

L04: Polarization of Plane Waves

EE3109 Applied Electromagnetics

Lecture Notes

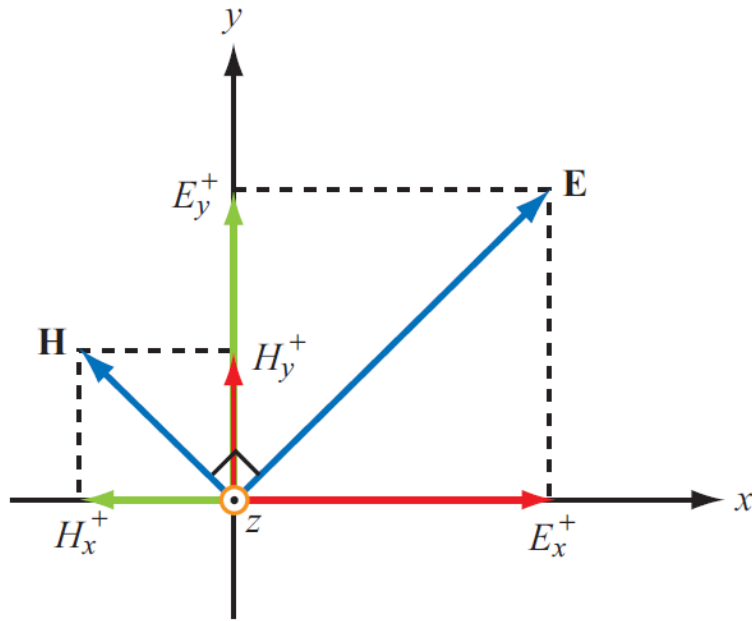
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Overview

- In this class we will introduce the concept of a **polarization** of a plane wave
- We will
 - **Define** what the “polarization” is
 - Introduce the **types** of polarization
 - **Analyze** the polarization of a plane wave
 - Introduce **applications** for polarization in modern science and technology

Wave decomposition



In general, a uniform plane wave traveling in the +z-direction may have both x- and y-components, in which case $\tilde{\mathbf{E}}$ is given by

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}} \tilde{E}_x^+(z) + \hat{\mathbf{y}} \tilde{E}_y^+(z), \quad (7.43a)$$

and the associated magnetic field is

$$\tilde{\mathbf{H}} = \hat{\mathbf{x}} \tilde{H}_x^+(z) + \hat{\mathbf{y}} \tilde{H}_y^+(z). \quad (7.43b)$$

Application of Eq. (7.39a) gives

$$\tilde{\mathbf{H}} = \frac{1}{\eta} \hat{\mathbf{z}} \times \tilde{\mathbf{E}} = -\hat{\mathbf{x}} \frac{\tilde{E}_y^+(z)}{\eta} + \hat{\mathbf{y}} \frac{\tilde{E}_x^+(z)}{\eta}. \quad (7.44)$$

By equating Eq. (7.43b) to Eq. (7.44), we have

$$\tilde{H}_x^+(z) = -\frac{\tilde{E}_y^+(z)}{\eta}, \quad \tilde{H}_y^+(z) = \frac{\tilde{E}_x^+(z)}{\eta}. \quad (7.45)$$

Wave Polarization

*The **polarization** of a uniform plane wave describes the locus traced by the tip of the **E** vector (in the plane orthogonal to the direction of propagation) at a given point in space as a function of time.*

Plane wave propagating along +z :

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\tilde{E}_x(z) + \hat{\mathbf{y}}\tilde{E}_y(z), \text{ with}$$

$$\text{If: } E_{x0} = a_x,$$

$$E_{y0} = a_y e^{j\delta},$$

$$\text{then } \tilde{\mathbf{E}}(z) = (\hat{\mathbf{x}}a_x + \hat{\mathbf{y}}a_y e^{j\delta})e^{-jkz},$$

and the corresponding instantaneous field is

$$\tilde{E}_x(z) = E_{x0}e^{-jkz},$$

$$\tilde{E}_y(z) = E_{y0}e^{-jkz},$$

$$\begin{aligned}\mathbf{E}(z, t) &= \Re \left[\tilde{\mathbf{E}}(z) e^{j\omega t} \right] \\ &= \hat{\mathbf{x}}a_x \cos(\omega t - kz) \\ &\quad + \hat{\mathbf{y}}a_y \cos(\omega t - kz + \delta).\end{aligned}$$

Polarization State

Polarization **state** describes the trace of **E** as a function of time at a fixed z .

E-field again:

$$\begin{aligned}\mathbf{E}(z, t) &= \Re \left[\tilde{\mathbf{E}}(z) e^{j\omega t} \right] \\ &= \hat{\mathbf{x}} a_x \cos(\omega t - kz) \\ &\quad + \hat{\mathbf{y}} a_y \cos(\omega t - kz + \delta)\end{aligned}$$

Magnitude of **E**

$$\begin{aligned}|\mathbf{E}(z, t)| &= [E_x^2(z, t) + E_y^2(z, t)]^{1/2} \\ &= [a_x^2 \cos^2(\omega t - kz) \\ &\quad + a_y^2 \cos^2(\omega t - kz + \delta)]^{1/2}\end{aligned}$$

Inclination Angle (w.r.t. x)

$$\psi(z, t) = \tan^{-1} \left(\frac{E_y(z, t)}{E_x(z, t)} \right)$$

(Have to choose the right branch...)

Linear Polarization:

$$\delta = 0 \text{ or } \delta = \pi$$

A wave is said to be linearly polarized if for a fixed z , the tip of $\mathbf{E}(z, t)$ traces a straight line segment as a function of time. This happens when $E_x(z, t)$ and $E_y(z, t)$ are **in-phase** (i.e., $\delta = 0$) or **out-of-phase** ($\delta = \pi$).

Under these conditions Eq. (7.50) simplifies to

$$\mathbf{E}(0, t) = (\hat{\mathbf{x}}a_x + \hat{\mathbf{y}}a_y) \cos(\omega t - kz) \quad (\text{in-phase}), \quad (7.53a)$$

$$\mathbf{E}(0, t) = (\hat{\mathbf{x}}a_x - \hat{\mathbf{y}}a_y) \cos(\omega t - kz) \quad (\text{out-of-phase}). \quad (7.53b)$$

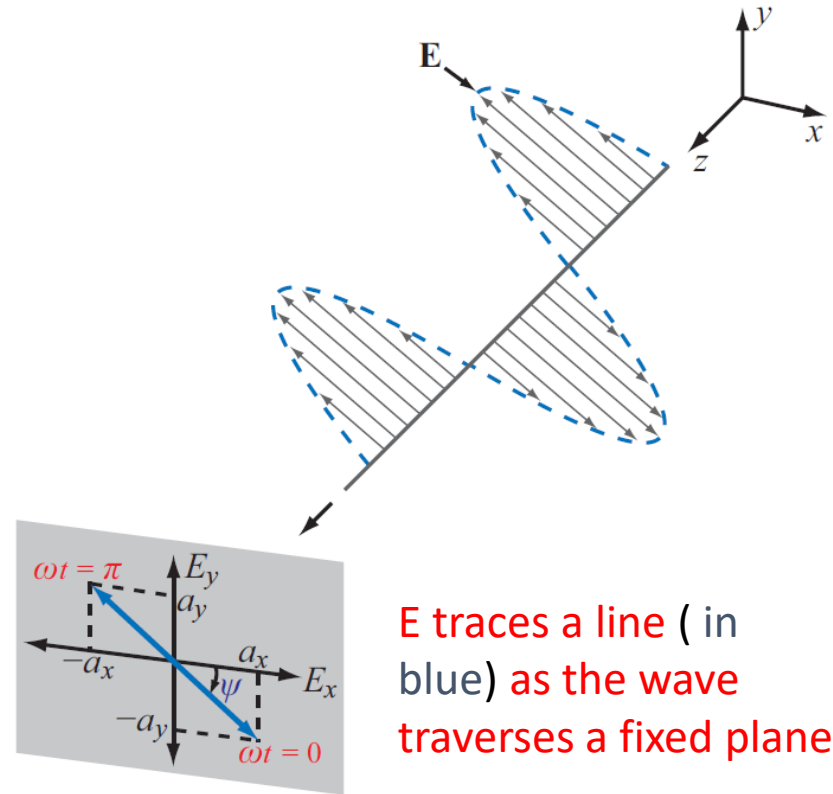
Let us examine the out-of-phase case. The field's magnitude is

$$|\mathbf{E}(z, t)| = [a_x^2 + a_y^2]^{1/2} |\cos(\omega t - kz)|, \quad (7.54a)$$

and the inclination angle is

$$\psi = \tan^{-1} \left(\frac{-a_y}{a_x} \right) \quad (\text{out-of-phase}). \quad (7.54b)$$

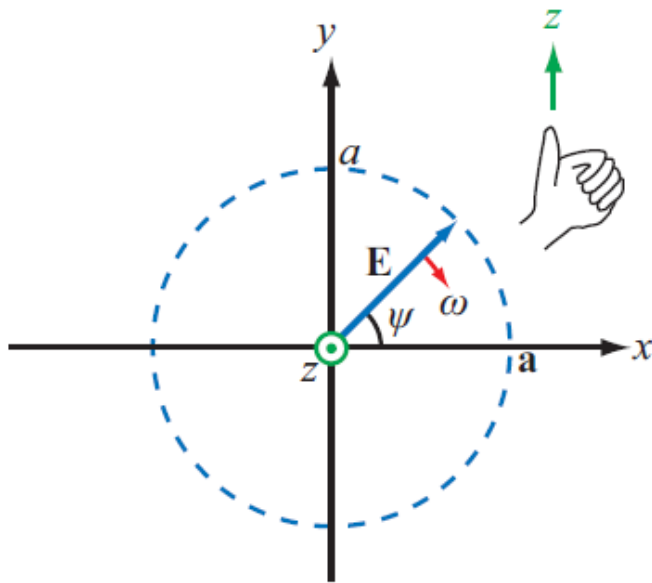
If $a_y = 0$, then $\psi = 0^\circ$ or 180° , and the wave is x -polarized; conversely, if $a_x = 0$, then $\psi = 90^\circ$ or -90° , and the wave is y -polarized.



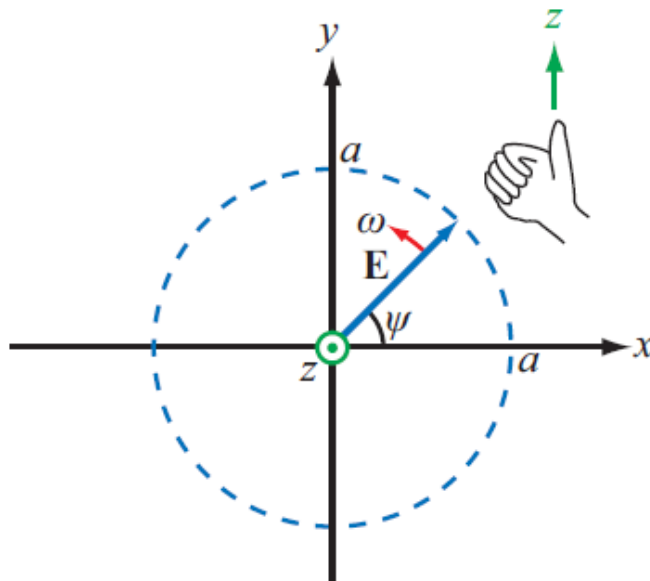
E traces a line (in blue) as the wave traverses a fixed plane

Polarization Handedness

Polarization handedness is defined in terms of the rotation of $\mathbf{E}$ as a function of time in a fixed plane orthogonal to the direction of propagation, which is opposite of the direction of rotation of $\mathbf{E}$ as a function of distance at a fixed point in time.



(a) LHC polarization



(b) RHC polarization

LH Circular Polarization:

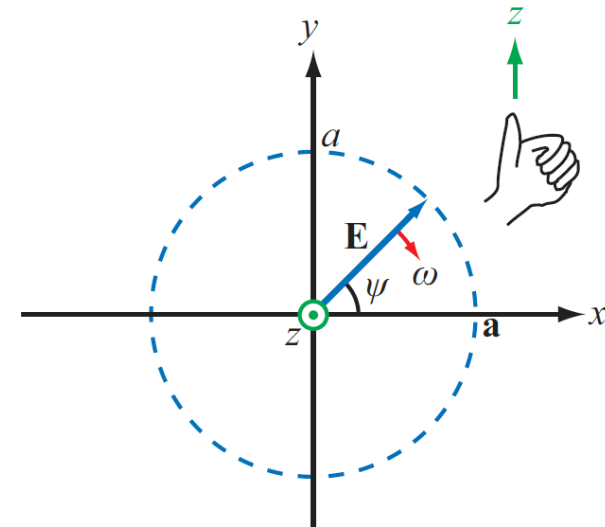
$$a_x = a_y = a \text{ AND } \delta = \pi/2$$

(a) Left-Hand Circular (LHC) Polarization

For $a_x = a_y = a$ and $\delta = \pi/2$, Eqs. (7.49) and (7.50) become

$$\begin{aligned}\tilde{\mathbf{E}}(z) &= (\hat{\mathbf{x}}a + \hat{\mathbf{y}}ae^{j\pi/2})e^{-jkz} \\ &= a(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-jkz},\end{aligned}$$

$$\begin{aligned}\mathbf{E}(z, t) &= \Re \left[\tilde{\mathbf{E}}(z) e^{j\omega t} \right] \\ &= \hat{\mathbf{x}}a \cos(\omega t - kz) + \hat{\mathbf{y}}a \cos(\omega t - kz + \pi/2) \\ &= \hat{\mathbf{x}}a \cos(\omega t - kz) - \hat{\mathbf{y}}a \sin(\omega t - kz).\end{aligned}$$



(a) LHC polarization

The corresponding field magnitude and inclination angle are

$$\begin{aligned}|\mathbf{E}(z, t)| &= \left[E_x^2(z, t) + E_y^2(z, t) \right]^{1/2} \\ &= [a^2 \cos^2(\omega t - kz) + a^2 \sin^2(\omega t - kz)]^{1/2} \\ &= \boxed{a},\end{aligned}\tag{7.56a}$$

$$\begin{aligned}\psi(z, t) &= \tan^{-1} \left[\frac{E_y(z, t)}{E_x(z, t)} \right] \\ &= \tan^{-1} \left[\frac{-a \sin(\omega t - kz)}{a \cos(\omega t - kz)} \right] \\ &= \boxed{-(\omega t - kz)}.\end{aligned}$$

RH Circular Polarization:

$$a_x = a_y = a \text{ AND } \delta = -\pi/2$$

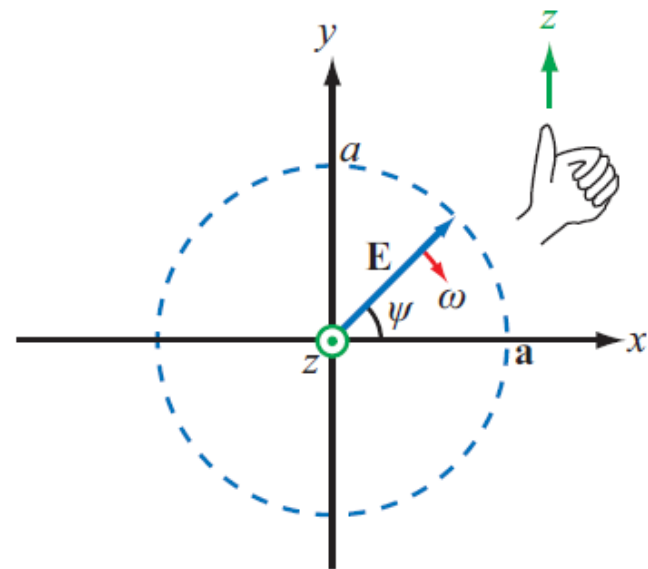
(b) Right-Hand Circular (RHC) Polarization

For $a_x = a_y = a$ and $\delta = -\pi/2$, we have

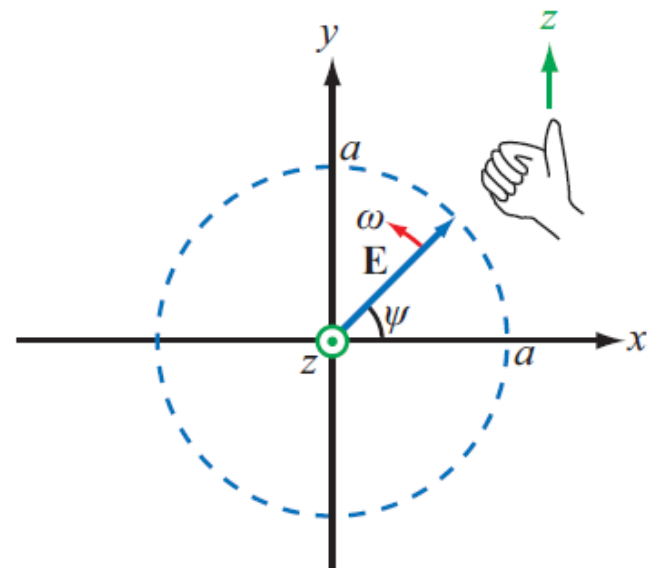
$$|\mathbf{E}(z, t)| = a, \quad \psi = (\omega t - kz).$$

Elliptical Polarization

(all other cases for a_x, a_y and δ)



(a) LHC polarization



(b) RHC polarization

Some examples

$$\begin{aligned}\mathbf{E}(z, t) &= \Re \left[\tilde{\mathbf{E}}(z) e^{j\omega t} \right] \\ &= \hat{\mathbf{x}} a_x \cos(\omega t - kz) \\ &\quad + \hat{\mathbf{y}} a_y \cos(\omega t - kz + \delta)\end{aligned}$$

A plane wave has above E-field distribution. Find its polarization state if:

- (a) $a_x = 3; a_y = 4; \delta = 180^\circ$
- (b) $a_x = 5; a_y = 5; \delta = 90^\circ$
- (c) $a_x = 1; a_y = 2; \delta = 90^\circ$
- (d) $a_x = 2; a_y = 2; \delta = \pi/4$

For each case, trace the time-evolution of the electric field along the plane $z = 0$.

Answers to Previous Slide

(a) Linear; (b) LH Circular; (c) LH Elliptical; (d) LH Elliptical

Example: (d) $a_x = 2; a_y = 2; \delta = \pi/4$

Applying Polarization: Liquid Crystal Displays

*Liquid crystals are neither a pure solid nor a pure liquid, but rather a hybrid of both. One particular variety of interest is the twisted **nematic** liquid crystal whose rod-shaped molecules have a natural tendency to assume a **twisted spiral** structure when the material is sandwiched between finely grooved glass substrates with orthogonal orientations.*

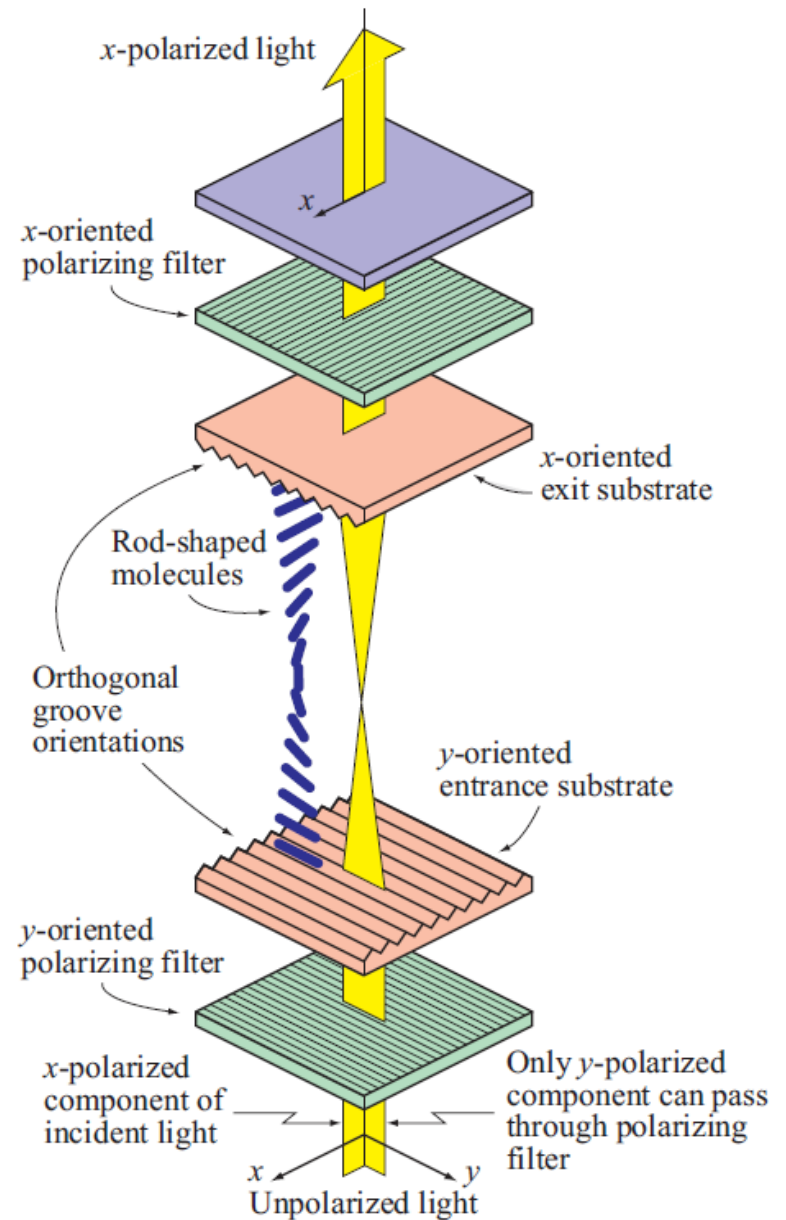


Figure TF14-1: The rod-shaped molecules of a liquid crystal sandwiched between grooved substrates with orthogonal orientations causes the electric field of the light passing through it to rotate by 90° .

Operation of a Single Pixel

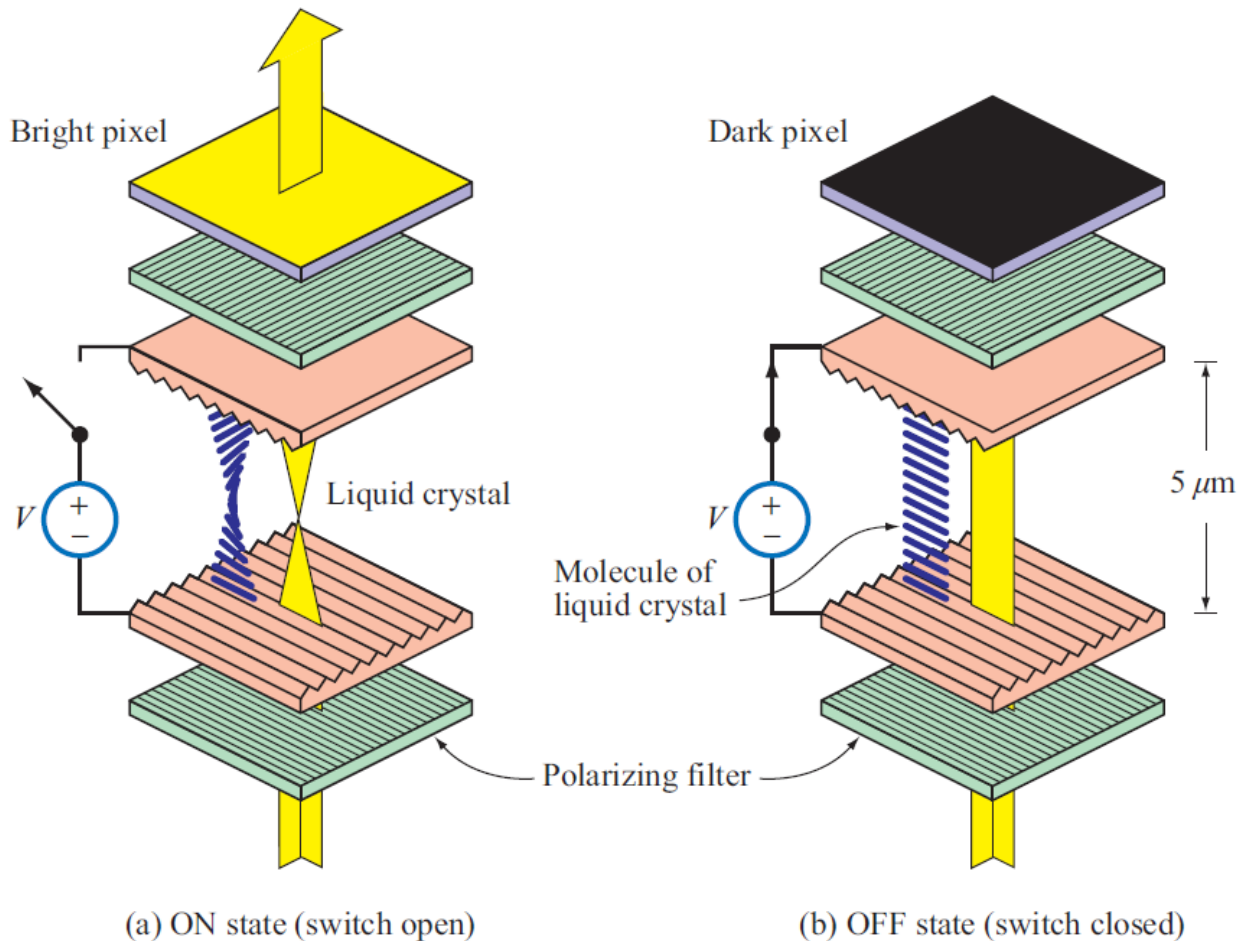


Figure TF14-2: Single-pixel LCD.

LCD 2-D Array

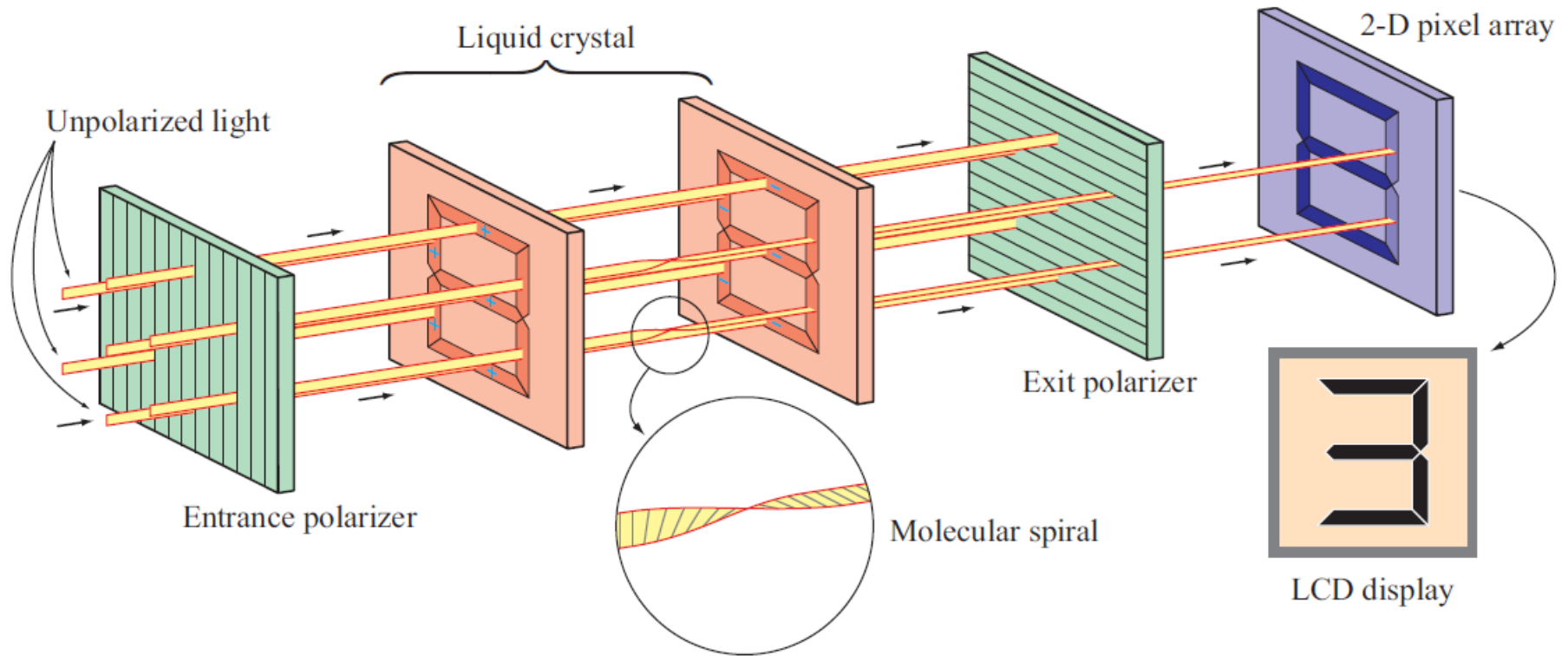


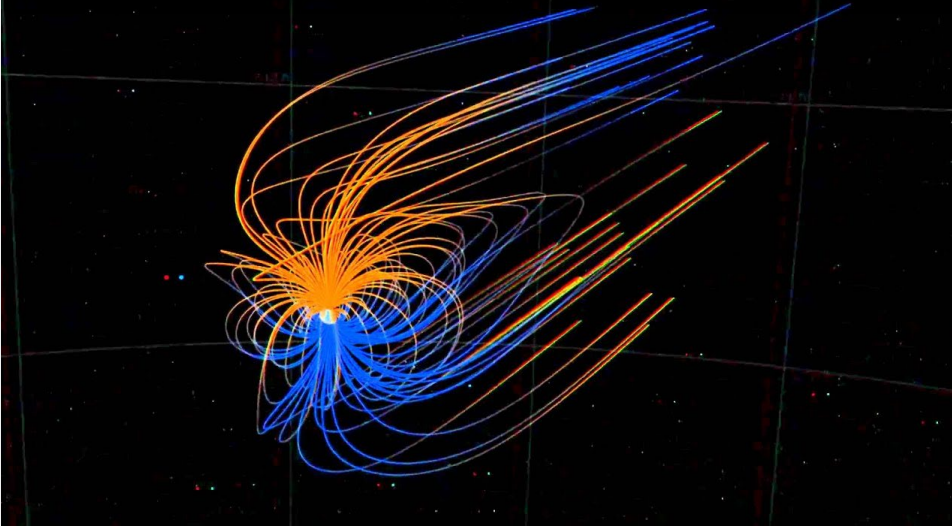
Figure TF14-3: 2-D LCD array.

Visualizing Polarization



<https://www.youtube.com/watch?v=8YkfEft4p-w>

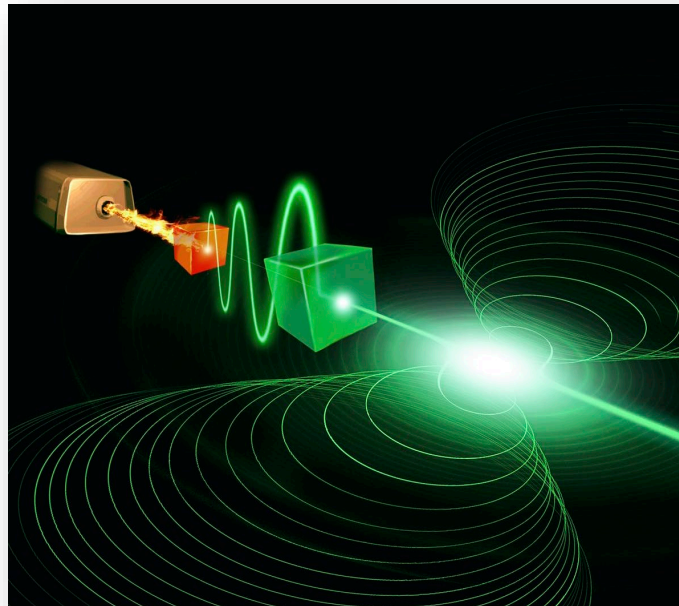
Polarization Technologies



<https://www.youtube.com/watch?v=k67OGI-ur3I>



https://www.esa.int/Applications/Telecommunications_Integrated_Applications/Successful_optical_data_relay_link_between_OIC_ET_S_and_Artemis



<https://theconversation.com/reimagining-the-laser-new-ideas-from-quantum-theory-could-herald-a-revolution-147436>

Polarization Technologies



<https://www.samsung.com/>

http:



<https://electronics.howstuffworks.com/3d-tv.htm>



<https://cumbriansky.files.wordpress.com/2009/03/satellite1.jpg>

Summary

- In this class, we considered the polarization of a plane wave. Particularly we learned
 - What types of polarizations there are
 - How to find the polarization of a plane wave
 - How to trace the time-evolution of electromagnetic field
 - Where polarization is used in modern technology