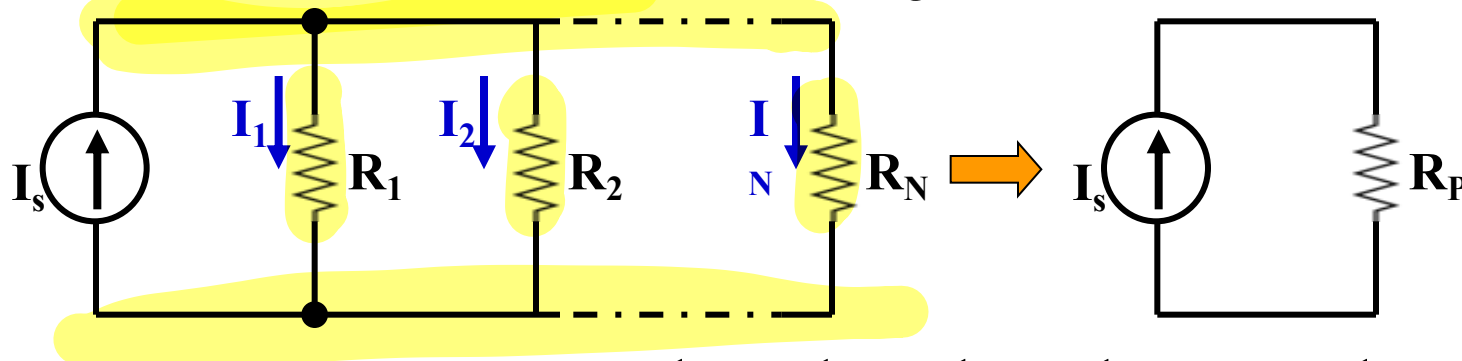


V. Resistive Networks

- Resistors are either arranged in parallel or in series or as combination of both

Parallel Network

Fig 15a: Parallel resistive network



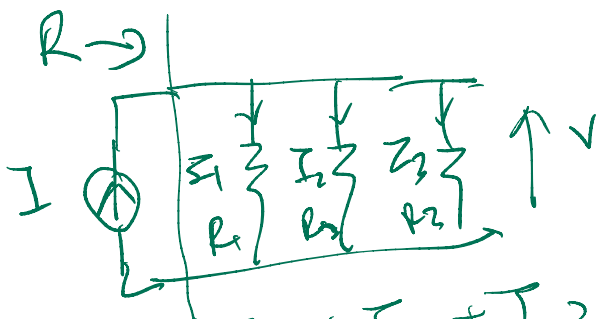
Equivalent Resistance R_P :

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$

Current Divider Rule

$$I_k = \frac{R_P}{R_k} I_s$$

- Current (I_s) from the source will have to be shared between the resistors (R_k) in each branch



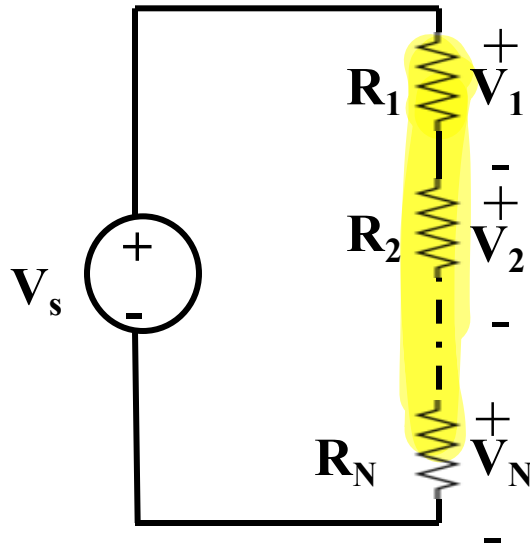
$$I = I_1 + I_2 + I_3 \quad \dots$$

$$\cancel{V} / R = \cancel{V} / R_1 + \cancel{V} / R_2 + \cancel{V} / R_3 \quad \dots$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \dots$$

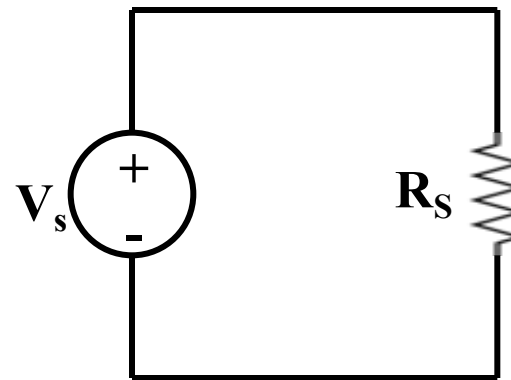
Series network (Highlights)

Series Network



Equivalent Resistance R_p :

Fig 15b: Series resistive network



$$R_s = R_1 + R_2 + \dots + R_N$$

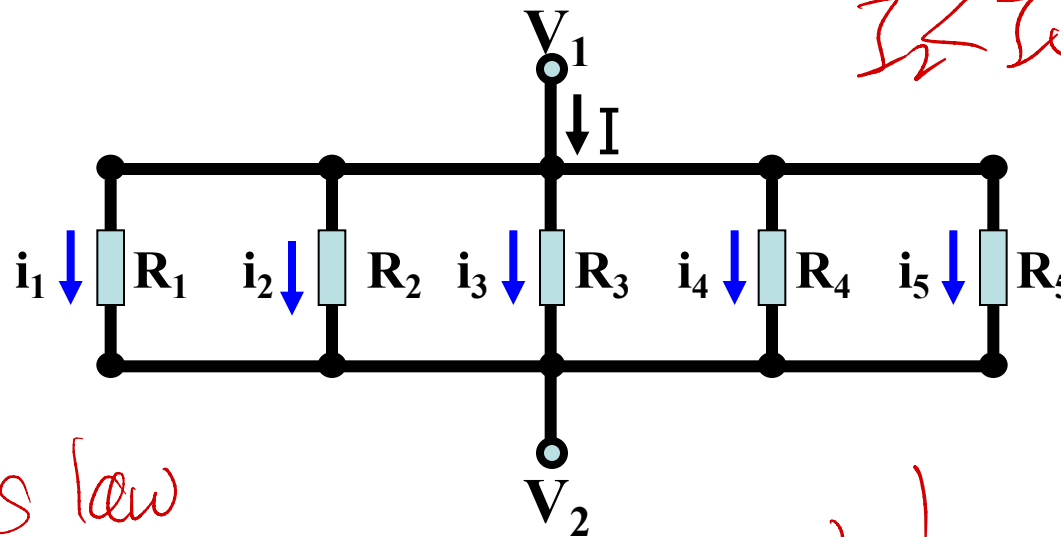
Voltage Divider Rule

$$V_k = \frac{R_k}{R_s} V_s$$

- Total voltage drop across all the resistors (V_s) in series is the sum total of voltage (V_k) drops across each resistor

Worked Example on Current Divider Rule

Rank the currents from largest to smallest if $R_2 > R_4 > R_1 > R_5 > R_3$



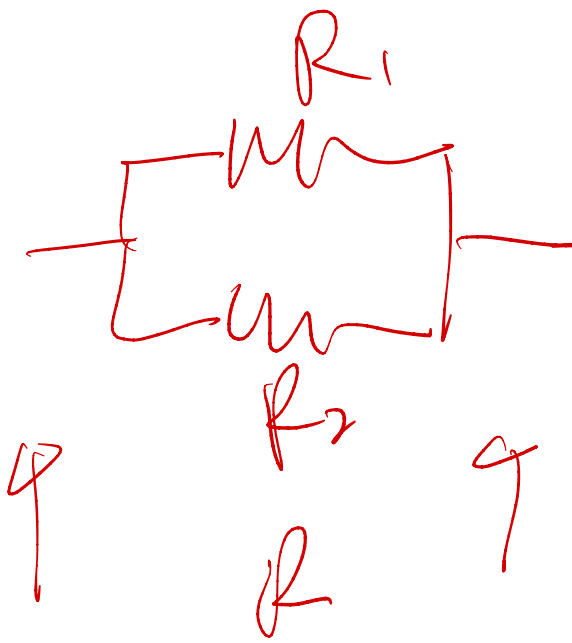
$$I \propto \frac{1}{R_1} \propto \frac{1}{R_2} \propto \frac{1}{R_3} \propto \frac{1}{R_4} \propto \frac{1}{R_5}$$

ohm's law

$$V = IR$$

↑
 $V_1 - V_2 = \text{constant}$

$$I \propto \frac{1}{R}$$



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

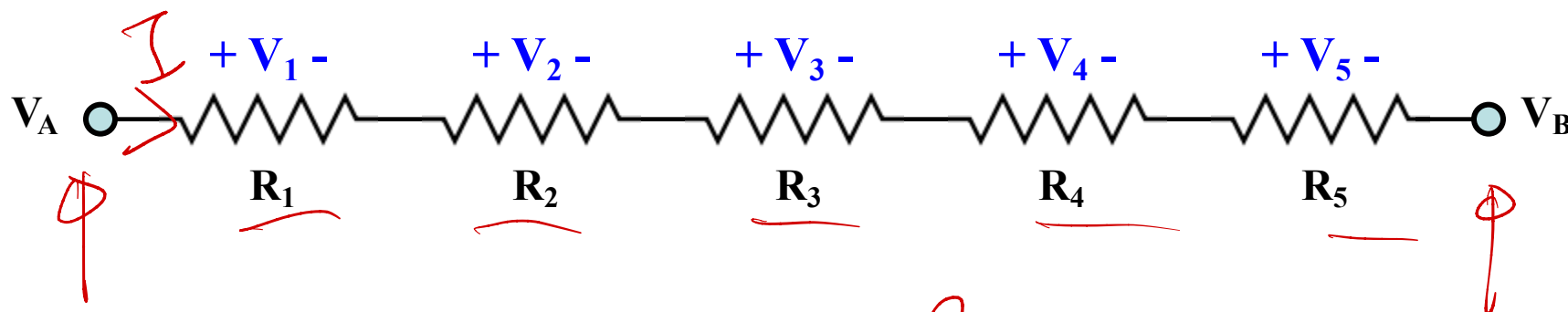
$$\frac{1}{R} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2} \quad \delta$$

Worked Example on Voltage Divider Rule

Rank the potential difference from largest to smallest

if $R_2 > R_4 > R_1 > R_5 > R_3$



$$V = IR$$

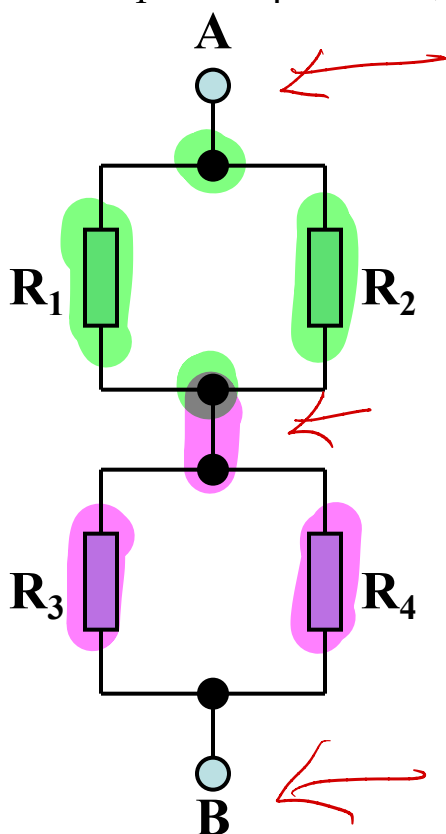
I constant

$$V \propto R$$

Worked Example 1 on Resistive Networks

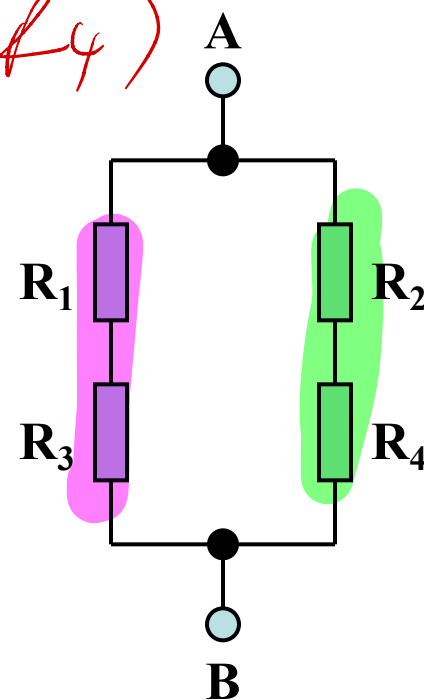
Find the effective resistance across A and B

$$R_1 = R_4 = 1\Omega, R_2 = R_3 = 3\Omega$$



$$R_{ab} = (R_1 // R_2) + (R_3 // R_4)$$

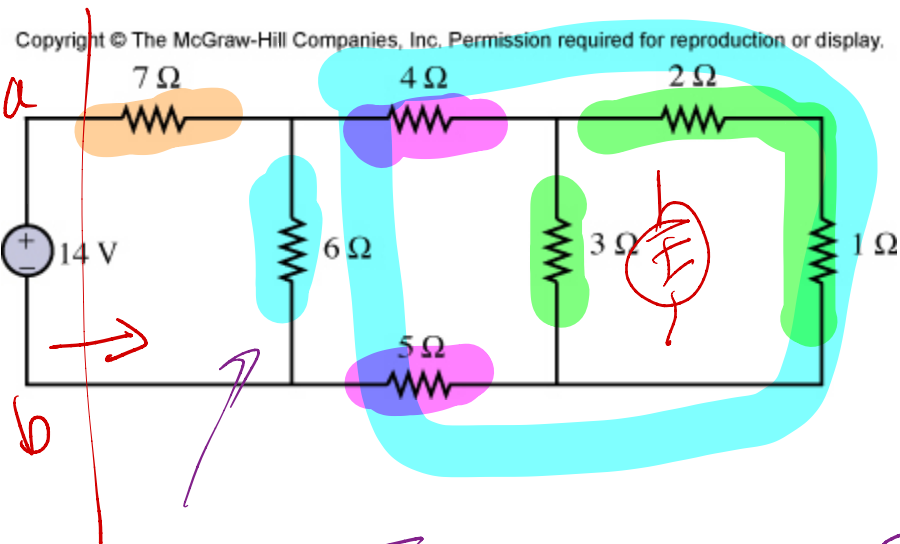
$$R_{ab} = (R_1 + R_3) // (R_2 + R_4)$$



Worked Example 2 on Resistive Networks

What is the equivalent resistance seen by the source?

What is power supplied by the source?

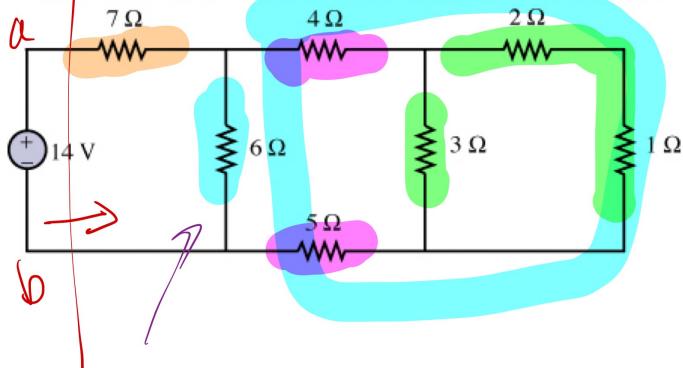


$$R_{ab} = \left\{ \left[3 \parallel (2+1) \right] + 4 + 5 \right\} \parallel 6 + 7$$

$$R_{ab} = 7 + \left[6 \parallel (4 + 5 + \underbrace{3 \parallel (2+1)}) \right]$$

$\downarrow 1.5$ $\downarrow 3$
 10.5 5.5

$$7 + \frac{6 \times 10.5}{6 + 10.5}$$



Handwritten notes below the circuit diagram:

A red box represents the equivalent circuit, containing a 14 V source and a resistor labeled R_{eq} . An arrow points from the text R_{eq} in the circuit diagram to this box.

$$P = \frac{V^2}{R_{eq}}$$

VI. Measuring Instruments - Voltmeter

- Voltmeter measures voltage across a circuit element



Fig 16: Image of typical voltage with a measuring range up to 30V

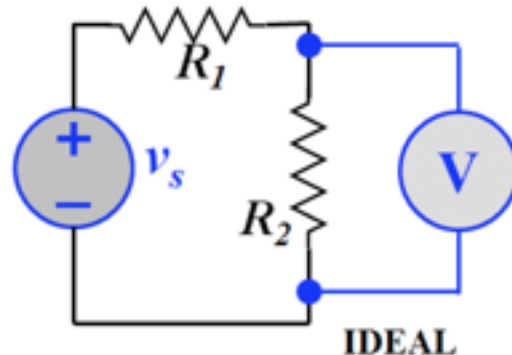


Fig 17a: Placement of an ideal voltmeter to measure the PD across R_2

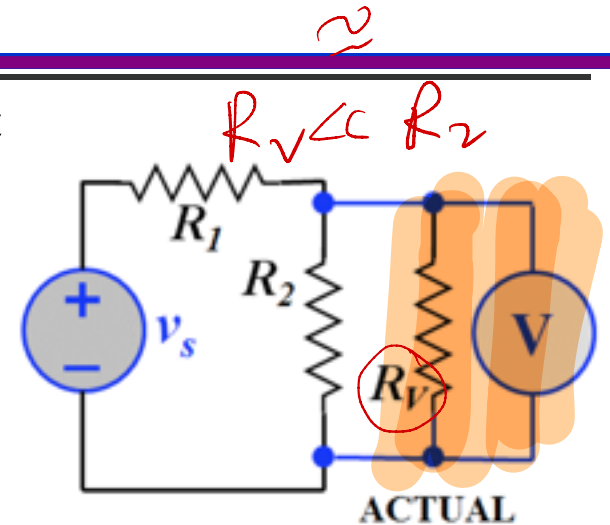


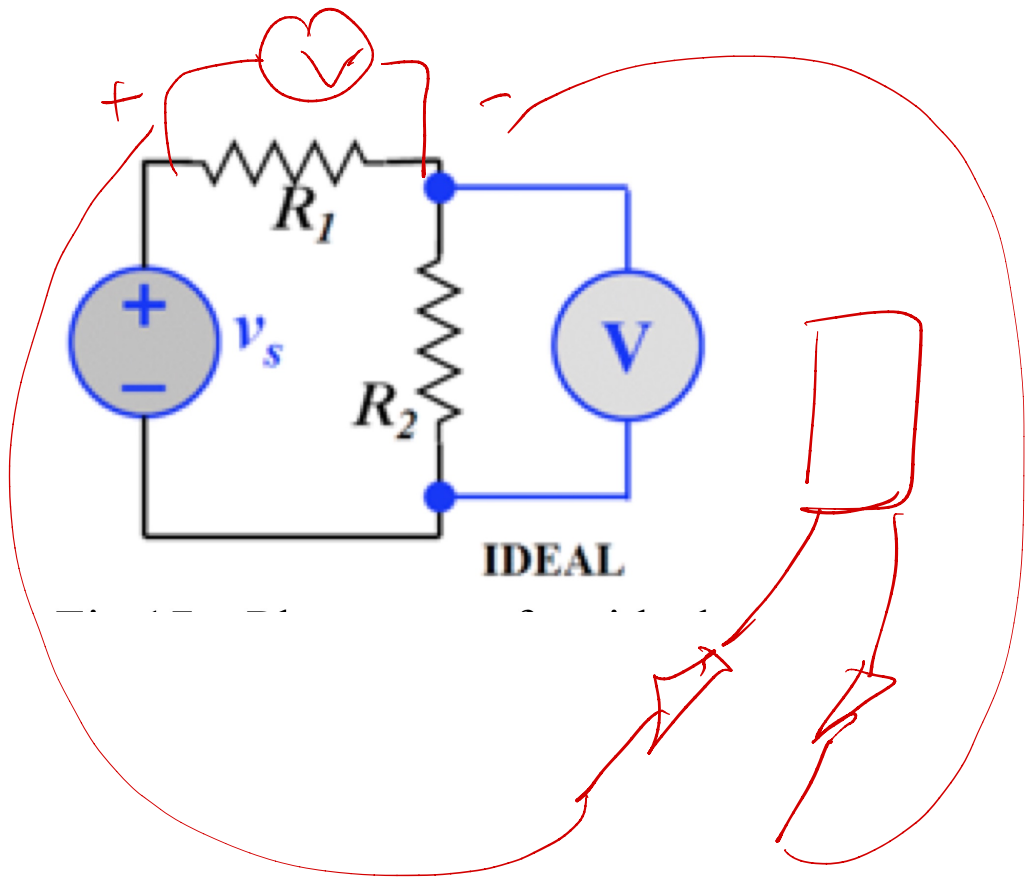
Fig 17b: Circuit model of how a real voltmeter behaves, which looks like a large parallel resistor (R_V)

How would you place the voltmeter in the circuits?

- Connected in **parallel** with the element being measured

In Fig 17a, voltmeter has no effect on original circuit.

In Fig 17b, voltmeter will steal current from R_2 . This will change the voltage measured across R_2 . Thus, the PD measured with a voltmeter is lower than the true value.



Worked Example on Voltmeters

Referring to Fig 17b, if $V_s = 5V$, and $R_v = 1M\Omega$, find the voltage across R_2 when

(i) $R_1 = R_2 = 1k\Omega$ $V = 2.5V$

(ii) $R_1 = R_2 = 1M\Omega$ $V = 1.67V$

$$V = V_s \frac{R_2 || R_v}{R_1 + R_2 || R_v}$$

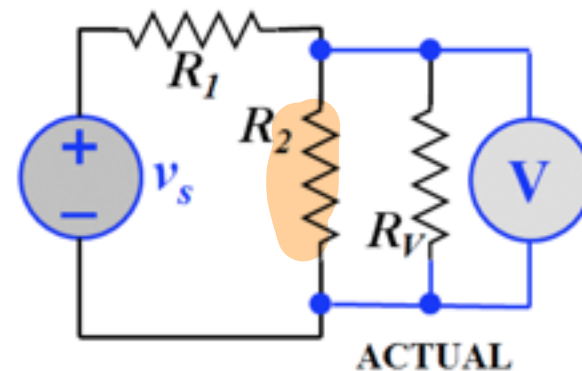


Fig 17b

General Solution

$$V_{measured} = v_s \frac{R_2 || R_v}{R_1 + R_2 || R_v} = v_s \frac{\frac{R_2 R_v}{R_2 + R_v}}{R_1 + \frac{R_2 R_v}{R_2 + R_v}} = v_s \frac{R_2 R_v}{R_1 R_2 + (R_1 + R_2) R_v} = \frac{v_s R_2}{(R_1 + R_2) + \frac{R_1 R_2}{R_v}}$$

- Ideal case for voltmeter: $R_v \rightarrow \infty$, $V_{ideal} = v_s \frac{R_2}{R_1 + R_2}$
- Actual case of voltmeter: measured voltage deviates more if resistance of other resistor(s) in the circuit is higher and more comparable to R_v

Ammeter

- Ammeter measures current



Fig 18: Image of typical current with a measuring range up to 20A

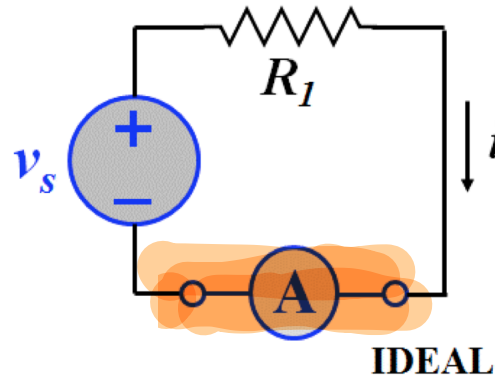


Fig 19a: Placement of an ideal ammeter to measure current through R_1

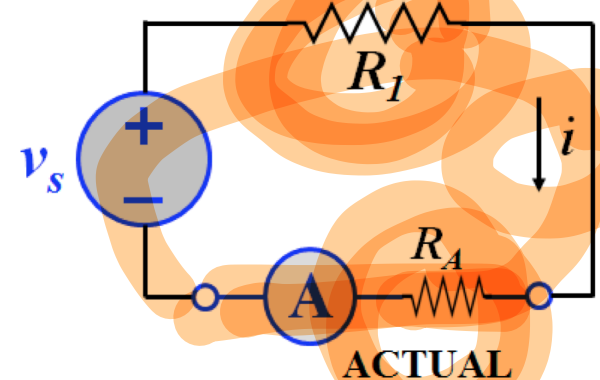


Fig 19b: Circuit model of how a real ammeter behaves, which looks like a small series resistor (R_A)

How would you place the ammeter in the circuits?

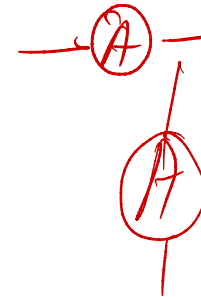
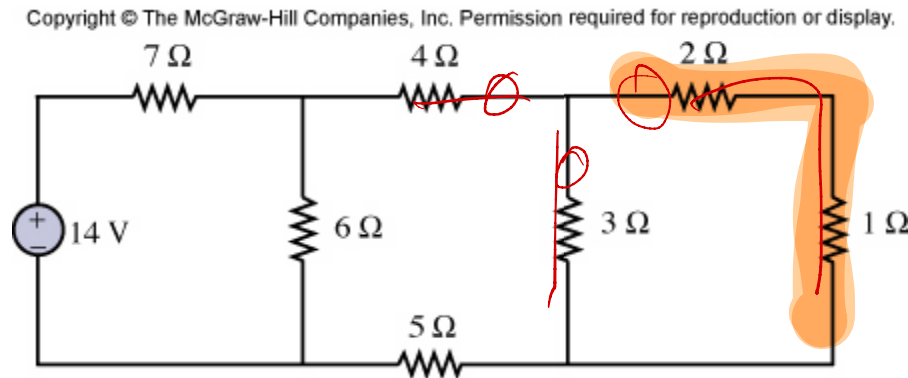
- Connected in **series** with the element being measured

In Fig 19a, ammeter has no effect on original circuit.

In Fig 19b, ammeter contains some resistance (R_A), adding to the total resistance, lowering the measured current. **Thus, the current measured with an ammeter is lower than the true value.**

Ammeter in Circuit

How would you place the ammeter in the circuits of the previous worked examples?



Ammeter Example

Referring to Fig 19b, if $V_s = 5V$, and $R_A = 1\Omega$,
find the loop current when

- (i) $R_1 = 1k\Omega$ $i = 4.995 \text{ mA}$
(ii) $R_1 = 2\Omega$ $i = 1.67A$

General Solution

$$i_{measured} = \frac{v_s}{R_1 + R_A}$$

- Ideal case for ammeter: $R_A \rightarrow 0$, $i_{ideal} = \frac{v_s}{R_1}$
- Actual case of ammeter: measured current deviates more if resistance of other resistor in the circuit is smaller and more comparable to R_A

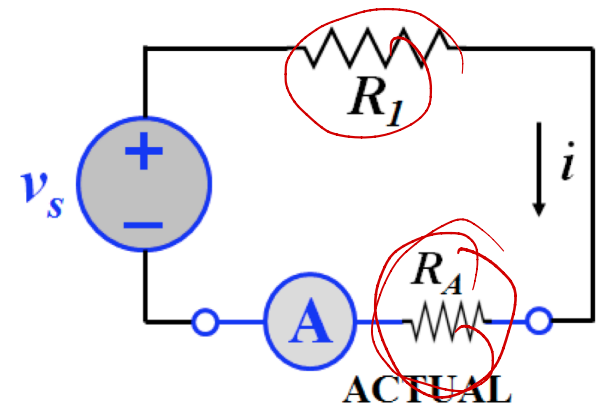


Fig 19b

1.2 Circuit Analysis Methods

- It is important to apply the 3 basic laws in circuit theory to analyze and design circuits
- This unit is organized into the following 3 sections
 - > Nodal voltage analysis (application of KCL and Ohm's law)
 - > Mesh current analysis (application of KVL and Ohm's law)
 - > Superposition

Alexander & Sadiku, "Fundamentals of Electric Circuits" 5th Edition Ch 3, 4.3

I. Nodal Voltage Analysis (NVA)

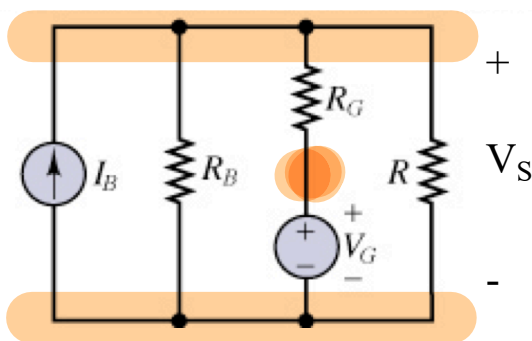
KCL & Ohm's law in action

- The method of nodal voltage analysis is an application of KCL and Ohm's law together
- The unknown variables that you will solve for are **node voltages**
- We will apply KCL at a node and express the unknown currents as unknown node voltages

Example on how to do NVA

Find the voltage across the current source, V_S :

Given: $I_B = 12\text{A}$, $V_G = 12\text{V}$, $R_G = 0.3\Omega$, $R_B = 1\Omega$, $R = 0.23\Omega$



Apply KCL at V_S :

$$I_B = \frac{V_S}{R_B} + \frac{V_S - V_G}{R_G} + \frac{V_S}{R}$$

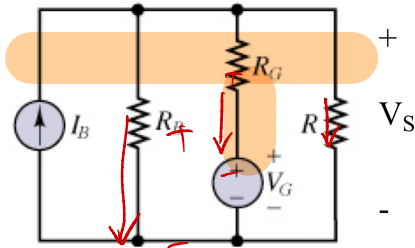
$$12 = \frac{V_S}{1} + \frac{V_S - 12}{0.3} + \frac{V_S}{0.23}$$

$$V_S = 5.99\text{V}$$

- Note that the negative terminal of V_G is used as a reference (0V) in the example
- The value of V_S is also referenced to this node
- The value of the reference node is not important since we are only interested in the voltage difference

Find the voltage across the current source, V_S :

Given: $I_B = 12A$, $V_G = 12V$, $R_G = 0.3\Omega$, $R_B = 1\Omega$, $R = 0.23\Omega$



$$I_B = \frac{V_S}{R_B} + \frac{V_S - V_G}{R_G} + \frac{V_S}{R}$$

$$\frac{1}{x} = 0V \quad V_S - V_G$$

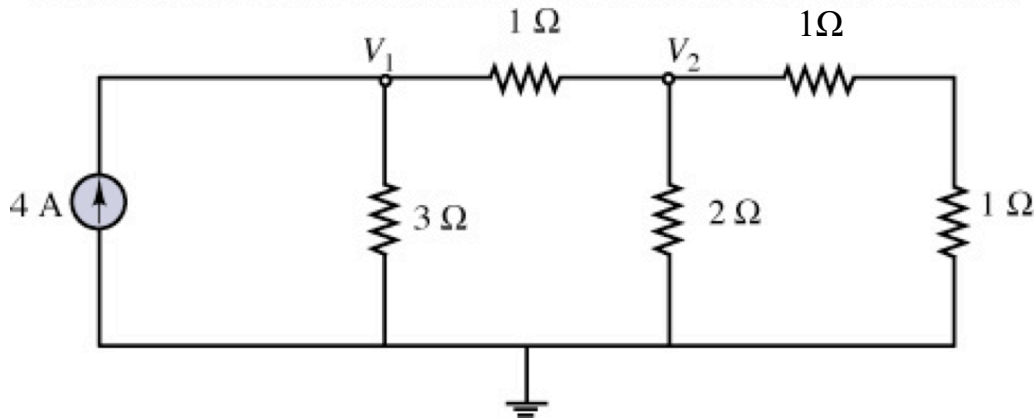
$$\frac{R_B}{R_B} = 1 \quad V_S \quad \frac{V_S - V_G}{R_G} \quad V_G$$

0
3

Worked Example on NVA 1

Use nodal voltage analysis to find V_1 and V_2 . (Answer: $V_1 = 4.8\text{V}$, $V_2 = 2.4\text{V}$)

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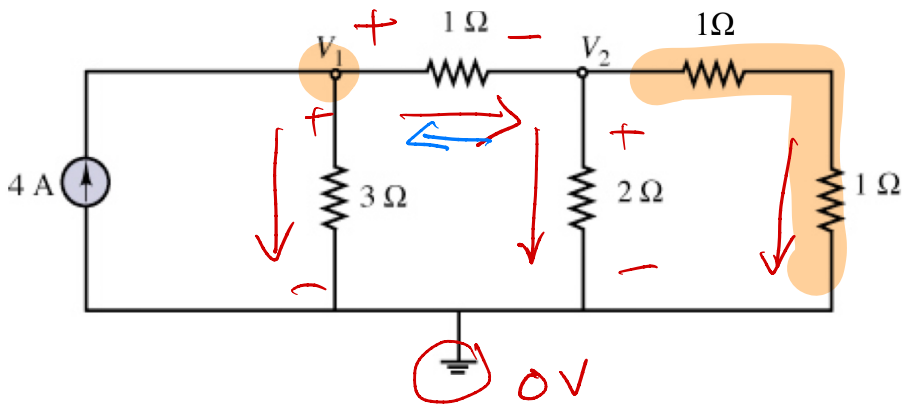


First method:

- Apply KCL at V_1 (1)
- Apply KCL at V_2 (2)
- Solve equations (1) and (2)

Alternate method:

- Combine all the resistors to find V_1 first
- Then combine the 3 resistors on the right followed by voltage divider rule to find V_2



node V_1

$$4 = \frac{V_1}{3} + \frac{V_1 - V_2}{1} \quad (1)$$

node V_2

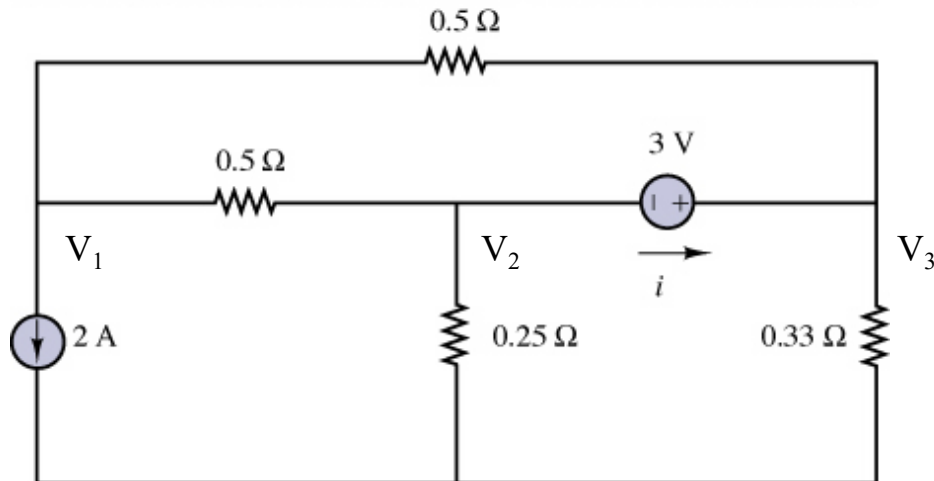
$$\frac{V_1 - V_2}{3} = \frac{V_2}{2} + \frac{V_2}{1+1} \quad (2)$$

Worked Example on NVA 2

Use nodal voltage analysis to find the current through the 3V source.

(Answer: $i = 8.31 \text{ A}$)

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Apply KCL at V_1 :

$$2 - \frac{V_2 - V_1}{0.5} - \frac{V_3 - V_1}{0.5} = 0$$

$$V_2 + V_3 - 2V_1 = 1 \quad (1)$$

Apply KCL at V_2 :

$$\frac{V_1 - V_2}{0.5} - \frac{V_2}{0.25} - i = 0$$

$$2V_1 - 6V_2 = i \quad (2)$$

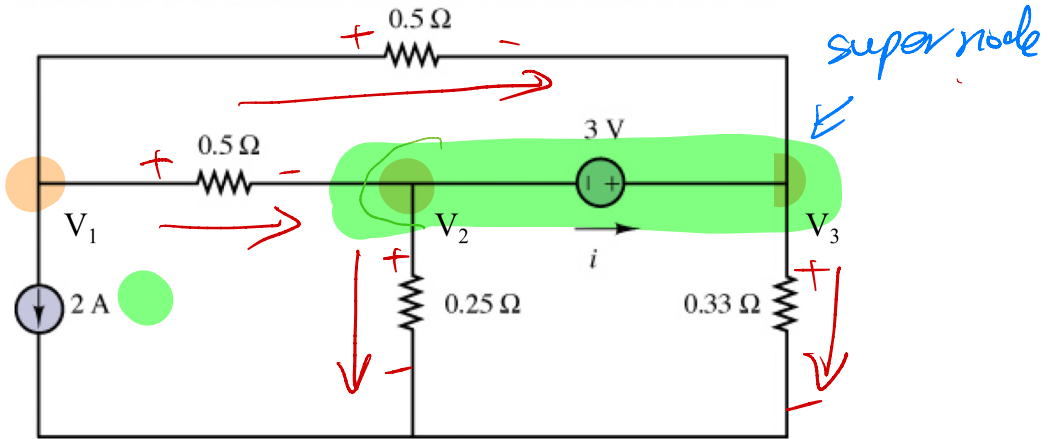
Apply KCL at V_3 :

$$\frac{V_3}{0.33} + \frac{V_3 - V_1}{0.5} = i$$

$$\frac{166}{33}V_3 - 2V_1 = i \quad (3)$$

Since a new variable is introduced in (2), we need one more equation.

$$V_3 - V_2 = 3 \quad (4)$$



node 1

$$0 = 2 + \frac{V_1 - V_2}{0.5} + \frac{V_1 - V_3}{0.5} \quad \text{--- (1)}$$

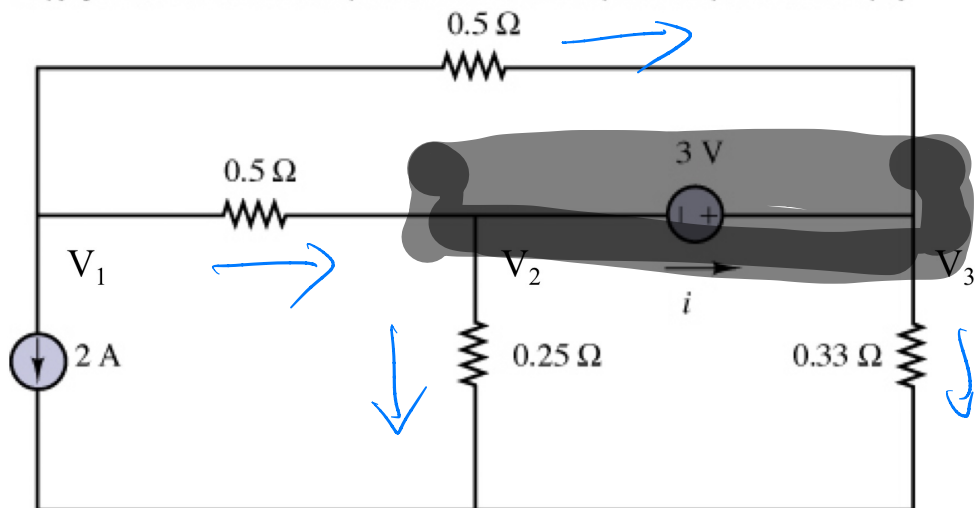
$$\frac{V_1 - V_2}{0.5} + \frac{V_1 - V_3}{0.5} = \frac{V_2}{0.25} + \frac{V_3}{0.33} \quad \text{--- (2)}$$

$$V_3 - V_2 = 3V \quad \text{--- (3)}$$



$$V_a > V_b$$

$$V_{ab} = V_a - V_b$$



Handwritten notes for nodal analysis:

Node V_1 (top node):

$$\sum \hat{i}_i = 0$$

Node V_2 (middle node):

$$\sum \hat{i}_i = 0$$

Node V_3 (bottom node):

$$\sum \hat{i}_i = 0$$

