3)t)] \quad \text{Volt}
 \end{aligned}$$

Taking the Fourier transform of the previous expression, we obtain

$$\begin{aligned}
 U(f) &= 50[\delta(f - 8 \times 10^5) + \delta(f + 8 \times 10^5)] \\
 &\quad + 25\left[\frac{1}{j}\delta(f - 8 \times 10^5 - 10^3) - \frac{1}{j}\delta(f + 8 \times 10^5 + 10^3)\right] \\
 &\quad - 25\left[\frac{1}{j}\delta(f - 8 \times 10^5 + 10^3) - \frac{1}{j}\delta(f + 8 \times 10^5 - 10^3)\right] \\
 &\quad + 125[\delta(f - 8 \times 10^5 - 2 \times 10^3) + \delta(f + 8 \times 10^5 + 2 \times 10^3)] \\
 &\quad + 125[\delta(f - 8 \times 10^5 + 2 \times 10^3) + \delta(f + 8 \times 10^5 - 2 \times 10^3)]
 \end{aligned}$$

Hence, the spectrum is shown in the figure below.



2) The extra carrier power is $P_{\text{carrier}} = \frac{100^2}{2} = 5000 \quad \text{W}$

The information signal power is $P_{\text{information signal}} = \frac{50^2}{2} + \frac{50^2}{2} + \frac{250^2}{2} + \frac{250^2}{2} = 65000 \quad \text{W}$

3) The power efficiency is $\eta = \frac{P_{\text{information signal}}}{P_{\text{carrier}} + P_{\text{information signal}}} \times 100\% \approx 92.857\%$

4) Note that when $t = 1/4000$, the information signal

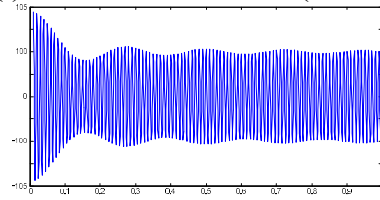
$$s(t) = s(1/4000) = \sin\left(\frac{2000\pi}{4000}\right) + 5\cos\left(\frac{4000\pi}{4000}\right) = 1 - 5 = -4$$

Hence

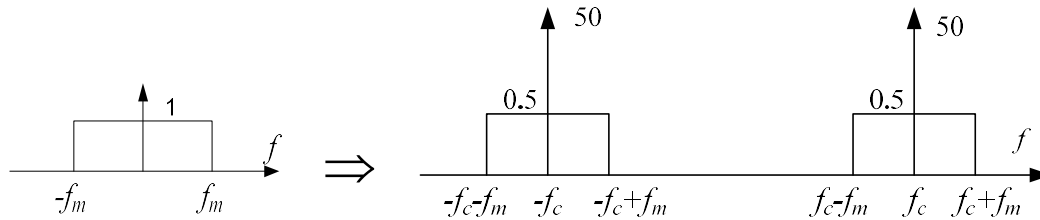
$$1 + s(1/4000) = -3 < 0$$

Therefore, the AM-DSB-C signal is an over-modulated signal, which can not be demodulated correctly with an envelop detector.

Q6) a) The plot of $s_{AM}(t)$ is shown as follows. (Illustration only. Scale may be wrong.)



Below are the amplitude spectrums of $s(t) = \frac{\sin 5\pi t}{\pi t}$ before (left) and after (right) modulation ($f_m=2.5\text{Hz}$). The adding of the carrier results in impulse functions at the carrier frequency.

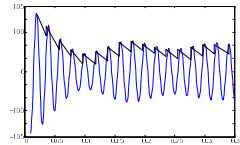


b) For the modulating signal $s(t) = \frac{\sin(5\pi t)}{\pi t} = 5 \frac{\sin(5\pi t)}{5\pi t}$, with maximum magnitude value = 5.

The AM signal can be rewritten as

$$s_{AM}(t) = 100 (1 + s(t)/100) \cos 2\pi f_c t,$$

So the modulation index $m = \frac{\max \left| \frac{\sin 5\pi t}{\pi t} \right|}{100} < 1$. It is not over modulation and can be demodulated using envelope detection. The output is as follows. (Illustration only.)



FM Solutions

Q7) (i) $\theta = 2\pi f_c t + \sin(2\pi f_m t) + 2\sin(4\pi f_m t)$. The instantaneous frequency is

$$f(t) = (1/2\pi)(d\theta/dt) = f_c + f_m \cos(2\pi f_m t) + 4f_m \cos(4\pi f_m t)$$

Therefore $f(t) - f_c = f_m \cos(2\pi f_m t) + 4f_m \cos(4\pi f_m t)$

The maximum of $f(t) - f_c$ takes place at $t=0$ as

$$\Delta f = \max(f(t) - f_c) = f_m + 4f_m = 5\text{kHz}$$

Q8) (i) $\beta = \Delta\omega/\omega_m = 100/(10 \times 10^3) = 0.01$. Since β is very small, this is a narrow band FM (NBFM) system. For NBFM, $B \approx 2(1+\beta)f_m \approx 2f_m$. Note that β is a function of signal amplitude. The bandwidth of NBFM is (approximately) independent of β and it is independent of signal amplitude as well. For this question: $B \approx 2f_m = 20\text{kHz}$.

(ii) $B \approx 2f_m = 20\text{kHz}$. (iii) $B \approx 2f_m = 40\text{kHz}$. (iv) $B \approx 2f_m = 40\text{kHz}$.

Q9) (i) The instantaneous frequency is:

$$(2\pi)^{-1} d\theta / dt = 10^7 + 4000 \cdot \cos(2000\pi t)$$

The maximum frequency deviation $\Delta f = 4 \times 1000 = 4\text{kHz}$

(ii)
$$P_t = \frac{\overline{\Phi}^2(t)}{R} = \frac{100^2}{2 \times 50} = 100\text{W}$$

(iii) $f_m = 1\text{kHz}$, $\beta = \Delta f / f_m = 4$, $f_c = 10^7 \text{Hz} = 10\text{MHz}$.

Therefore 10 MHz is the carrier. The average power is obtained by setting $n=0$:

$$P_0 = P_t \times J_0^2(4) = 100(-0.4)^2 = 16\text{W} \Rightarrow 16/100 = 16\%.$$

(iv) Using Carson's rule: $B \approx 2(\Delta f + f_m) = 2(\beta + 1)f_m = 10\text{kHz}$

Q10) a) $s(t) = 5\text{sinc}(5\pi t)$. The maximum frequency deviation $= k \times \max(s(t)) = 0.01 \times 5 = 0.05\text{Hz}$.

b) Let $\theta(t) = 2\pi k \int_{-\infty}^t s(t) dt$.

So $s_{FM}(t) = A \cos(2\pi f_c t + \theta(t)) = A \cos(2\pi f_c t) \times \cos(\theta(t)) - A \sin(2\pi f_c t) \times \sin(\theta(t))$.

Applying $\cos(x) \approx 1$ and $\sin(x) \approx x$, for $x \ll 1$, we have

$$s_{FM}(t) \approx A \cos(2\pi f_c t) - A \sin(2\pi f_c t) \times \theta(t) = A \cos(2\pi f_c t) - A \sin(2\pi f_c t) \times 2\pi k \int_{-\infty}^t s(t) dt.$$

This is the form of Armstrong FM modulation.

Q11) a) Since the frequency of the wide band FM signal is 16 times that of the NBFM signal, 4 frequency doublers would be required.

b) $\beta = 16\beta' = 1.6$.

c) The maximum frequency deviation of the wide band FM signal is

$$\Delta f = \beta \times f_m = 1.6 \times 4\text{kHz} = 6.4\text{kHz}$$

Hence, the frequency sensitivity is $k = \Delta f / A_m = 64 \text{Hz/V}$.

Q12) (a) The instantaneous frequency $f(t)$ is given by

$$f(t) = (d/dt) (2\pi f_c t + 6\sin(2\pi f_m t)) / 2\pi = f_c + 6f_m \cos(2\pi f_m t)$$

(b) The modulation index is $\beta = 6$. Based on Carson's Rule, we have $B = 2(1 + \beta)f_m \leq 1.4\text{MHz}$.

Therefore, the maximum value for f_m is 100kHz.

(c) The total power is $P = 0.5 \times (30)^2 = 450$.

Therefore, the power at the frequency $\pm f_c \pm f_m$ is $P_1 = 2P \times J_1^2(6) = 68.906601$.

Q13) (a) $B = 2(\beta + 1)f_m = 240\text{kHz}$.

(b) $\beta' = \beta/8 = 0.625$
 $f_c' = f_c/8 = 10\text{MHz}$.

Q14) a) BPSK

b) (binary) FSK