

EE3109 Applied Electromagnetics
2024/25 Semester B
Week 4 Tutorial Solutions

1. (Ulaby 7.9) An RHC-polarized wave with a modulus of 2 (V/m) is traveling in free space in the negative z-direction. Write the expression for the wave's electric field vector, given that the wavelength is 3 cm.

Solution

First we calculate the values of the relevant wave parameters:

$$\lambda = 3\text{cm} = 0.03\text{m}; \quad f = \frac{c}{\lambda} = \frac{3 \times 10^8}{0.03} = 10^{10} \text{ Hz} = 10 \text{ GHz};$$

$$k = \frac{2\pi}{\lambda} = 209.4 \text{ rad/m} \quad \text{and} \quad \omega = 2\pi f = 2\pi \times 10^{10} \text{ rad/s}.$$

The modulus of a circularly polarized wave travelling in $\pm z$ direction is given as:

$$\mathbf{E}(z, t) = \sqrt{E_x^2(z, t) + E_y^2(z, t)} = \sqrt{a^2 \cos^2(\omega t \mp kz) + a^2 \sin^2(\omega t \mp kz)} = a = 2 \text{ V/m}$$

For an RHC wave travelling in the positive z direction;

$$\mathbf{E}(z, t) = \hat{\mathbf{x}}a \cos(\omega t - kz) + \hat{\mathbf{y}}a \cos(\omega t - kz - \pi/2)$$

However, now we have the wave propagating in the negative z direction. Refer to Fig. 1 and apply the right-hand rule to see that, when the direction of wave propagation changes from +z to -z, the curl direction of your fingers will also change from counter-clockwise (Fig. 1a) to clockwise (Fig. 1b). One way to affect this change is to transform into a new co-ordinate system $(x', y', z') = (y, x, -z)$. Here the primed co-ordinates represent the new system. Note that (x', y', z') still form a right-hand triplet as required.

The corresponding RHC wave, travelling in the $+z'$ direction (which is the $-z$ direction), is:

$$\begin{aligned} \mathbf{E}(z, t) &= \hat{\mathbf{x}}'a \cos(\omega t - kz') + \hat{\mathbf{y}}'a \cos(\omega t - kz' - \pi/2) \\ &= \hat{\mathbf{y}}a \cos(\omega t + kz) + \hat{\mathbf{x}}a \cos(\omega t + kz - \pi/2) \\ &= \hat{\mathbf{x}}a \sin(\omega t + kz) + \hat{\mathbf{y}}a \cos(\omega t + kz) \end{aligned}$$

Substitute in the values of the parameters to get:

$$\mathbf{E}(z, t) = \hat{\mathbf{x}} 2 \sin(2\pi \times 10^{10}t + 209.4z) + \hat{\mathbf{y}} 2 \cos(2\pi \times 10^{10}t + 209.4z) \text{ V/m}$$

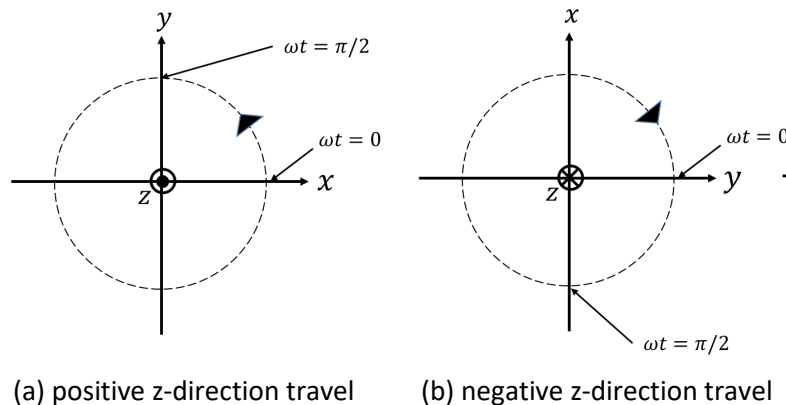


Fig. 1 Locus of $\mathbf{E}$ versus time for a RHC wave

Q2. Sol'n.

7.10 For a wave characterized by the electric field

$$\mathbf{E}(z, t) = \hat{\mathbf{x}}a_x \cos(\omega t - kz) + \hat{\mathbf{y}}a_y \cos(\omega t - kz + \delta)$$

identify the polarization state, determine the polarization angles (γ, χ) , and sketch the locus of $\mathbf{E}(0, t)$ for each of the following cases:

- (a) $a_x = 3 \text{ V/m}$, $a_y = 4 \text{ V/m}$, and $\delta = 0$
- (b) $a_x = 3 \text{ V/m}$, $a_y = 4 \text{ V/m}$, and $\delta = 180^\circ$
- (c) $a_x = 3 \text{ V/m}$, $a_y = 3 \text{ V/m}$, and $\delta = 45^\circ$
- (d) $a_x = 3 \text{ V/m}$, $a_y = 4 \text{ V/m}$, and $\delta = -135^\circ$

Solution:

$$\psi_0 = \tan^{-1}(a_y/a_x), \quad [\text{Eq. (7.60)}],$$

$$\tan 2\gamma = (\tan 2\psi_0) \cos \delta \quad [\text{Eq. (7.59a)}],$$

$$\sin 2\chi = (\sin 2\psi_0) \sin \delta \quad [\text{Eq. (7.59b)}].$$

Case	a_x	a_y	δ	ψ_0	γ	χ	Polarization State
(a)	3	4	0	53.13°	53.13°	0	Linear
(b)	3	4	180°	53.13°	-53.13°	0	Linear
(c)	3	3	45°	45°	45°	22.5°	Left elliptical
(d)	3	4	-135°	53.13°	-56.2°	-21.37°	Right elliptical

- (a) $\mathbf{E}(z, t) = \hat{\mathbf{x}}3 \cos(\omega t - kz) + \hat{\mathbf{y}}4 \cos(\omega t - kz).$
- (b) $\mathbf{E}(z, t) = \hat{\mathbf{x}}3 \cos(\omega t - kz) - \hat{\mathbf{y}}4 \cos(\omega t - kz).$
- (c) $\mathbf{E}(z, t) = \hat{\mathbf{x}}3 \cos(\omega t - kz) + \hat{\mathbf{y}}3 \cos(\omega t - kz + 45^\circ).$
- (d) $\mathbf{E}(z, t) = \hat{\mathbf{x}}3 \cos(\omega t - kz) + \hat{\mathbf{y}}4 \cos(\omega t - kz - 135^\circ).$

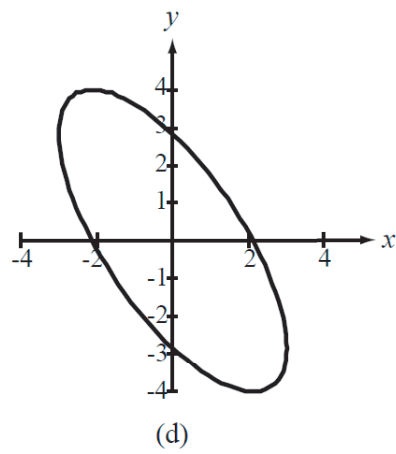
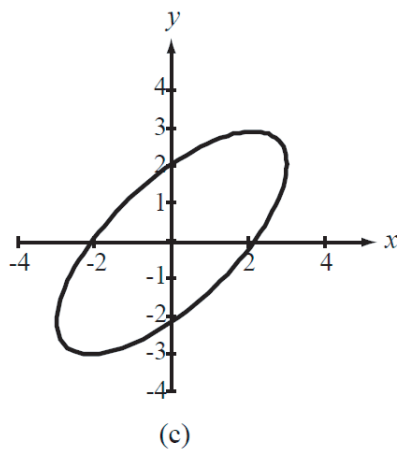
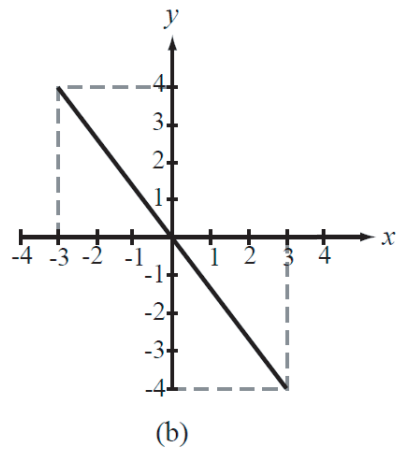
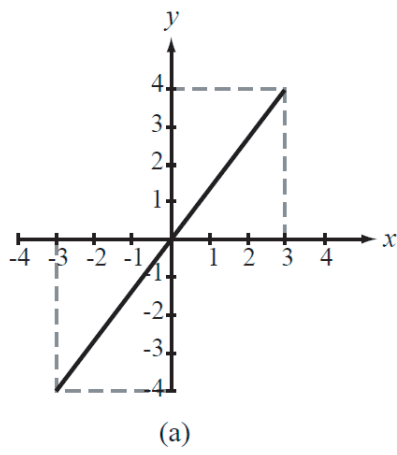


Figure P7.10 Plots of the locus of $\mathbf{E}(0, t)$.

Q3. Sol'n.

7.15 A linearly polarized plane wave of the form $\tilde{\mathbf{E}} = \hat{\mathbf{x}}a_x e^{-jkz}$ can be expressed as the sum of an RHC polarized wave with magnitude a_R , and an LHC polarized wave with magnitude a_L . Prove this statement by finding expressions for a_R and a_L in terms of a_x .

Solution:

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}}a_x e^{-jkz},$$

$$\text{RHC wave: } \tilde{\mathbf{E}}_R = a_R(\hat{\mathbf{x}} + \hat{\mathbf{y}}e^{-j\pi/2})e^{-jkz} = a_R(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-jkz},$$

$$\text{LHC wave: } \tilde{\mathbf{E}}_L = a_L(\hat{\mathbf{x}} + \hat{\mathbf{y}}e^{j\pi/2})e^{-jkz} = a_L(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-jkz},$$

$$\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_R + \tilde{\mathbf{E}}_L,$$

$$\hat{\mathbf{x}}a_x = a_R(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) + a_L(\hat{\mathbf{x}} + j\hat{\mathbf{y}}).$$

By equating real and imaginary parts, $a_x = a_R + a_L$, $0 = -a_R + a_L$, or $a_L = a_x/2$, $a_R = a_x/2$.
